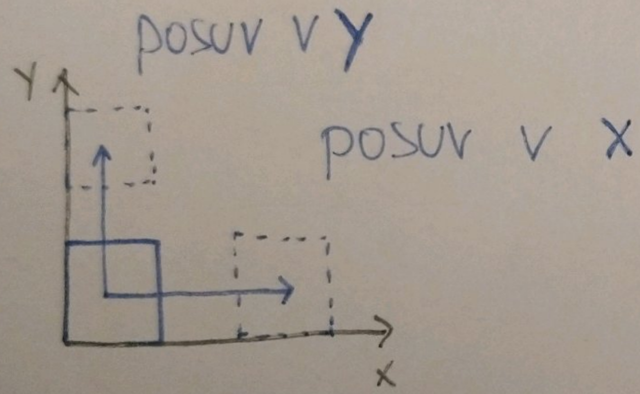
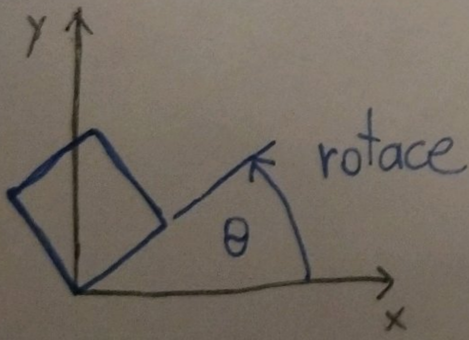
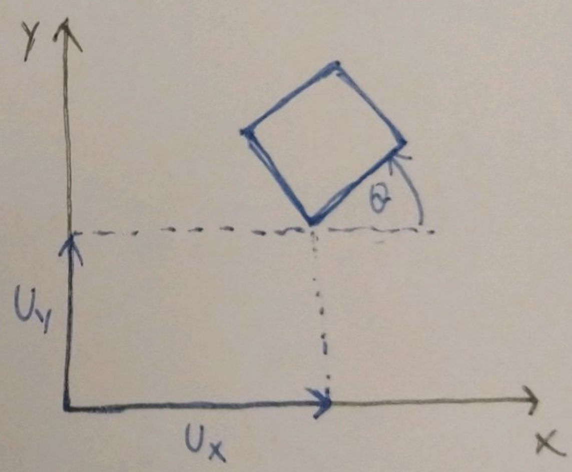
Stupně volnosti tělesa ve 2D

2x posuv + rotace





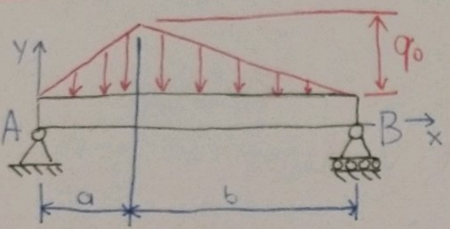


4. cvičení

MCH*Z

Příklad 1a

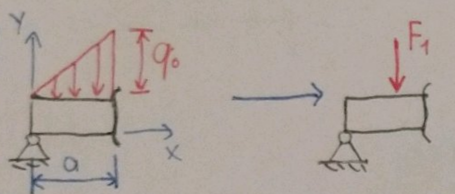
– určit momentové účinky vzhledem k levé podpěře (nahraďte působení spojitého zatížení q_0 jedinou silou a určete vzdálenost od A)



$a = 2\ m,\ b = 4\ m,\ q_0 = 750\ N/m$

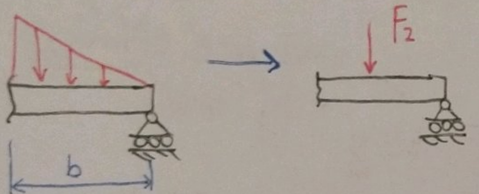
// velikost trojúhelníku (plocha pod křivkou)

1)



$$F_1 = \frac{q_0 a}{2} = \frac{750 \cdot 2}{2} = 750\ N \qquad x_{F_1} = \frac{2}{3} a = 1{,}33\ m$$

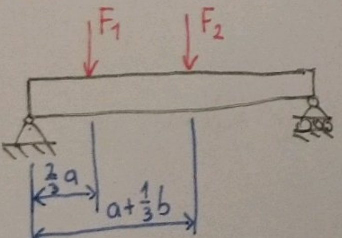
2)



$$F_2 = \frac{q_0 b}{2} = \frac{750 \cdot 4}{2} = 1500\ N \qquad x_{F_2} = a + \frac{1}{3} b = 3{,}33\ m$$

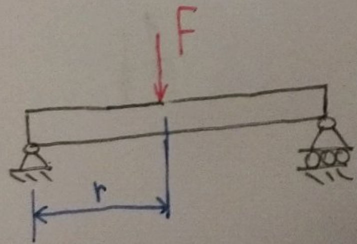
$$F = F_1 + F_2 = \underline{\underline{2250\ N}}$$

momentové účinky



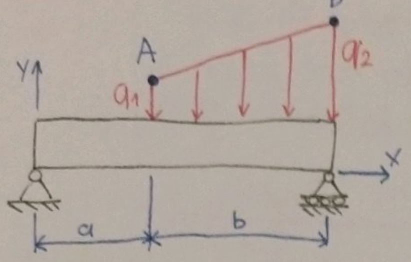
$$M = F_1 x_{F_1} + F_2 x_{F_2} = 6000\ Nm$$

$$M = F r \Rightarrow r = \frac{M}{F} = \frac{6000}{2250} = \underline{\underline{2{,}67\ m}}$$



Příklad 1b

– nahraďte spojité zatížení ekvivalentní silou (k levé podpoře)



$a = 2\,m$, $b = 4\,m$, $q_1 = 2{,}5\ kN/m$, $q_2 = 5\ kN/m$

rovnice přímky

$$q(x) = c_1 x + c_2$$

$A[a, q_1]$, $B[a+b, q_2]$

$$q_1 = c_1 a + c_2 \quad (1)$$

$$q_2 = c_1(a+b) + c_2 \quad (2)$$

$$(1)-(2):\ q_1 - q_2 = c_1 a - c_1(a+b)$$

$$= c_1(a - a - b)$$

$$= -c_1 b$$

směrnice přímky
~ tg (úhlu)

$$c_1 = \frac{q_2 - q_1}{b} = 625\ \frac{N}{m}$$

$$c_2 = q_1 - c_1 a = 2500 - 625 \cdot 2 = 1250\ N$$

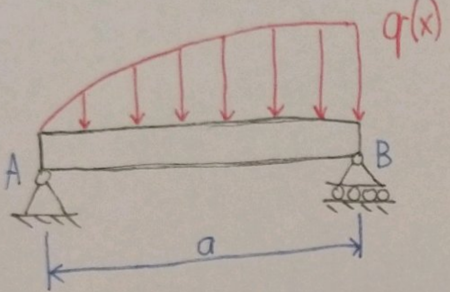
$$F = \int_a^{a+b} q(x)\,dx = \int_a^{a+b} (c_1 x + c_2)\,dx = \left[k\frac{x^2}{2} + q x\right]_a^{a+b} = \frac{k}{2}(a+b)^2 + q(a+b) - \frac{k}{2}a - q a = 14{,}5\ kN$$

$$M = \int_a^{a+b} q(x)\,x\,dx = \int_a^{a+b} c_1 x^2 + c_2 x\ dx = \left[c_1\frac{x^3}{3} + c_2\frac{x^2}{2}\right]_a^{a+b} = -58\ kNm$$

$$r = \frac{M}{F} = 4\,m$$

Příklad 1d

$a = 16\ \mathrm{m}\ ,\ q(x) = 200\sqrt{x}\ \mathrm{N m^{-1}}$



$$F = \int_0^a q(x)\,dx = \int_0^a 200\,x^{\frac{1}{2}}\,dx = \left[200\,\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^a =$$

$$= \left[\frac{400}{3}\,x^{\frac{3}{2}}\right]_0^a = \frac{400}{3}\sqrt{a^3} = \frac{400}{3}\sqrt{16^3} = \underline{\underline{8533\ \mathrm{N}}}$$

$$x_T = \frac{\int_a^b x\,q(x)\,dx}{\int_a^b q(x)\,dx} = \frac{81920}{8533} = \underline{\underline{9{,}6\ \mathrm{m}}}$$

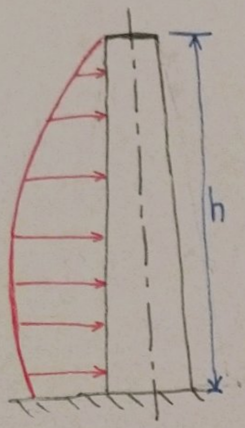
$$\int_0^a x\,200\,x^{\frac{1}{2}}\,dx = \int_0^a 200\,x^{\frac{3}{2}}\,dx = \left[200\,\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^a =$$

$$= \left[\frac{400}{5}\sqrt{x^5}\right]_0^a = \frac{400}{5}\sqrt{16^5} = \underline{\underline{81920\ \mathrm{Nm}}}$$

$$M_A = F\,x_T = 8533 \cdot 9{,}6 = \underline{\underline{81{,}9\ \mathrm{kNm}}}$$

Příklad 2 - Komín $h = 30\,m$ fouká vítr, $q(y) = 5000 \cos\left(\frac{\pi y}{60}\right)$.

U: F, M v místě základů



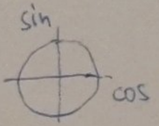
$$F = \int_0^h q(y)\,dy = \int_0^h 5000 \cos\left(\frac{\pi y}{60}\right) dy = \left| \begin{array}{l} u = \frac{\pi y}{60} \\ du = \frac{\pi}{60} dy \\ dy = \frac{60}{\pi} du \end{array} \right| =$$

$$= 5000 \int_0^{\frac{\pi h}{60}} \cos(u) \frac{60}{\pi} du = 5000 \frac{60}{\pi} \left[\sin(u)\right]_0^{\frac{\pi h}{60}} =$$

$$= 5000 \frac{60}{\pi} \Big(\sin\left(\frac{\pi h}{60}\right) - \underbrace{\sin(0)}_{0} \Big) = 5000 \frac{60}{\pi} \sin\left(\frac{\pi 30}{60}\right) = \underline{\underline{95492\ N}}$$

$$M = \int_0^h y\, q(y)\,dy = \int_0^h 5000\, y \cos\left(\frac{\pi y}{60}\right) dy = \left| \begin{array}{l} u = \frac{\pi y}{60} \quad \dots\ y = \frac{60}{\pi} u \\ du = \frac{\pi}{60} dy \\ dy = \frac{60}{\pi} du \end{array} \right| =$$

$$= \int_0^{\frac{\pi h}{60}} 5000 \frac{60}{\pi} u \cos(u) \frac{60}{\pi} du = 5000 \left(\frac{60}{\pi}\right)^2 \int_0^{\frac{\pi h}{60}} u \cos(u)\,du =$$



$$= 5000 \left(\frac{60}{\pi}\right)^2 \left[u \sin(u) + \cos(u)\right]_0^{\frac{\pi h}{60}} =$$

$$= 5000 \left(\frac{60}{\pi}\right)^2 \Big[\underbrace{\frac{\pi h}{60}}_{\frac{\pi}{2}} \underbrace{\sin\left(\frac{\pi h}{60}\right)}_{1} + \underbrace{\cos\left(\frac{\pi h}{60}\right)}_{0} - \underbrace{\cos(0)}_{1} \Big] = \underline{\underline{1041\ kNm}}$$

$$\int_0^a u \cos(u)\,du = \left[u \sin(u)\right]_0^a - \int_0^a \sin(u)\,du =$$

$$= \left[u \sin(u)\right]_0^a - \left[-\cos(u)\right]_0^a = \left[u \sin(u) + \cos(u)\right]_0^a$$

$$\int u v' = [uv] - \int u' v$$