Waves

Wave equation, interference, standing wave, propagation of waves in space. Energy and wave intensity, acoustics.

Waves

- Mechanic (sound, ultrasound, noise, vibration of machines, earthquakes, tsunami wave,…)
- Elektromagnetic (TV and radio transmission, microwaves, light)
- Gravitation
- Quantum particles and their wave functions

Oscillations and waves

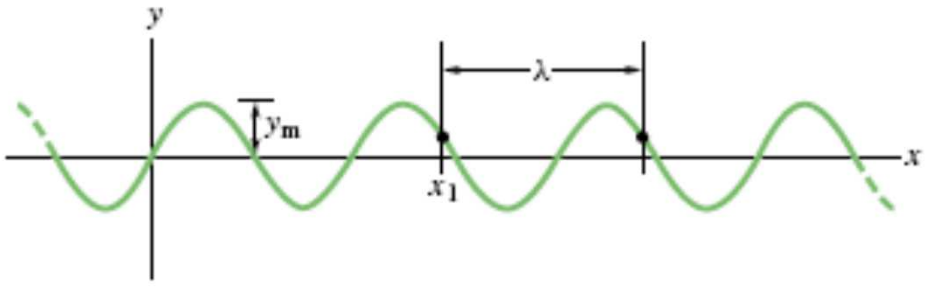
Wave is common oscillating motion of space elements

Every point oscillates, but with variable phase in space, superposition of these oscillations is wave

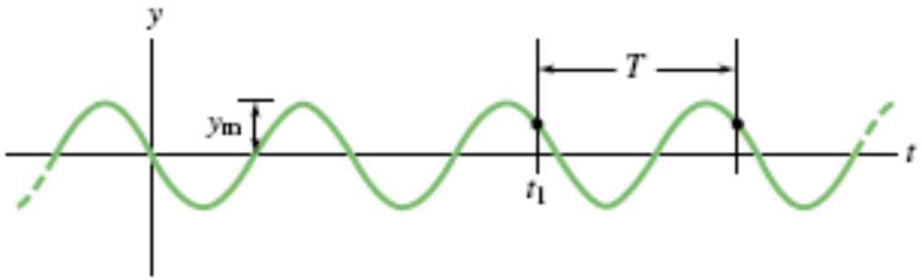
Points oscillate “on site” without any displacement together with propagated wave

Wave – parameters

Amplitude, phase, wave vector, wavelength, angular frequency, frequency, period



(a)



(b)

$$y(r,t) = y_m \sin(\omega t - k \cdot r)$$

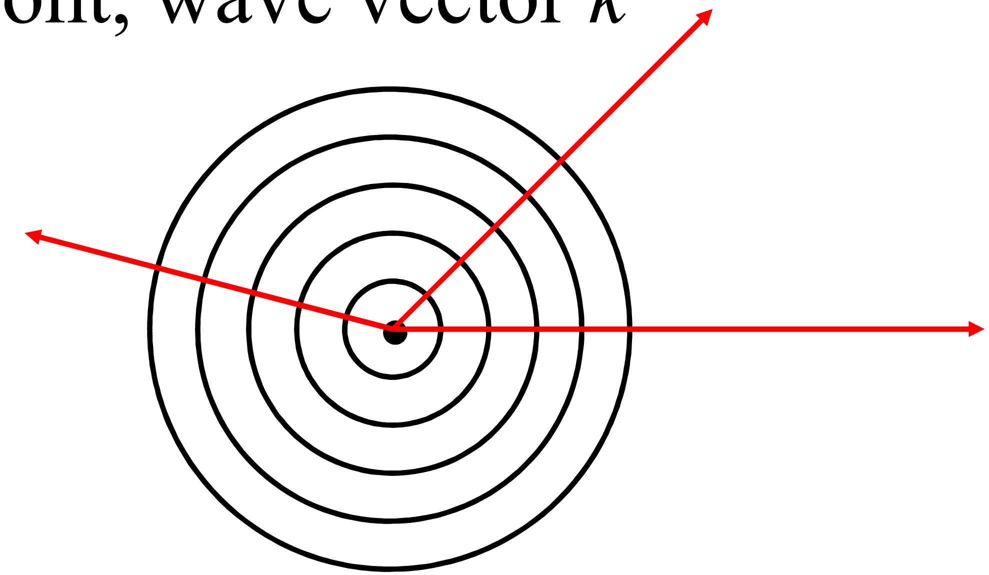
$$k = \frac{2\pi}{\lambda}, k = \frac{\omega}{c}$$

$$\lambda = cT$$

Wavefront, ray

Wavefront travels by phase velocity *c*

Ray – direction of wave, perpendicular to wavefront, wave vector *k*



Wavefront

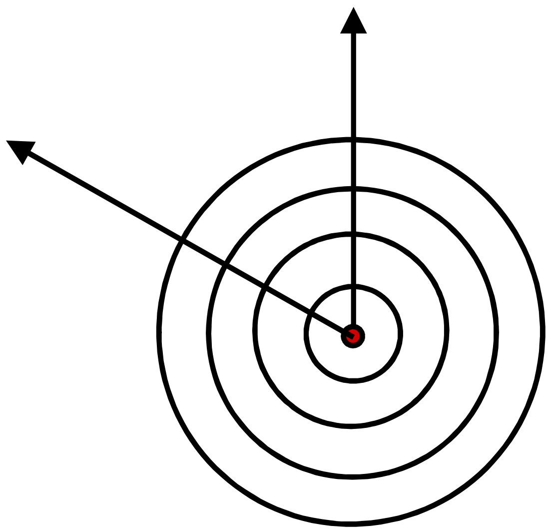
- Phase, points with equal phase = wavefront

$$\varphi = \omega t - kx$$

- Phase velocity

$$\lambda = cT = \frac{c}{f}$$

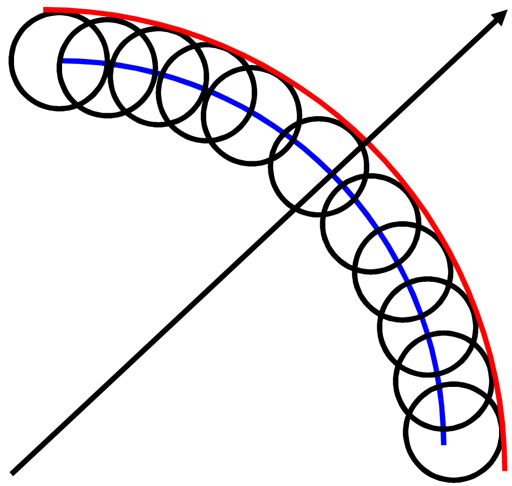
$$\omega = \frac{2\pi}{T} = 2\pi f$$



Wavefronts are spherical planes in an isotropic space, they are almost planar far from wave source

Huyghens' principle

Every point on the wavefront is a new source of waves. Resulting wavefront is built by superposition of all these incremental waves.



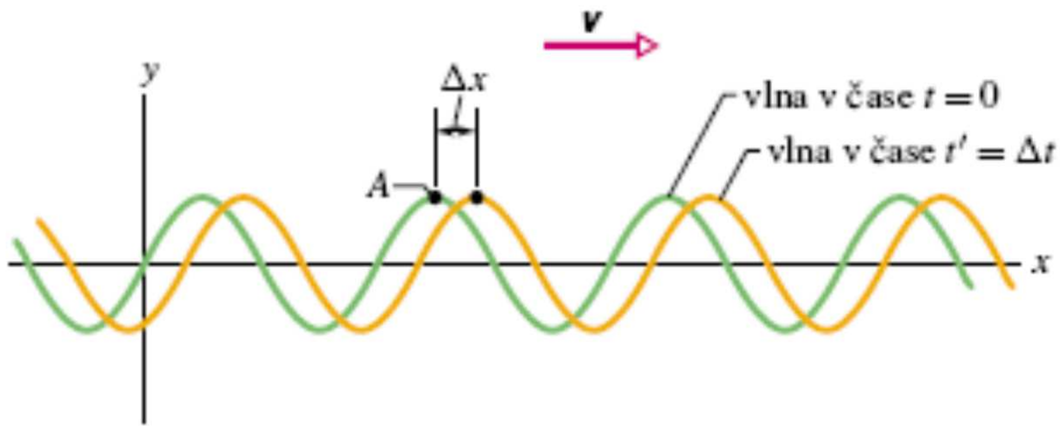
Phase velocity

Wavefront position in space is given by

$$\omega t - kx = konst.$$

Derivative by time

$$c = \frac{dx}{dt} = \frac{\omega}{k}$$



Wave polarization

- Longitudinal (e.g. sound in liquids and gasses), displacements of points are in the same direction as the wave propagation direction

- Transversal (e.g. waves on water surface, ultrasound in solid state, etc.), displacements of points are perpendicular to the wave propagation direction

Waves on string

- Deflection of string (transversal wave)
- Phase velocity on string

$$v = \sqrt{\frac{F}{\mu}}$$

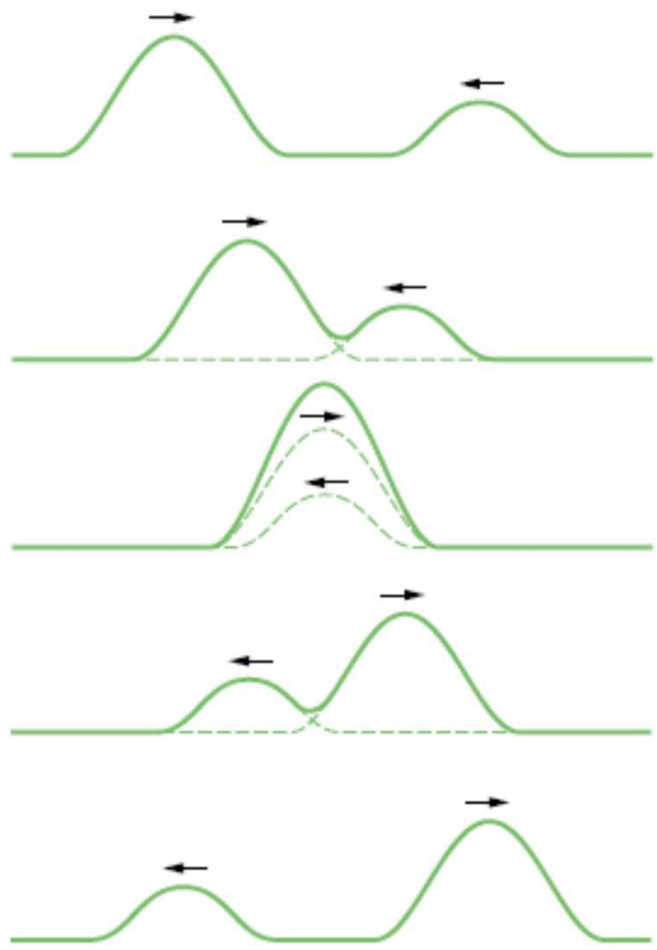
F … force in string [N]

μ ... linear mass density of string [kg m^{-1}]

Superposition of waves

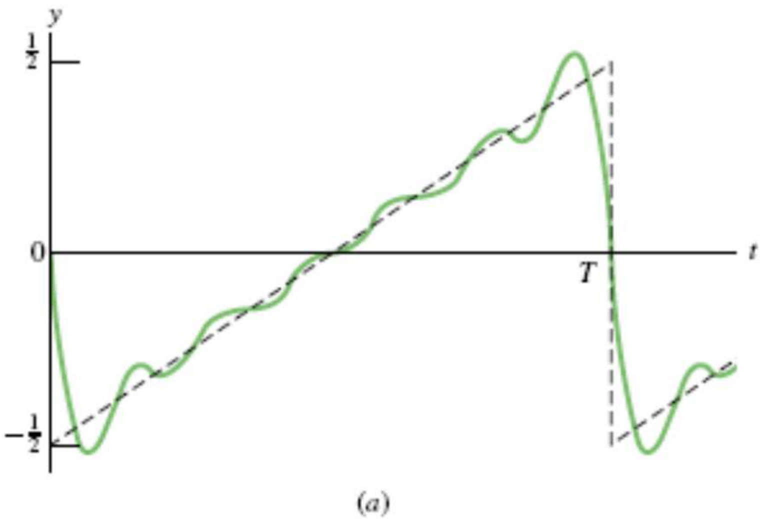
Superposition principle
= sum of displacements
from both waves

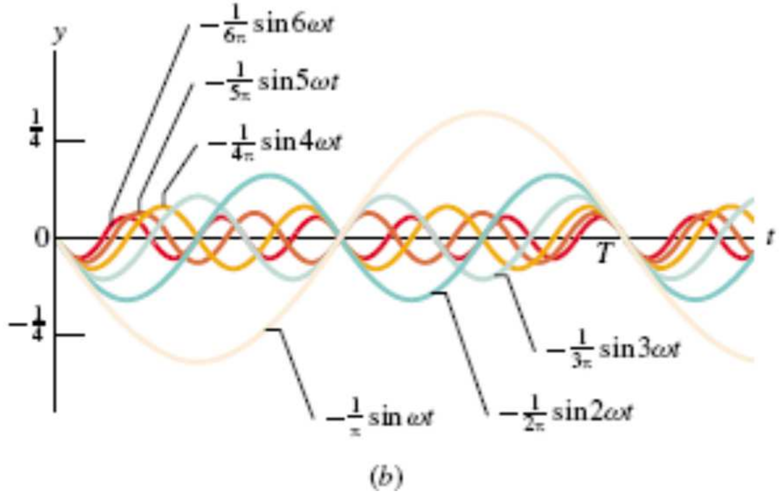
$$y(x,t) = y_1(x,t) + y_2(x,t)$$



Fourier analysis

Every wave could be superposed from the harmonic (sinusoidal) waves of different amplitude





Interference
– waves of the same frequency

Superposition of two waves, same frequency

$$y_1 = y_m \sin(\omega t - kx)$$

$$y_2 = y_m \sin(\omega t - kx + \varphi)$$

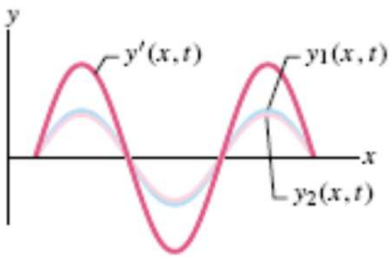
Superposed wave

$$y = y_1 + y_2 = 2y_m \cos(\frac{1}{2}\varphi)\sin(\omega t - kx + \frac{1}{2}\varphi)$$

Amplitude depends on phase difference

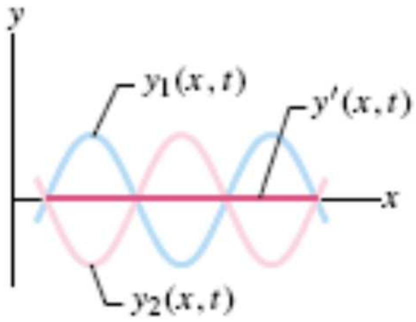
Amplitude of superposed wave

Constructive interference



(a)

Destructive interference



(b)

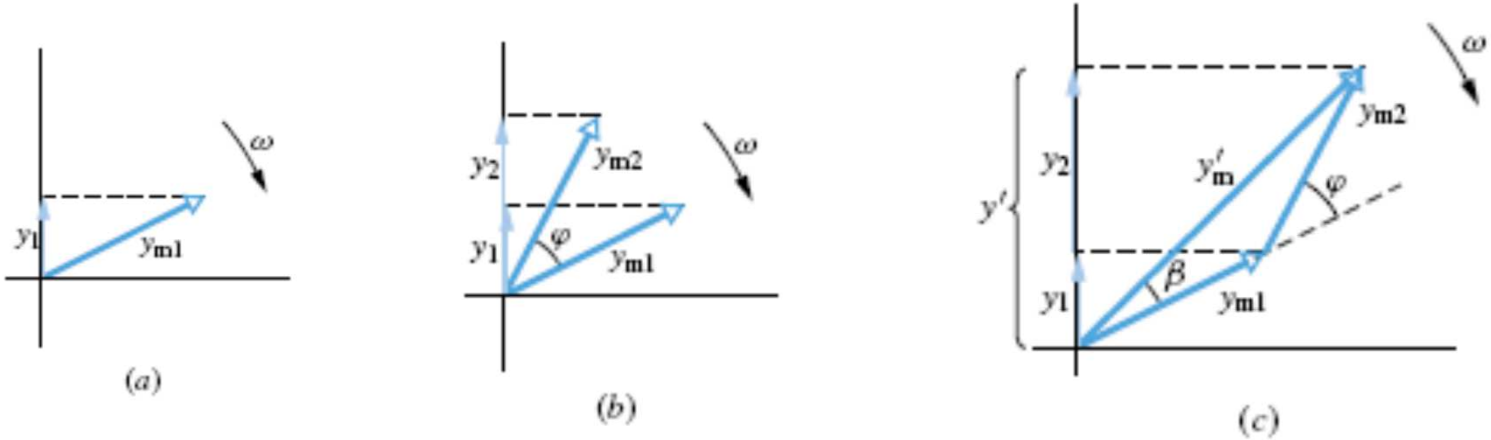
Tabulka 17.1 Fázové rozdíly a jim odpovídající druh interference[a]

FÁZOVÝ ROZDÍL VE STUPNÍCH	FÁZOVÝ ROZDÍL V RADIÁNECH	DRÁHOVÝ ROZDÍL VE VLN. DÉLKÁCH	AMPLITUDA VÝSLEDNÉ VLNY	DRUH INTERFERENCE
0	0	0	$2y_m$	úplně konstruktivní
120	$2\pi/3$	0,33	y_m	částečná
180	π	0,50	0	úplně destruktivní
240	$4\pi/3$	0,67	y_m	částečná
360	2π	1,00	$2y_m$	úplně konstruktivní
865	15,1	2,40	$0{,}60y_m$	částečná

[a] Interferují dvě identické harmonické vlny o amplitudě y_m, postupující souhlasným směrem.

Phase vector diagram

Superposition of waves with the same frequency

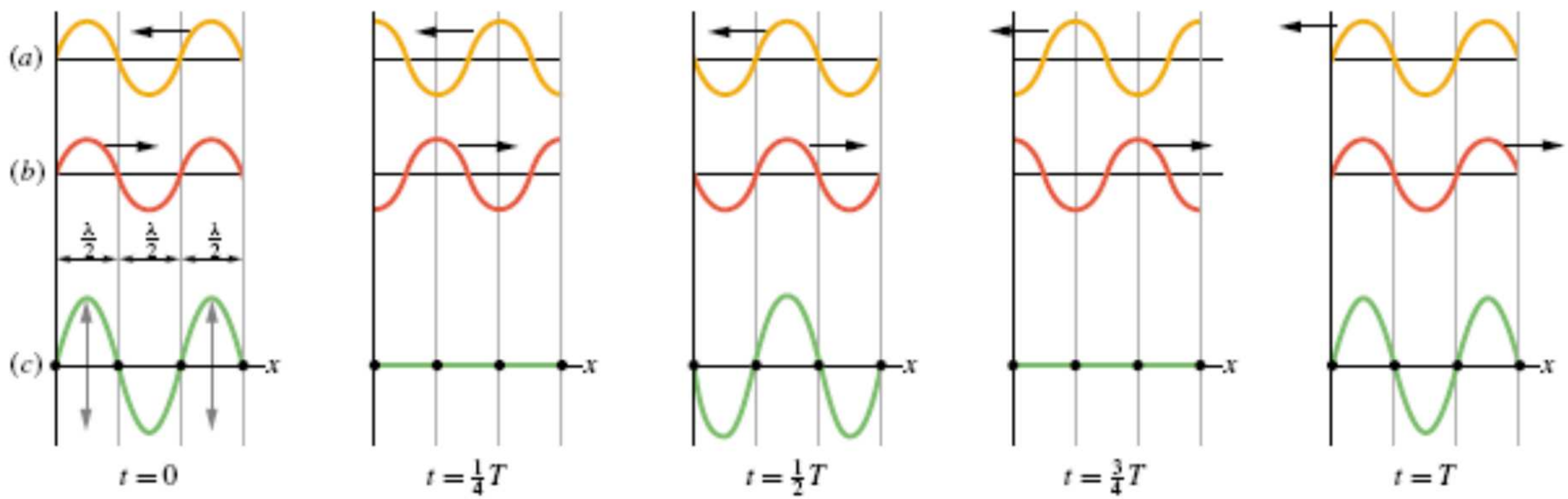


$$y_m^2 = y_{m1}^2 + y_{m2}^2 - 2y_{m1}y_{m2}\cos(180^\circ - \varphi)$$

$$\tan(\beta) = \frac{y_{m2}\sin\varphi}{y_{m1} + y_{m2}\cos\varphi}$$

Standing wave

Superposition of two waves travelling with the same speed in the opposite direction



Nodes and antinodes

Standing wave displacement

$$y_1(x,t) = y_m \sin(kx - \omega t)$$

$$y_2(x,t) = y_m \sin(kx + \omega t)$$

$$y = y_1 + y_2 = 2y_m \sin(kx)\cos(\omega t)$$

Nodes

$$\sin(kx) = 0$$

$$kx = n\pi, \quad x = n\frac{\lambda}{2},$$

$$n = 0,1,2,3,\ldots$$

Antinodes

$$\sin(kx) = \pm 1$$

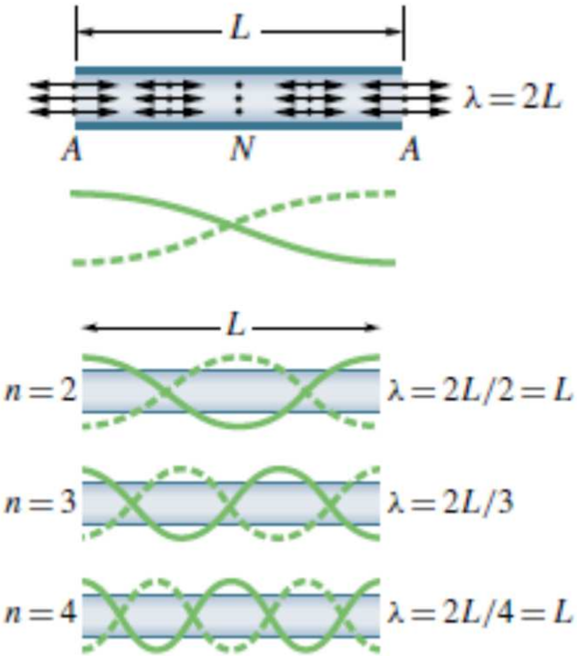
$$kx = \left(n + \frac{1}{2}\right)\pi, \quad x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2},$$

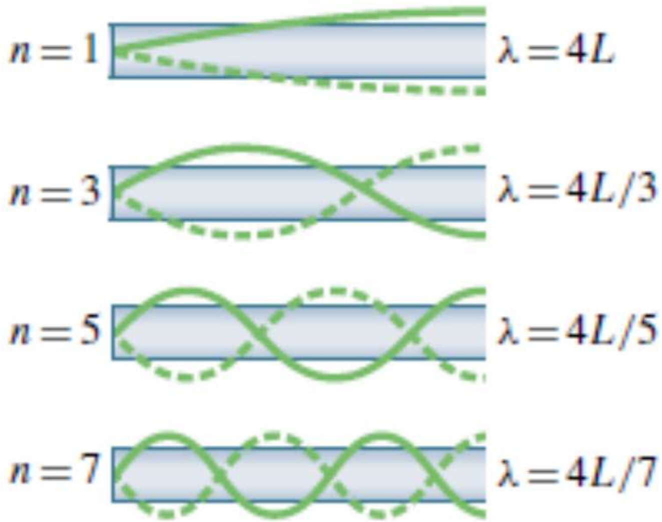
$$n = 0,1,2,3,\ldots$$

Standing wave in resonant tube

Trumpets, flutes, etc. – fundamental and higher overtones (harmonics)

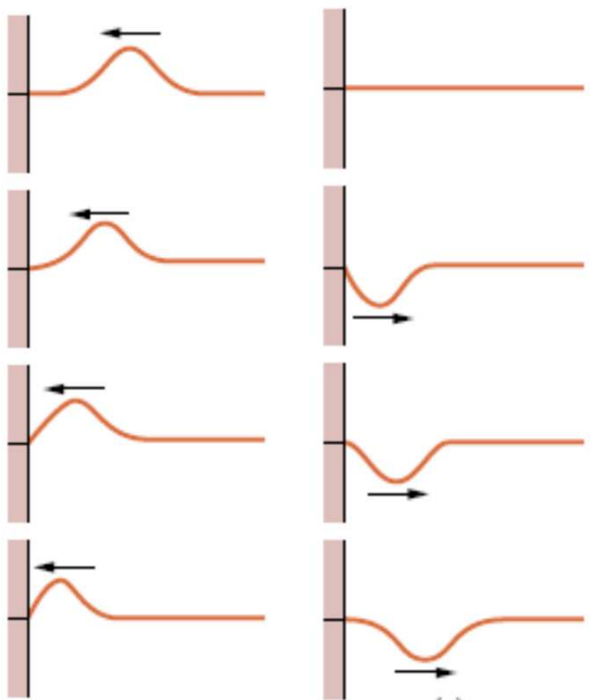
Open end – antinode, closed end - node





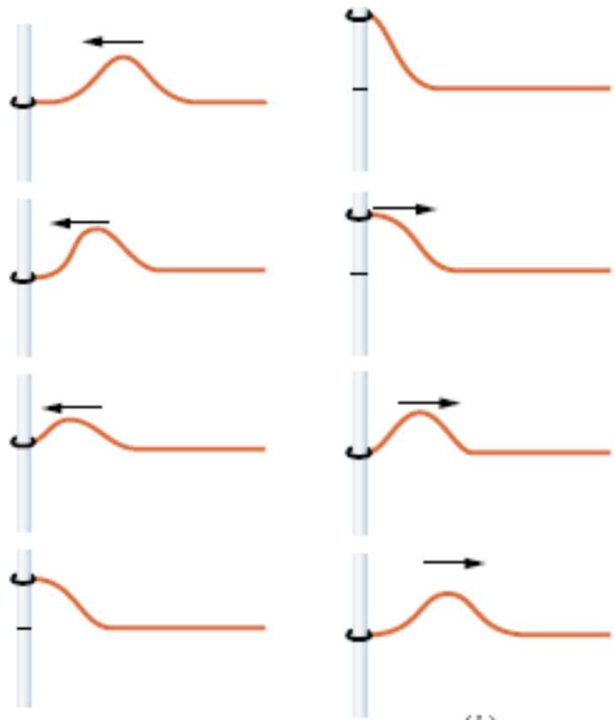
Reflection of wave at the end of waveguide

Fixed end – in phase



(a)

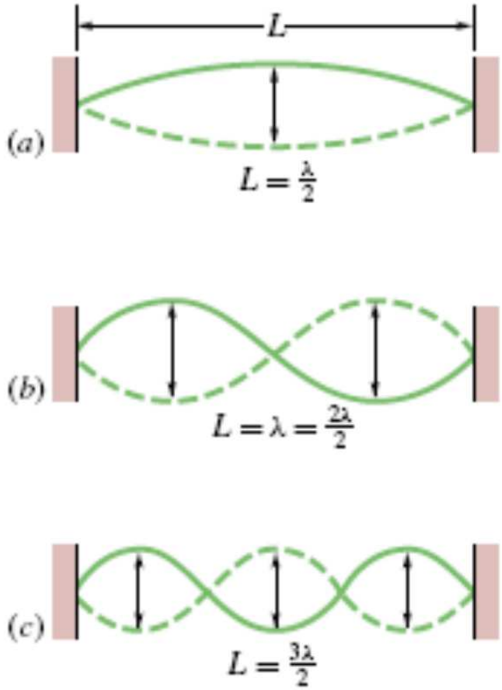
free end – opposite phase



(b)

Free vibrations of string

Constructive interference of superposed waves



Frequencies

$$\lambda = \frac{2L}{n}, \quad n = 1,2,3\ldots$$

$$f = \frac{v}{\lambda} = n\frac{v}{2L}$$

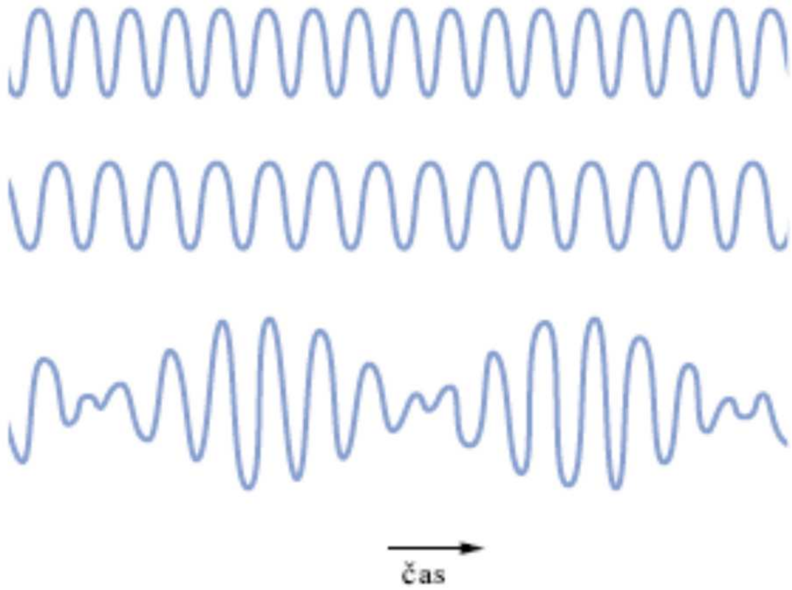
harmonic frequencies

overtones=harmonics

Beats

Superposition of two waves with near frequencies

$$y_1 = y_m \sin(\omega_1 t - k_1 x)$$
$$y_2 = y_m \sin(\omega_2 t - k_2 x)$$
$$\omega = \frac{1}{2}(\omega_1 + \omega_2), \quad \Delta\omega = \frac{1}{2}(\omega_1 - \omega_2)$$
$$k = \frac{1}{2}(k_1 + k_2), \quad \Delta k = \frac{1}{2}(k_1 - k_2)$$



$$y = y_1 + y_2 = 2y_m \cos(\Delta\omega t - \Delta k x)\sin(\omega t - kx)$$

Frequency of beats

Angular frequency of beats

$$\omega_{beats} = 2\Delta\omega$$

Frequency of beats

$$f_{beats} = \left| f_1 - f_2 \right|$$

Group velocity

„Wave packet“

$$\Delta\omega t - \Delta k x = konst.$$

Propagation velocity of „packet“ = group velocity

$$v_{gr} = \frac{d\omega}{dk}$$

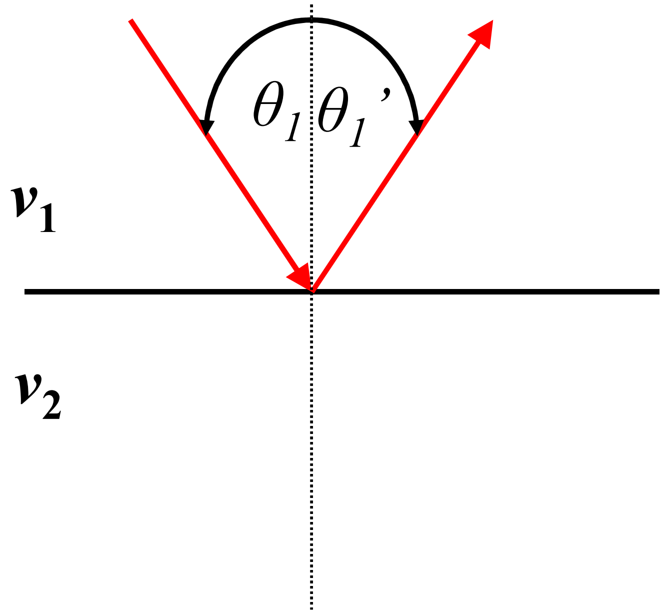
Dispersion

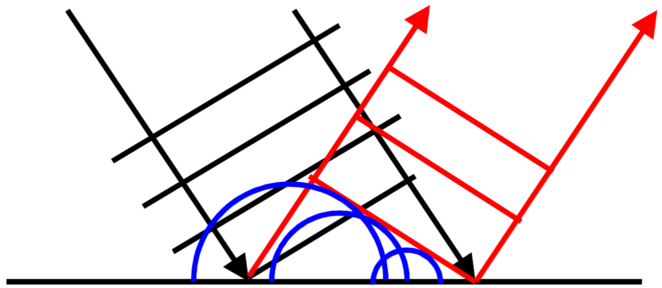
If not $\omega = kc$, then it is $v_{gr} \neq c$

Wave reflection

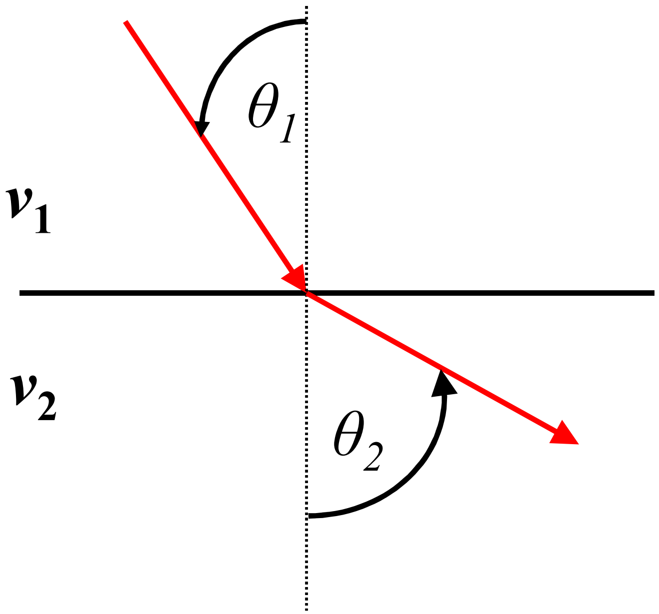
Law of reflection

$$\theta_1 = \theta_1'$$





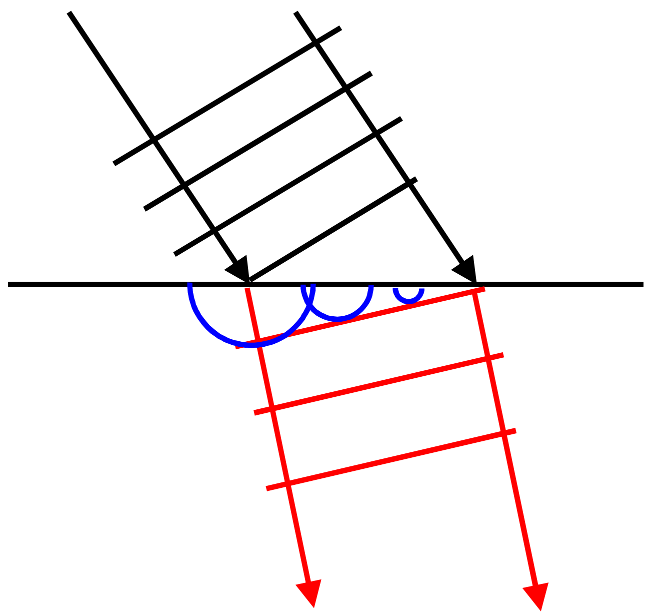
Wave refraction



Snellius

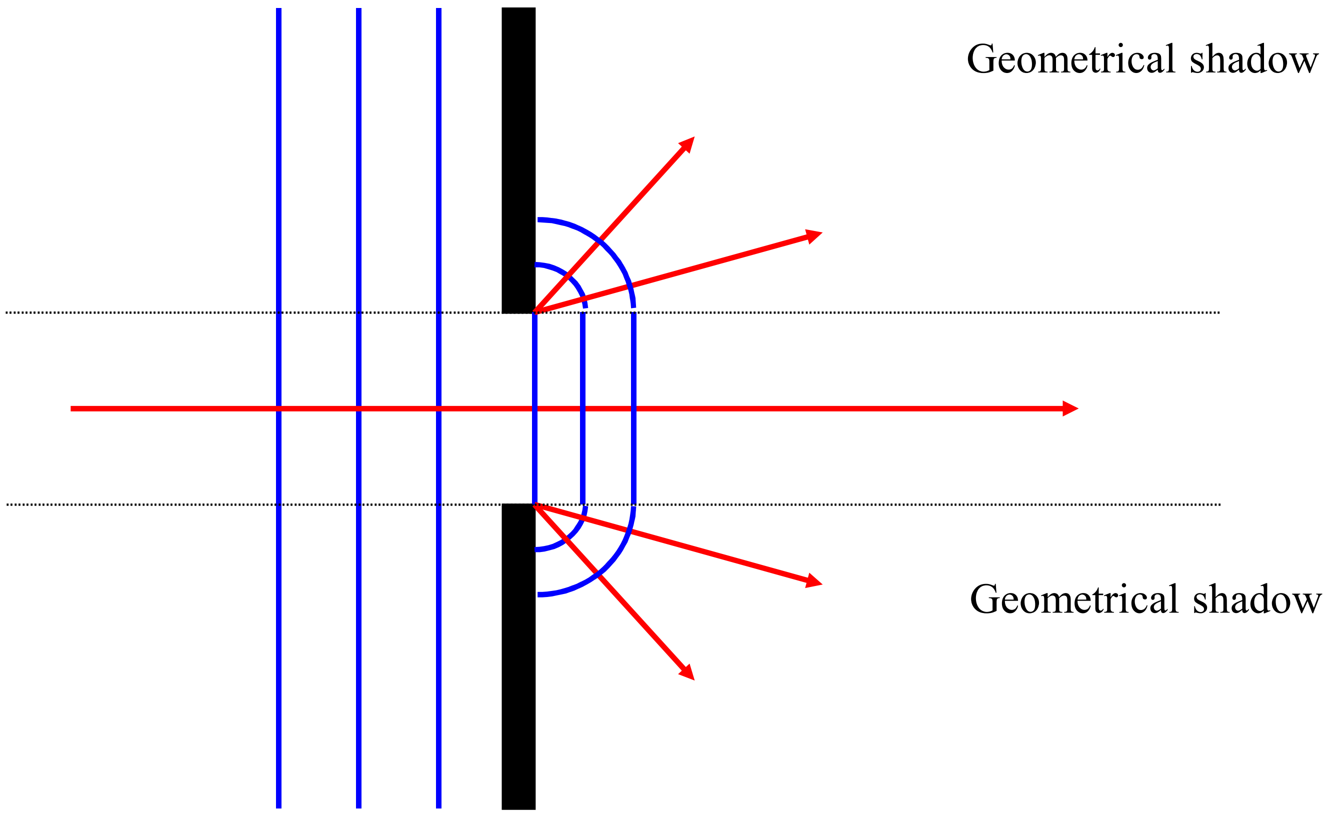
Law of refraction

$$\frac{\sin(\theta_1)}{\sin(\theta_2)} = \frac{v_1}{v_2}$$



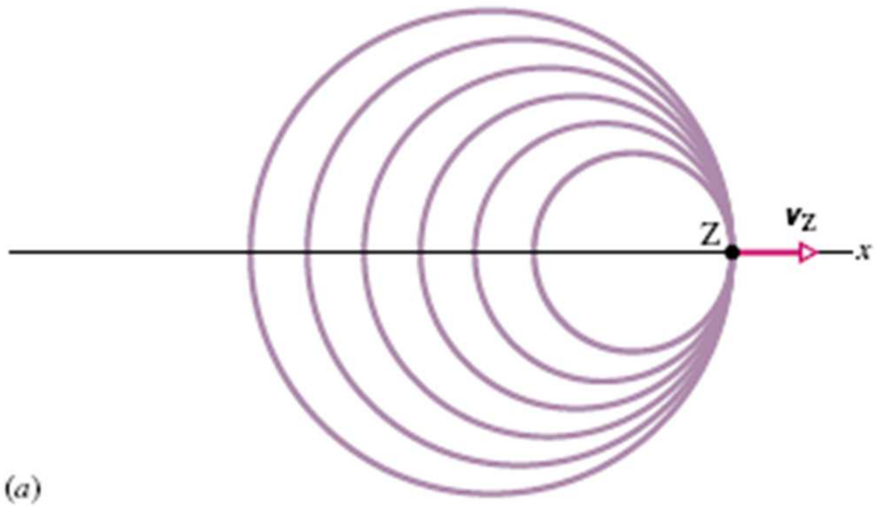
Diffraction of waves on slit

Waves are propagated in the geometrical shadow



Shock wave

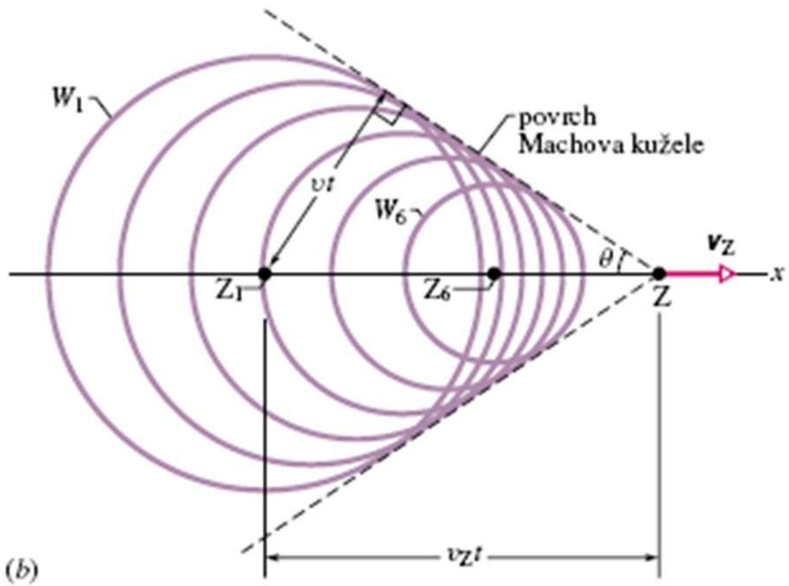
Concentration of wave fronts = concentration of sound wave pressure



(a)

Mach number and sound barrier

Crests of wavefronts are concentrated to built shock wave (sound barrier)



Mach number

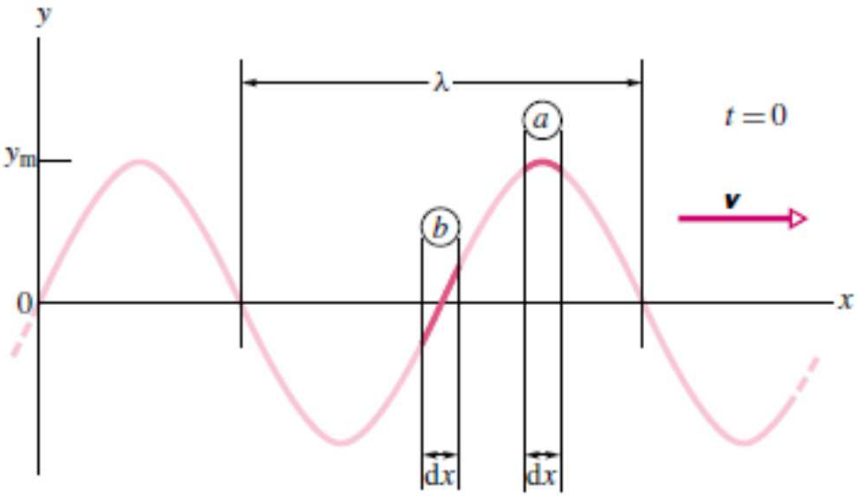
$$\frac{v_Z}{v}$$

Mach cone

$$\sin\theta = \frac{vt}{v_Z t} = \frac{v}{v_Z}$$

Wave energy

1D waveguide – kinetic and potential energies



$$y(x,t) = y_m \sin(kx - \omega t)$$

$$dE_k = \tfrac{1}{2}\, dm\, u^2 \qquad dm = \mu dx$$

$$u = \frac{\partial y}{\partial t} = -\omega y_m \cos(kx - \omega t).$$

$$\overline{E_k} = \frac{1}{\lambda}\int_0^{\lambda} dE_k.$$

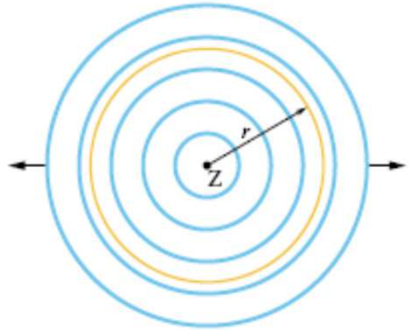
Mean energy per 1m string length

$$\overline{E}_k = \frac{1}{4}\mu\omega^2 y_m^2$$

Mean value of kinetic and potential energy is the same.

Wave intensity

- Mean power $\overline{P} = (\overline{E_k} + \overline{E_p})v = 2\overline{E_k}v$



$$\overline{P} = \frac{1}{2}\mu v \omega^2 y_m^2$$

- Wave intensity = power per 1m^2 of wavefront

$$I = \frac{P}{S} = \frac{P}{4\pi r^2}$$

Wave intensity decreases with increasing size of wavefront!

Acoustics

Mechanical waves – longitudinal only in air, longitudinal and transversal in solids

- Sound (20Hz-20kHz)
- Ultrasound (above 20kHz)
- Infrasound (below 20Hz)

Sound

- Frequency 20Hz-20kHz
- Phase velocity
- Wavelengths in air

Tabulka 18.1 **Rychlost zvuku**

PROSTŘEDÍ	$\frac{v}{\mathrm{m \cdot s^{-1}}}$	PROSTŘEDÍ	$\frac{v}{\mathrm{m \cdot s^{-1}}}$	PROSTŘEDÍ	$\frac{v}{\mathrm{m \cdot s^{-1}}}$
Plyny[a]		*Pevné látky*[a]		*Kapaliny*[a]	
Vzduch (0 °C)	331	Hliník	6 420	Voda (0 °C)	1 402
Vzduch (20 °C)	343	Ocel	5 941	Voda (20 °C)	1 482
Helium	965	Žula	6 000	Mořská voda[b]	1 522
Vodík	1 284				

[a] 0 °C a tlak 1 atm, pokud neuvedeno jinak.
[b] Při 20 °C a salinitě 3,5 %.

Phase velocity of sound

- In solids

$$v = \sqrt{\frac{E}{\rho}}$$

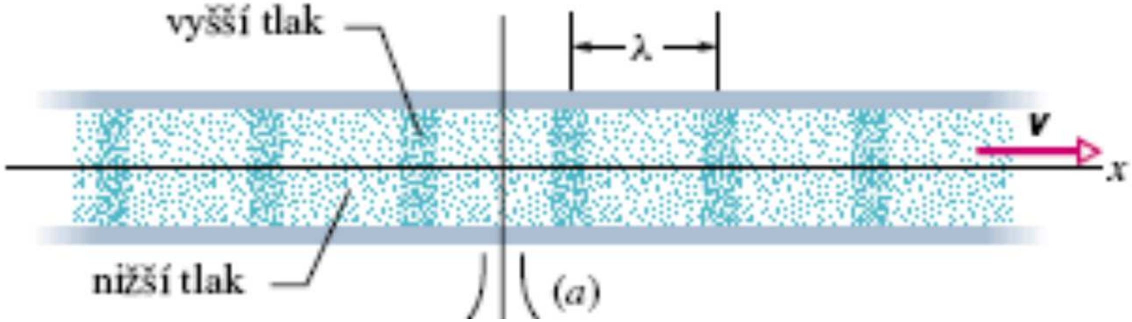
- In liquids

$$v = \sqrt{\frac{K}{\rho}}$$

Modulus of volume compressibility $K = -V\left(\frac{\Delta p}{\Delta V}\right)$

Propagation of sound wave

Longitudinal wave of acoustic pressure differences (10^{-5} - 10^{1}Pa)

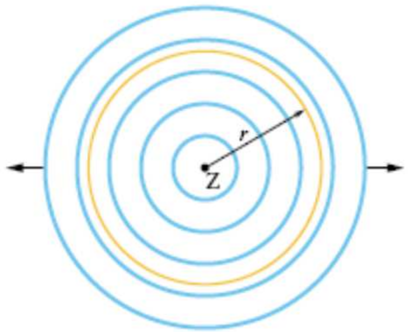


Medium particles oscillate in the direction of wave propagation

Sound intensity

- Acoustic power per 1m^2 of wavefront surface

$$I = \frac{P}{S} = \frac{P}{4\pi r^2}$$



- Sound intensity

$$I = \frac{1}{2}\rho v \omega^2 u_m^2$$

- Acoustic impedance of material

$$Z_a = \rho v$$

Sound level

Ratio of intensity and its reference in logarithmic scale

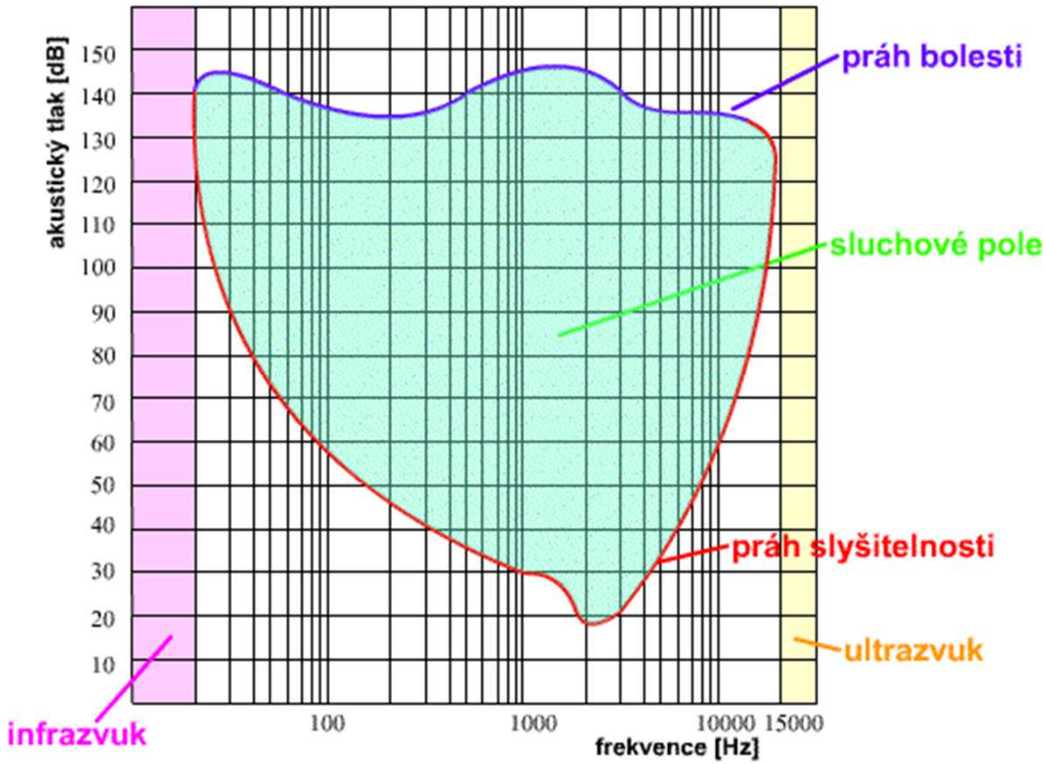
$$L = (10dB)\log\frac{I}{I_0}, \quad I_0 = 10^{-12}Wm^{-2}$$

Tabulka 18.2 Některé hladiny intenzity zvuku v dB

Práh slyšitelnosti	0	Rockový koncert	110
Ševelení listů	10	Práh bolesti	120
Běžný hovor	60	Proudový motor	130

Threshold of hearing and pain

homen.vsb.cz/~ber30/texty/varhany/anatomie/pistaly_akustika.htm



Literatura

Pictures used from the book:

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Fyzika. Brno: VUTIUM, 2000. díl 2
Mechanika - Termodynamika