Electrostatics.
Electrical current.

Potential and field intensity, electrical forces, capacity, energy of charged capacitor. Current conductivity in liquids and gasses, electrolysis, Faraday’s law of electrolysis.

Electrostatics

Electrical charge

- Electrical forces, electric field
- Electric current

Property related to the matter particles – electrons, protons etc.

Charge could not be created or perished, but only distributed

Electrical charge

- Free – e.g. electrons
- Bound – e.g. dipoles

Elementary charge $e = 1.602 \cdot 10^{-19}\,C$

Charge density – surface, volume

Faraday cage – shielding of electric field

Creation of electric charges

- Friction
- Temperature change – pyroelectricity
- Pressure – piezoelectricity
- Light – photovoltaic effect
- Electromagnetic induction
- Electrostatic induction
- ….

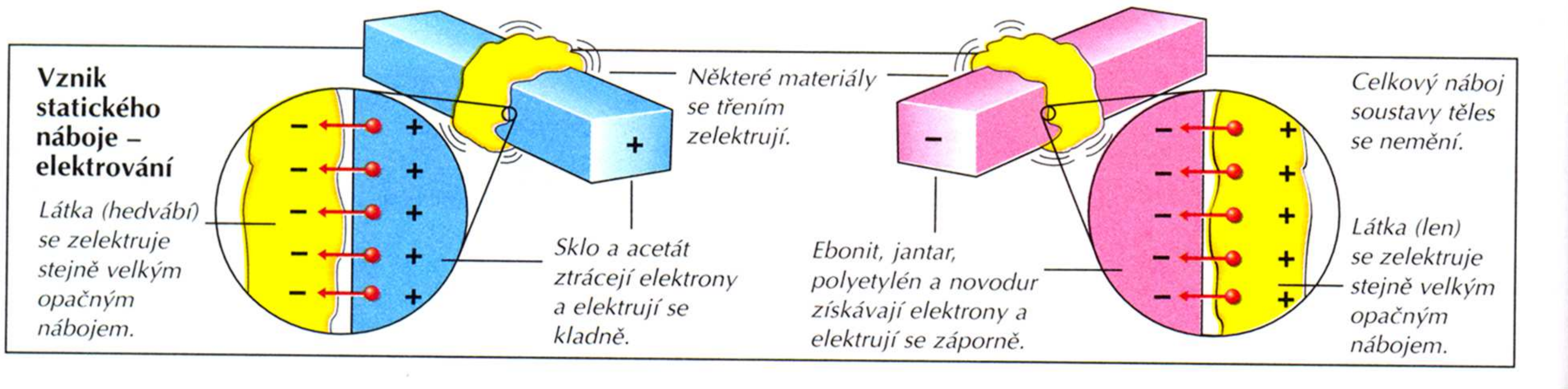
Frictional charges on textiles

Dissociation of polar chemical bonds like

$-COO^{-}$, $-NH_3^{+}$

One substance is positively and second negatively charged

Amount of charge is related to chemical nature of bonds



Frictional electricity sequence

+

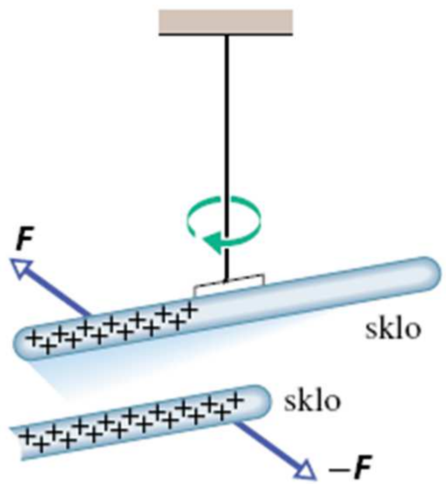
Wool
Polyamide (Nylon)
Viscose
Cotton
Natural silk
Acetate
Polyvinylalcohol
Polyester (Dacron)
Polyacrylonitrile (Orlon)
Polyvinylchloride
Polyethylene
Polytetrafluor (Teflon)

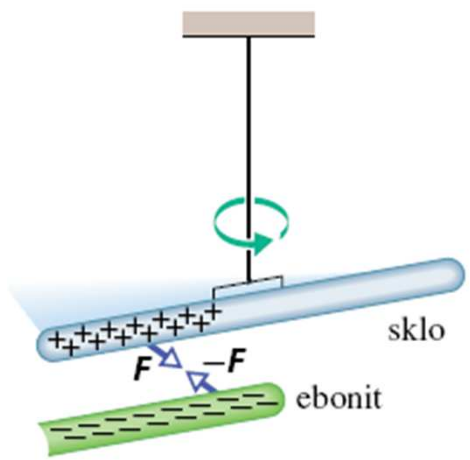
-

Fiber at higher position in sequence is charged positively at mutual contact

Electrical charges

- Positive
- Negative



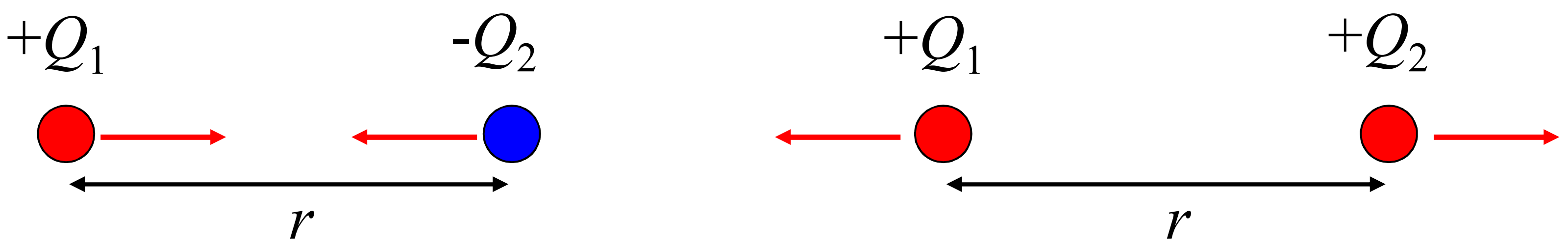


Electrical force

Coulomb's law

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2}, \quad \varepsilon_0 = 8.854 \cdot 10^{-12}\, Fm^{-1}$$

Attractive or repulsive force



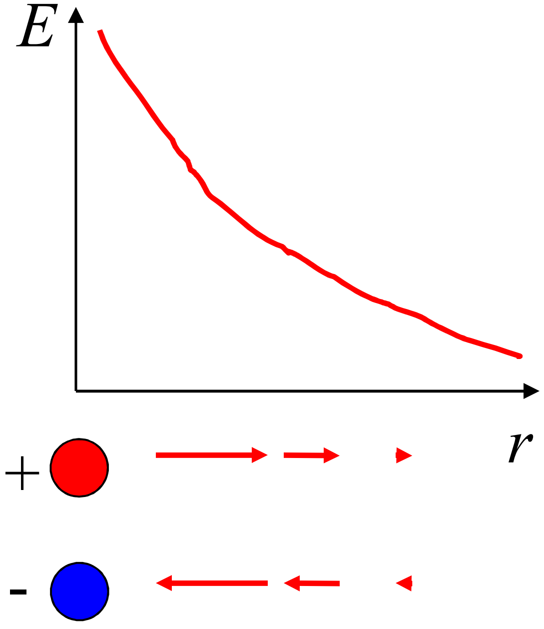
Electric field

Electric field intensity

$$E = \frac{F}{Q'} \quad [V/m]$$

For point charge

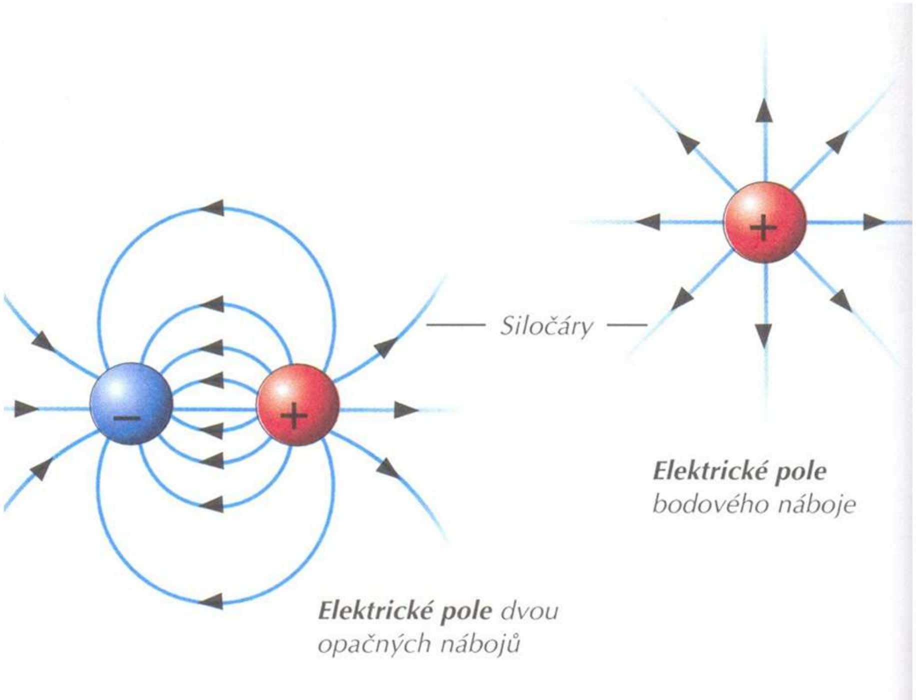
$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$



Field lines

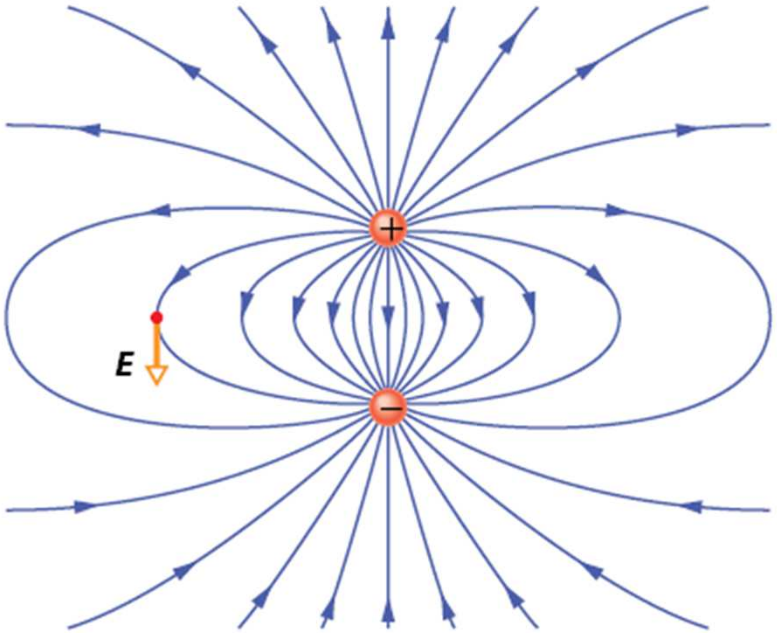
Oriented lines, force is tangential to it

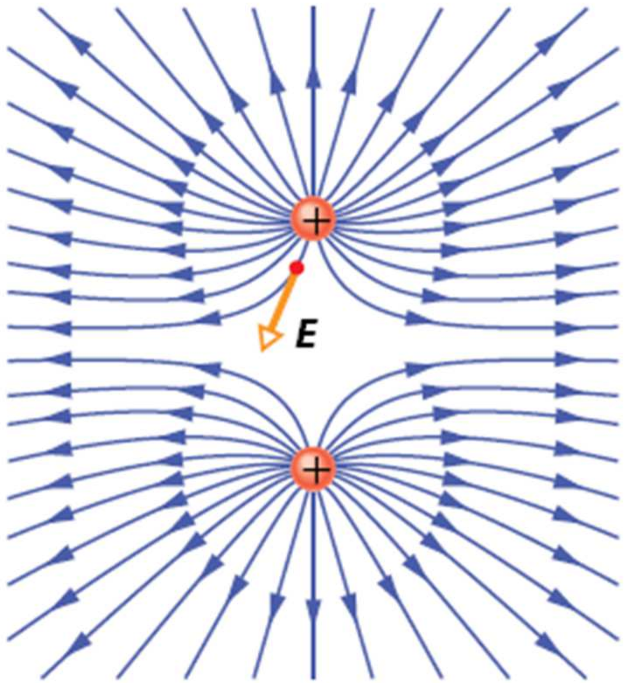
Field lines input conductor surface in perpendicular direction – arching in the vicinity of sharp edges



Dipole field lines

Dipole - compare two charges of the same polarity!





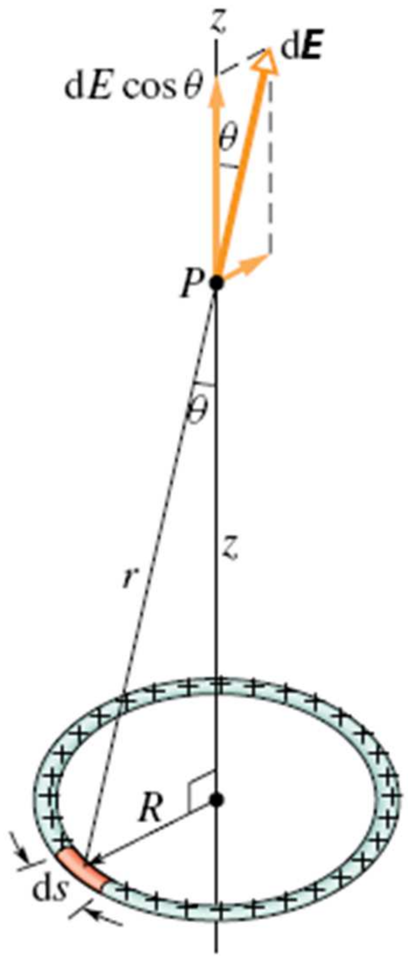
Field superposition

For electric forces, field intensity and potential

Resulting force from the set of charges is vector sum of contributions from individual charges.

$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} + \mathbf{F}_{14} + \ldots + \mathbf{F}_{1n}$$

Field intensity for charged ring



Field intensity by integration

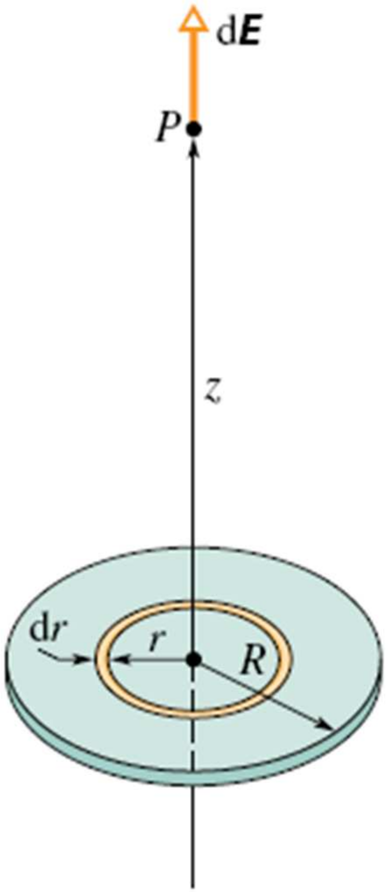
Q=τ(2πR), τ…linear density of charge

$$E = \int \mathrm{d}E \cos\theta = \frac{1}{4\pi\varepsilon_0} \frac{z\tau}{(z^2 + R^2)^{3/2}} \int_0^{2\pi R} \mathrm{d}s =$$

$$= \frac{z\tau(2\pi R)}{4\pi\varepsilon_0 (z^2 + R^2)^{3/2}}$$

$$E = \frac{Qz}{4\pi\varepsilon_0 (z^2 + R^2)^{3/2}}$$

Field intensity of charged disc



Field intensity of disc

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

far from the disc

$$E = \frac{\sigma}{2\varepsilon_0}$$

Gauss's law of electrostatics

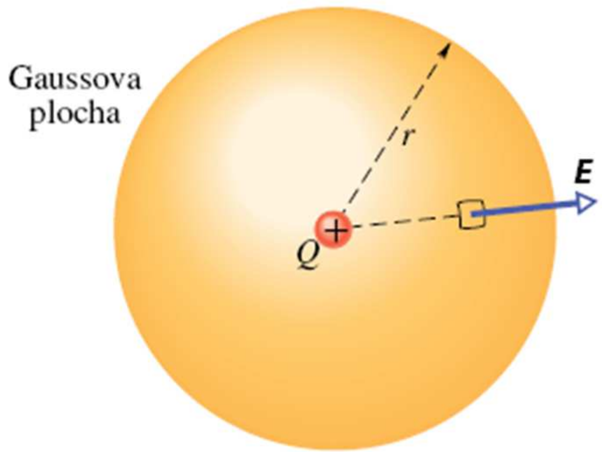
Electric flux

$$\Phi_E = \oint_{\mathscr{S}} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{S}$$

Gauss's law – total electric flux on closed surface is given by the net charge enclosed within that surface divided by ε_0

$$\varepsilon_0 \oint \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{S} = Q$$

Coulomb's and Gauss's laws



$$\varepsilon_0 E \oint \mathrm{d}S = Q$$

$$\varepsilon_0 E (4\pi r^2) = Q$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

Charged and isolated conductor

All free charge is on the surface of conductor

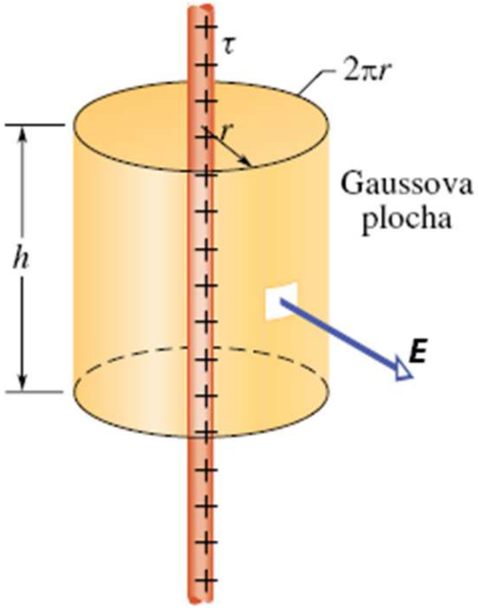
Gauss's law

Electric field near conducting surface is perpendicular to it

$$E = \frac{\sigma}{\varepsilon_0}$$

Field intensity for charged wire

Gauss's law

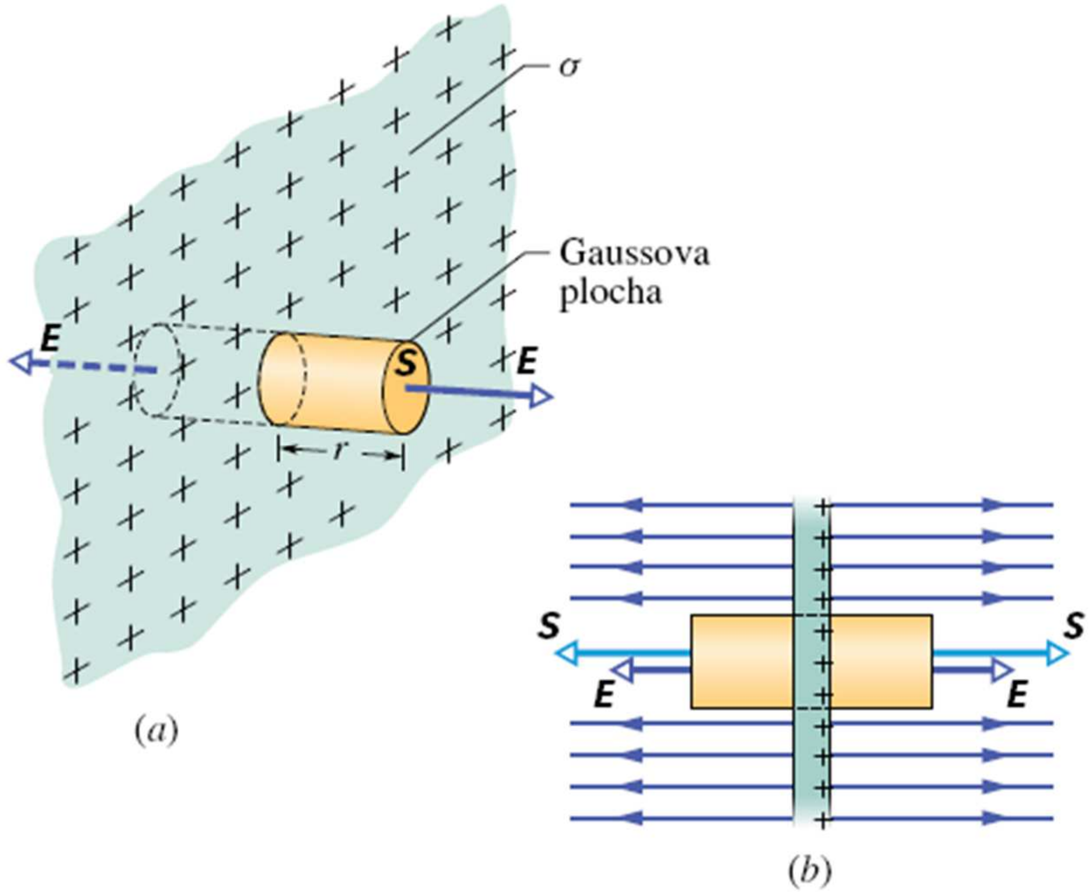


$$E = \frac{\tau}{2\pi\varepsilon_0 r}$$

Charged plane

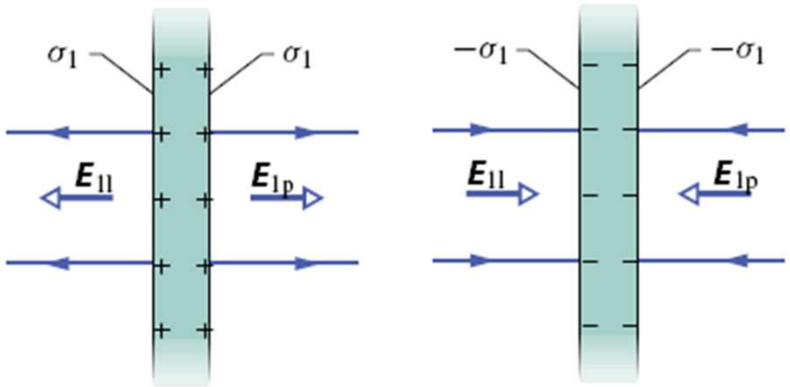
Gauss's law

$$E = \frac{\sigma}{2\varepsilon_0}$$

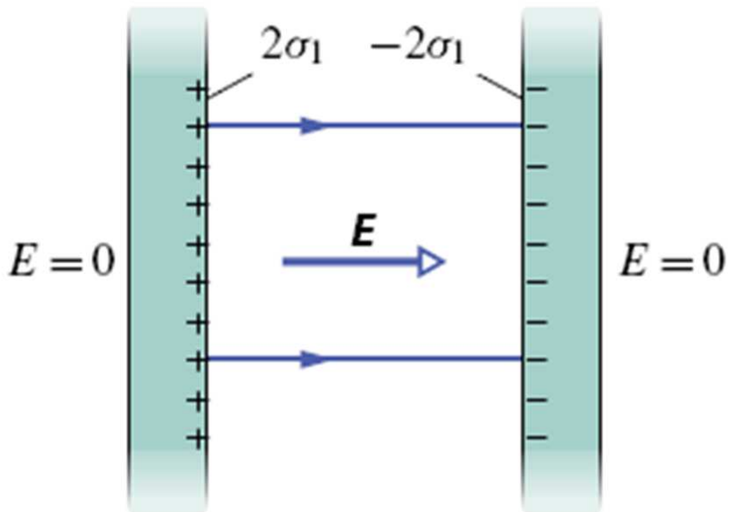


Charged plate capacitor

Two plates charged by opposite polarity charges



$$E = \frac{2\sigma_1}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$



Electric potential, voltage

Potential = work needed to place a charge at the given point in an electric field

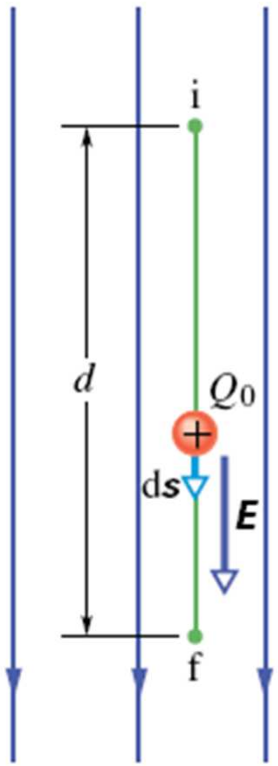
$$\varphi = \frac{A}{Q'} \quad [V]$$

Voltage $U = \varphi_2 - \varphi_1$

Equipotential surfaces – concentric spherical shells for point charge, perpendicular to field lines

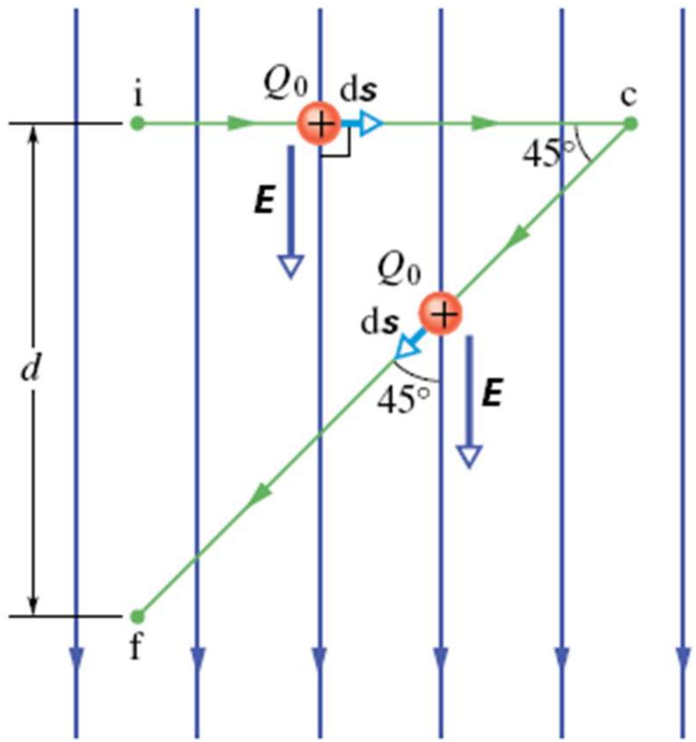
Work of electrical forces

Conservative forces = work independent from specific path, dependent on initial and final position

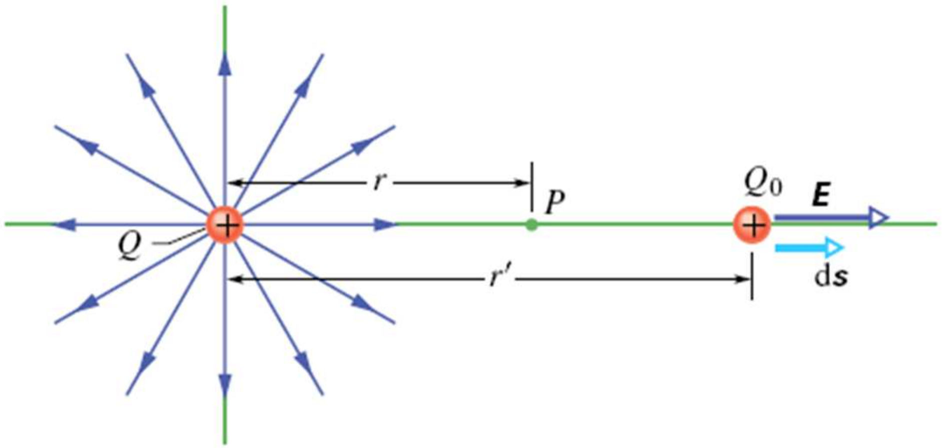


same

work!



Potential of point charge



Potential

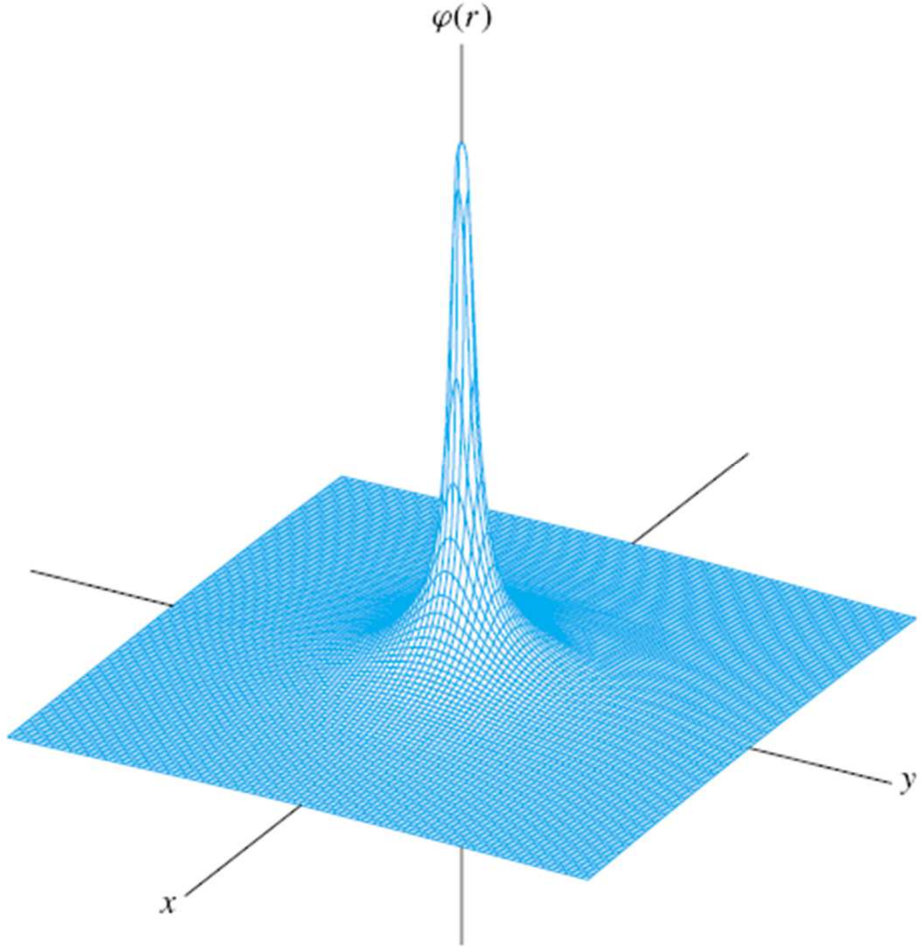
- Positive charge

$$\varphi(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

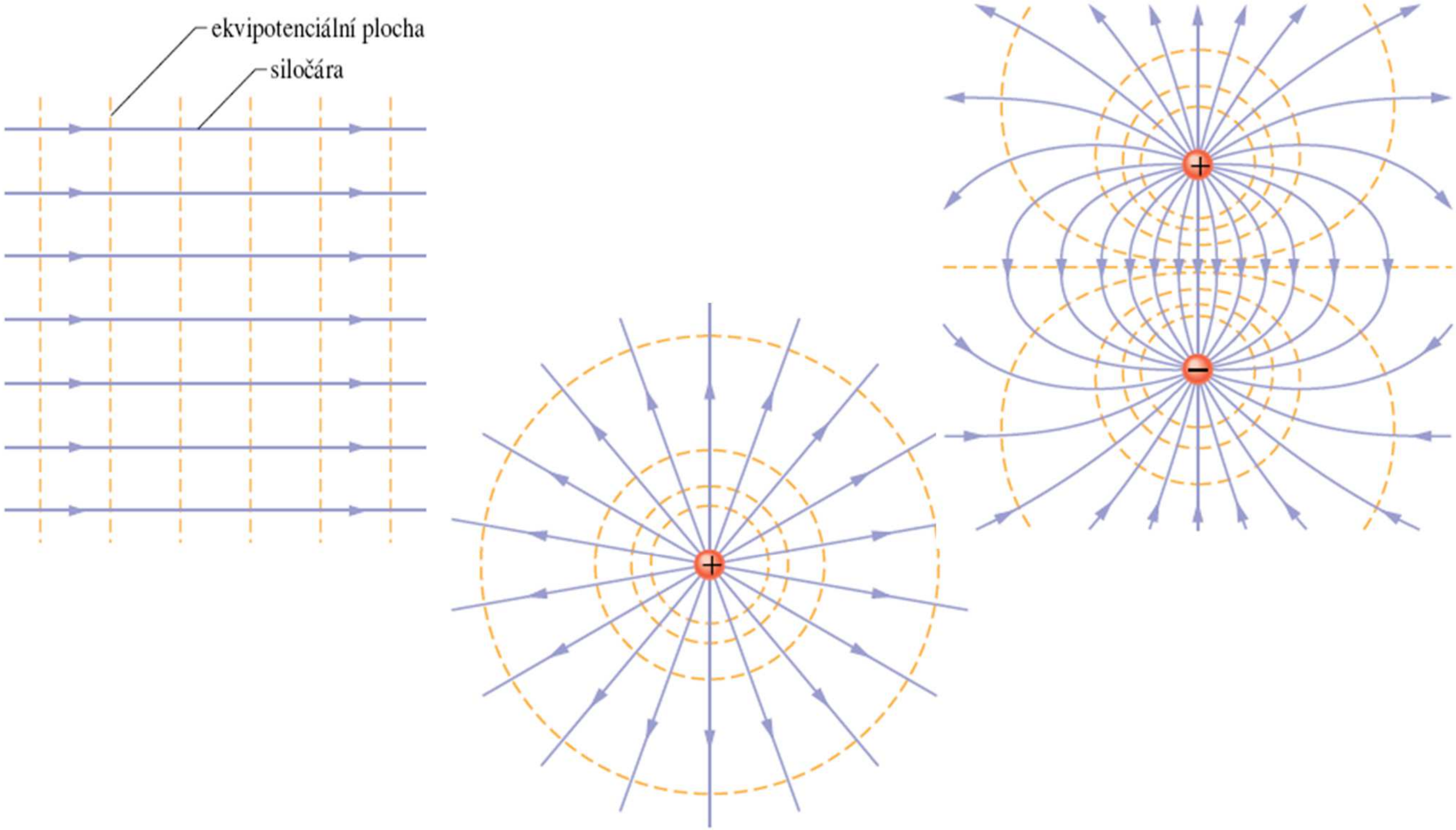
- negative charge

$$\varphi(r) = -\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

Positive point charge - potential

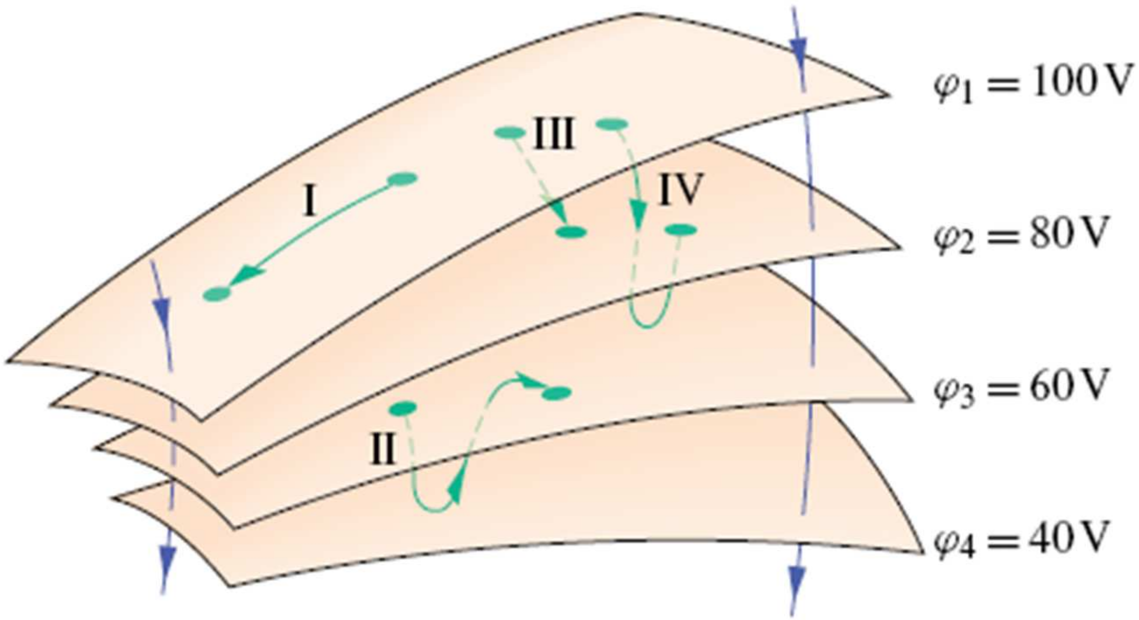


Equipotential surface



Work and equipotential surfaces

Equipotential surface = same potential value



General relation between field intensity and potential

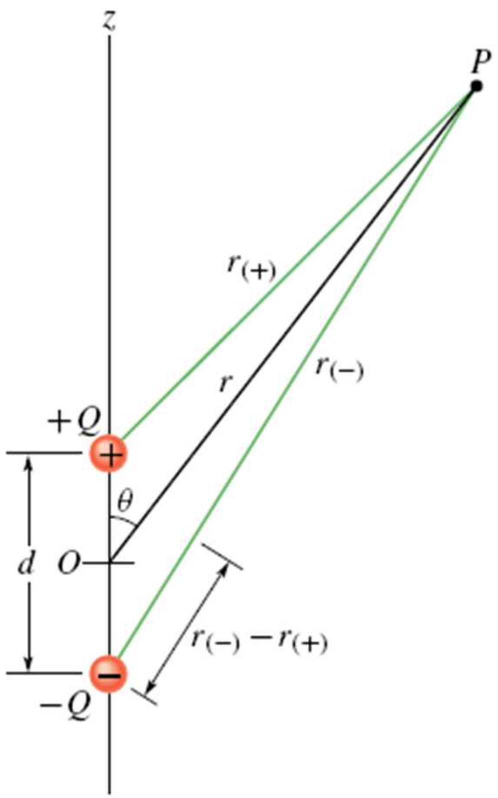
Field intensity

$$E = -grad\varphi = -\left(\frac{\partial\varphi}{\partial x} \quad \frac{\partial\varphi}{\partial y} \quad \frac{\partial\varphi}{\partial z} \right)$$

Field potential

$$\varphi_2 - \varphi_1 = -\int E \cdot dr$$

Electric dipole field intensity



sum of potentials from single charges

$$\varphi = \sum_{i=1}^{2} \varphi_i = \varphi_{(+)} + \varphi_{(-)} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r_{(+)}} + \frac{-Q}{r_{(-)}}\right)$$

for $d \ll r$ it is

$$\varphi = \frac{Q}{4\pi\varepsilon_0}\frac{d\cos\theta}{r^2}$$

at dipole axis $p = Qd$

$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3}$$

Potential of continuous charge

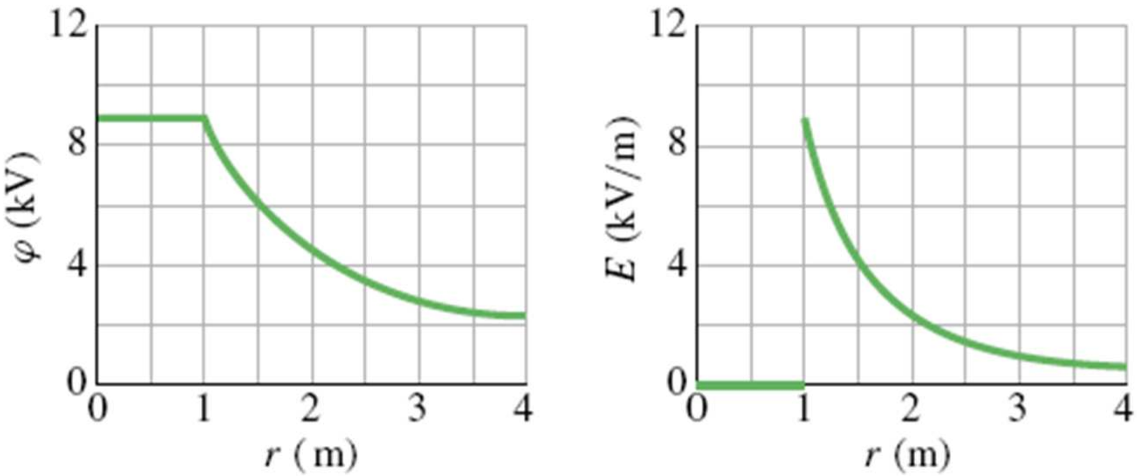
Integration of charge elements contributions

$$\mathrm{d}\varphi = \frac{1}{4\pi\varepsilon_0}\frac{\mathrm{d}Q}{r}$$

$$\varphi = \int \mathrm{d}\varphi = \frac{1}{4\pi\varepsilon_0}\int \frac{\mathrm{d}Q}{r}$$

Potential of charged conductor

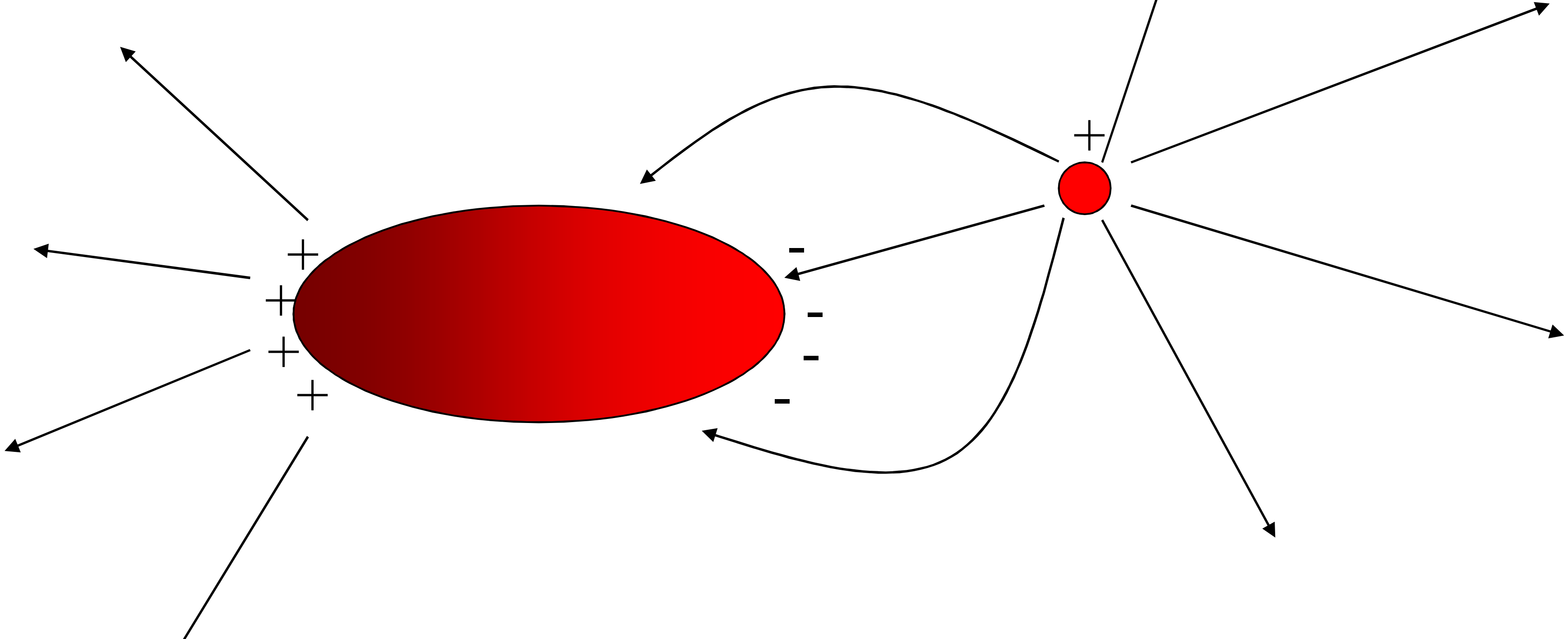
E=0 V inside conductor (Faraday cage)



All points of conductor are at the same potential

Electrostatic induction

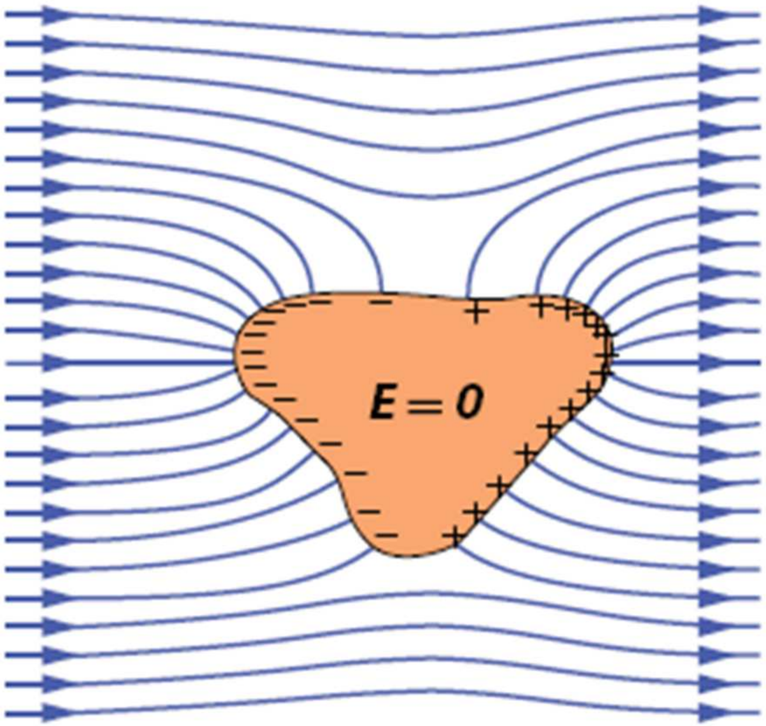
Redistribution of free charges in matter



Separation and segregation of charges

Conductor in external field

Zero field inside, perpendicular field lines at the surface



Conductor capacity

Voltage and charge on conductor are proportional

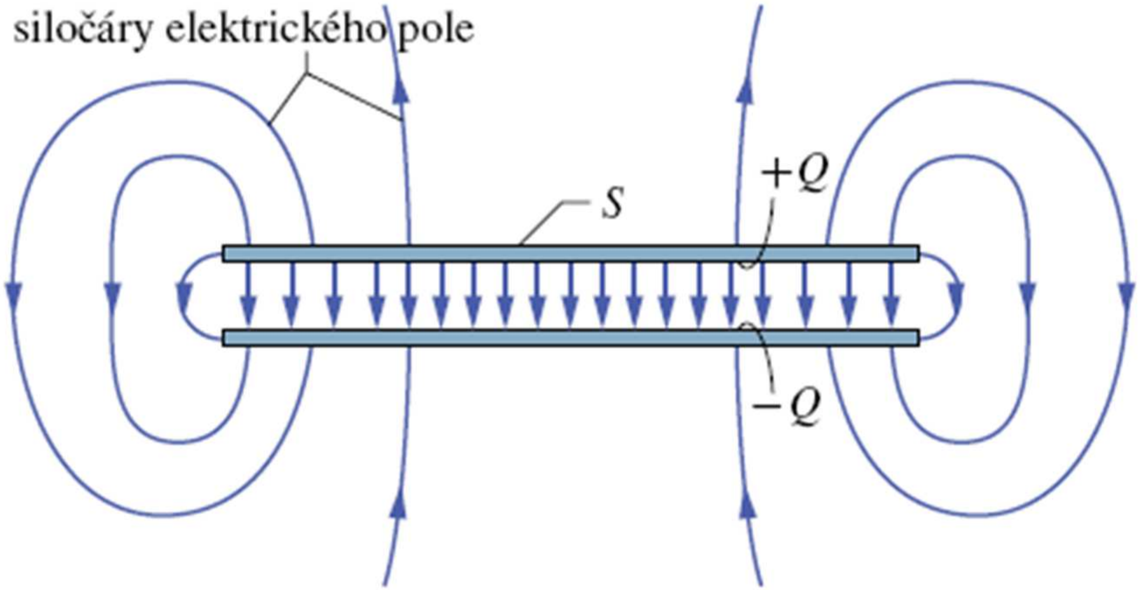
$$Q = CU$$

Conductor (body) capacity [C]=F (Farad)

$$C = \frac{Q}{U}$$

Capacity of plate capacitor

Related to its dimensions and material between plates



$$C = \frac{\varepsilon_0 S}{d}$$

Various capacitors

Capacity

Single charged conductive sphere

$$C = 4\pi\varepsilon_0 R$$

Spherical capacitor

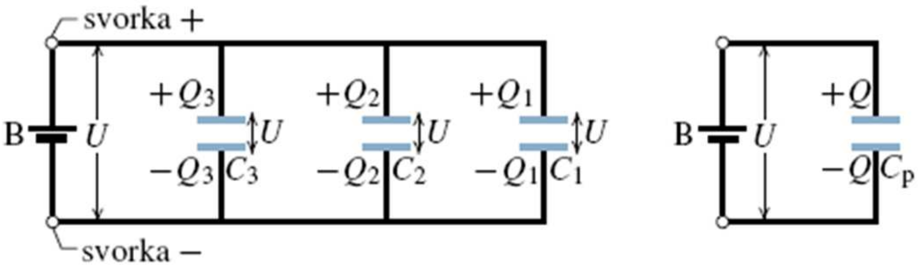
$$C = 4\pi\varepsilon_0 \frac{ab}{b-a} \qquad b > a$$

Cylindrical capacitor

$$C = \frac{2\pi\varepsilon_0 L}{\ln(b/a)} \qquad b > a,\ L \gg b.$$

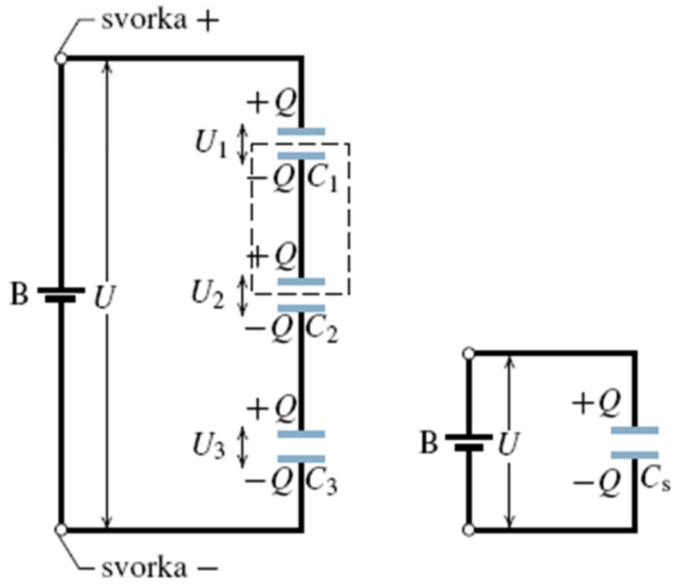
Combination of capacitors

- Parallel



$$C_p = \sum_{j=1}^{n} C_j$$

- Series



$$\frac{1}{C_s} = \sum_{j=1}^{n} \frac{1}{C_j}$$

Electric field energy of charged capacitor

Charging of capacitor needs work to overcome electric forces – energy stored in charged capacitor

$$E_{\mathrm{el}} = \frac{Q^2}{2C} = \tfrac{1}{2} C U^2$$

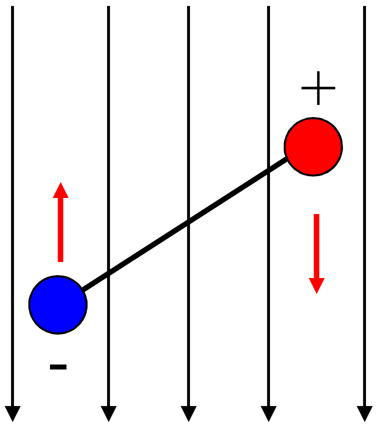
Volume density of energy

$$w_{\mathrm{el}} = \frac{E_{\mathrm{el}}}{V} = \tfrac{1}{2} \varepsilon_0 E^2$$

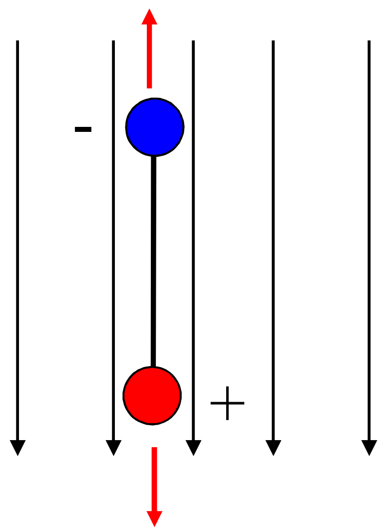
Electric dipole

Opposite polarity charges – e.g. charges within molecular bonds

Dipole in an external field



unstable position



stable position

Conductors and dielectrics

- Conductors – free charges, high mobility, high current conductivity
- Dielectrics – bound charges, low mobility, current conductivity by displacement movement of bound charges

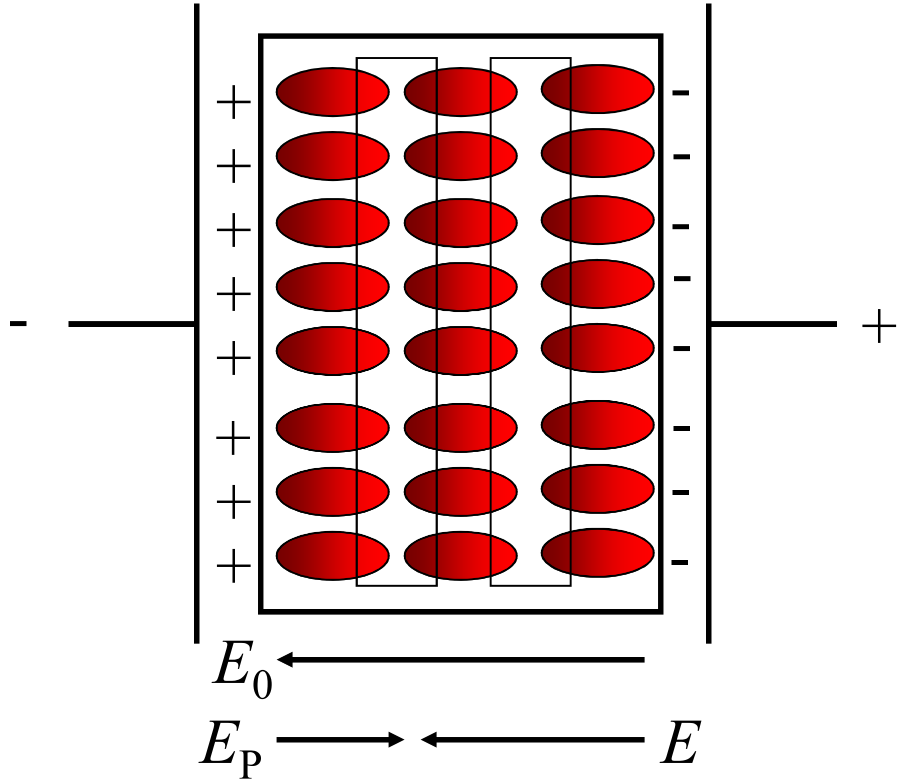
Dielectrics could be polarized by the dipole moment redistribution and allignment

Polar and nonpolar dielectrics

- Polar dielectrics – molecules are dipoles, they orient in an external field (e.g. water)
- Nonpolar dielectrics – molecules are neutral, dipole moments are created by external field (e.g. some crystals)

Polarisation of dielectrics

Electric field created inside matter, polarity opposite to external field



Total field

$$E = E_0 + E_P$$

Electric displacement

Dipole moment density

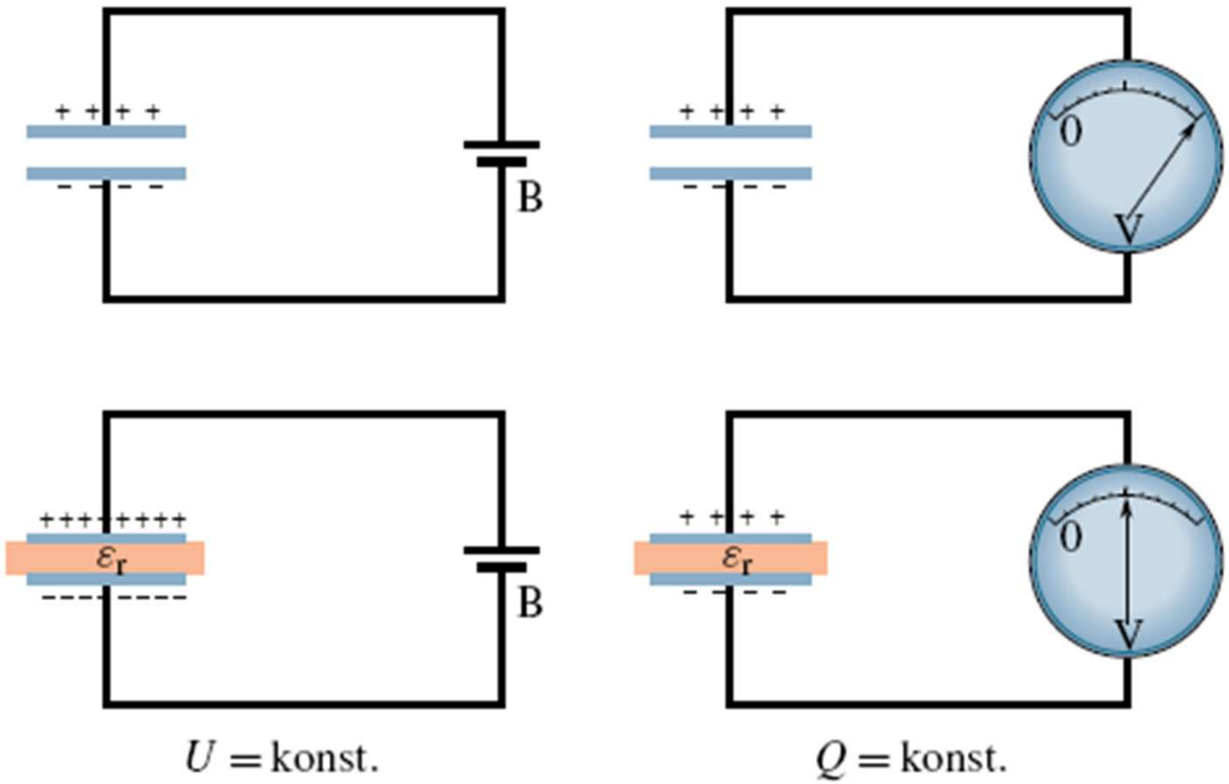
$$P = \varepsilon_0(\varepsilon_r - 1)E$$

Relative permittivity $\varepsilon_r \geq 1$

Electric displacement = surface density of charge

$$D = \varepsilon_r \varepsilon_0 E = \varepsilon_0 E + P$$

Charged capacitor with dielectric



Capacity of capacitor with dielectrics

Body capacity = charge to increase potential by 1V

$$C = \frac{\Delta Q}{\Delta U} \quad [F]$$

Plate capacitor capacity $$C = \varepsilon_r \varepsilon_0 \frac{S}{d}$$

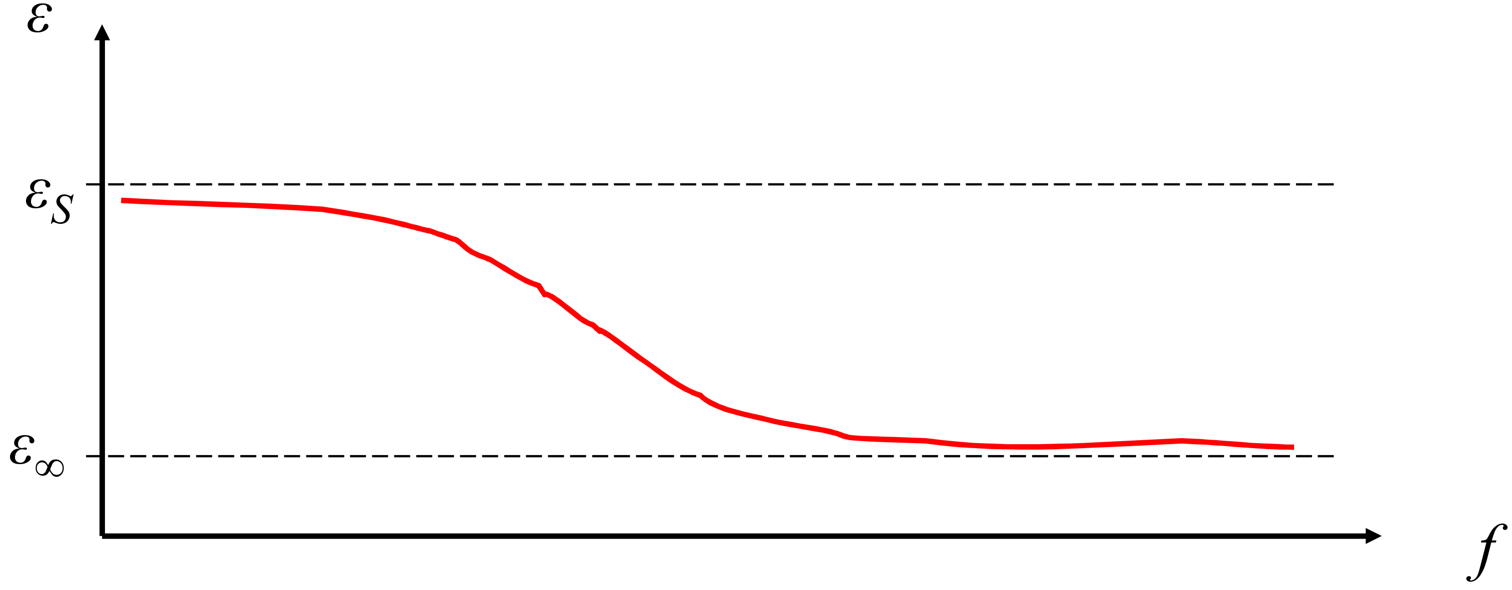
Energy of plate capacitor

$$\frac{1}{2} C U^2 = \frac{1}{2} Q U = \frac{1}{2} \frac{Q^2}{C}$$

Permittivity as a function of frequency

Static and dynamic permittivity

Orientational dynamics of dipole moment system



Electric current

Mobility of free charges in conductor

Electric current

$$I = \frac{\Delta Q}{\Delta t} \rightarrow \frac{dQ}{dt} \quad [A]$$

Electric current is the same at any conductor cross-section, function of conductor size and material, voltage

Ohm's law

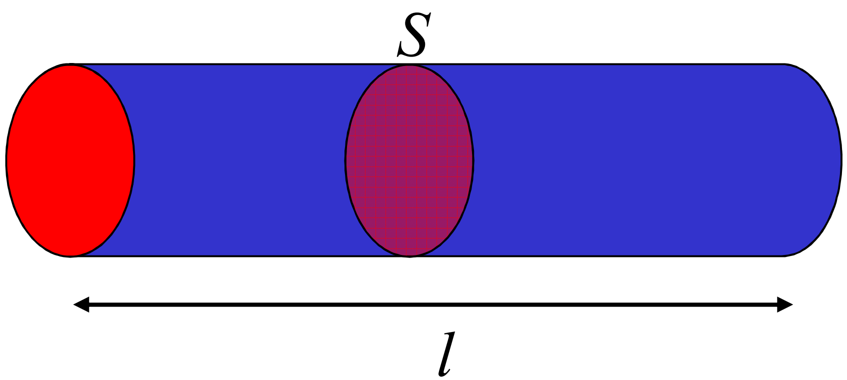
Linear approximation of voltage-current relation

$$U = RI$$

Electric resistance $R = \rho \frac{l}{S}$ [Ω]

Resistivity

ρ [Ωm]



Resistivities

Dielectrics $[10^{8}\Omega m]$		Metals $[10^{-8}\Omega m]$		Textile fibers $[10^{8}\Omega m]$	
Celluloid	2	Fe	8.81	Cotton	10^{6}
Rubber	10^{6}	Cu	1.555	Viscose silk	10^{7}
Paper	10^{2}	Al	2.45	Acetate silk	10^{11}
Polystyren	10^{7}			Wool	10^{8}
PVC	10^{5}			Nylon	10^{9}- 10^{12}

Electric resistance as a function of temperature

Linear function for metals

$$R_t = R_0(1+\alpha\Delta t)$$

Temperature coefficient of resistance

$$\alpha \quad [10^{-3} K^{-1}]$$

Exponential decay for semiconductors

Power in electric current circuits

- Circuit power

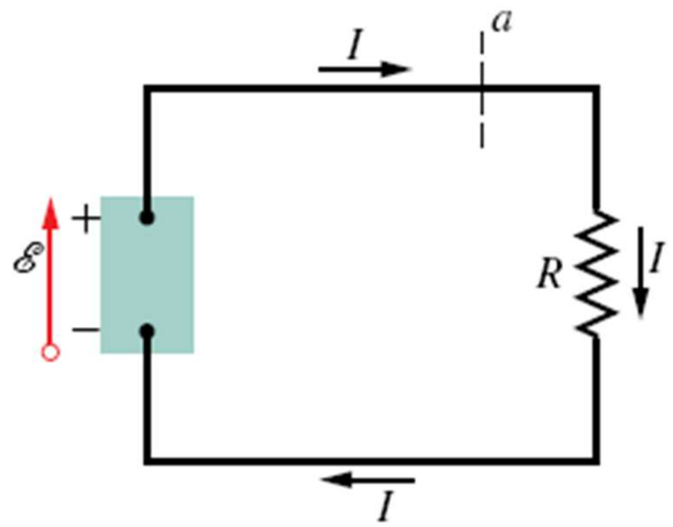
$$P = U I$$

- Lost power = Joule heat dissipation at resistances

$$P = I^2 R = \frac{U^2}{R}$$

DC circuits of electric current

- Junctions
- Branches
- Loops



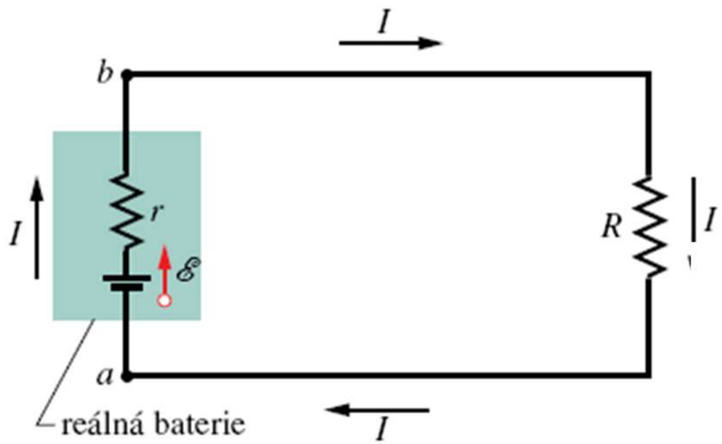
- Resistors
- Capacitors
- EMF sources (battery)

Kirchhoff's rules

- First rule (junctions) – the sum of all currents entering the junction must equal the currents leaving the junction
- Second rule (loops) – the sum of the changes in potential around any closed loop of a circuit must be zero

+ sign rule for currents and potential changes

Simple electric circuit – EMF + resistor

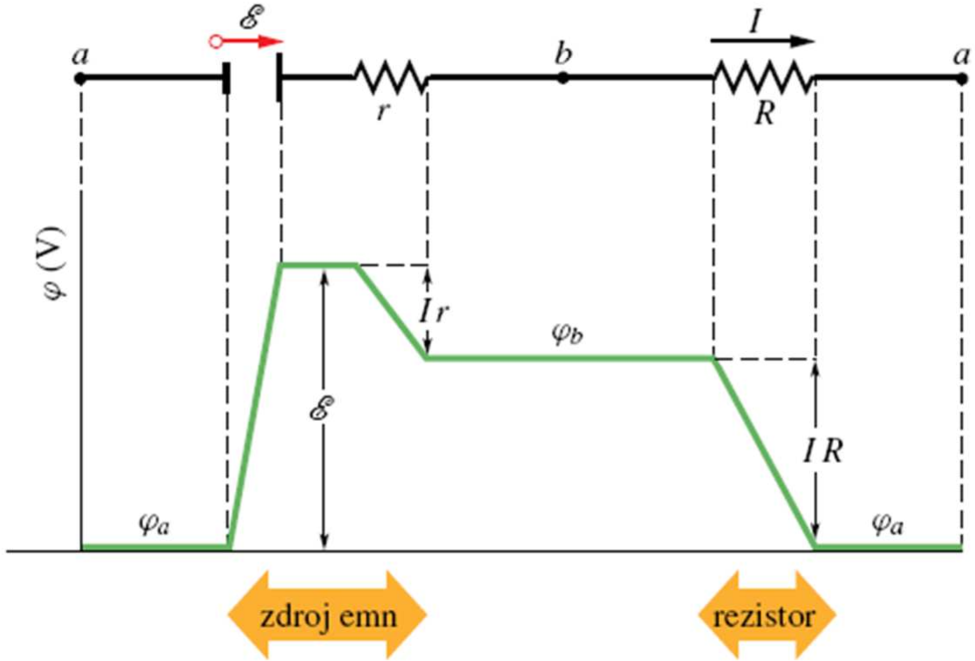


$$\mathscr{E} - Ir - IR = 0$$

$$I = \frac{\mathscr{E}}{R + r}$$

Terminal voltage

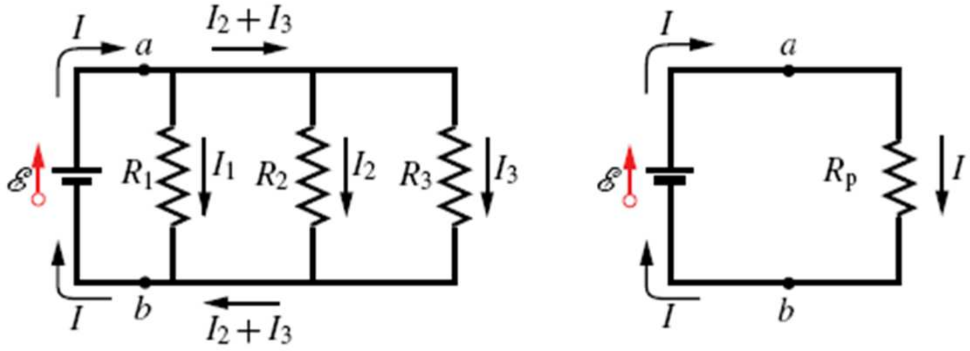
$$U = \mathscr{E} - Ir. \qquad U = \mathscr{E}\frac{R}{R + r}$$



Combination of resistors

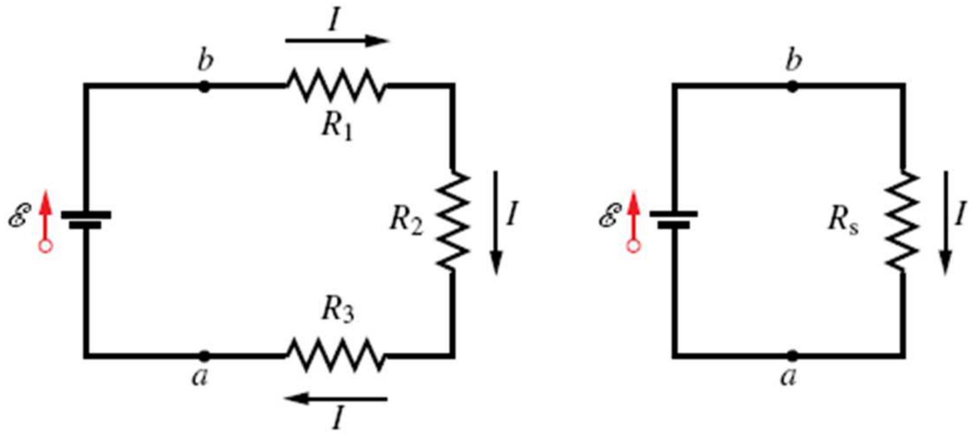
- Parallel

$$\frac{1}{R_p} = \sum_{j=1}^{n} \frac{1}{R_j}$$



- Series

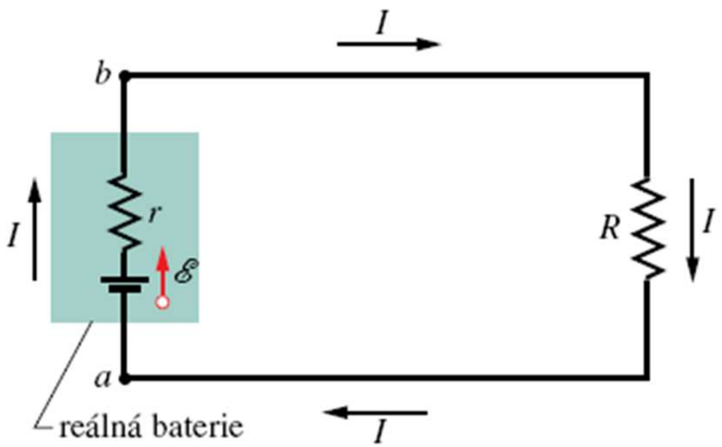
$$R_s = \sum_{j=1}^{n} R_j$$



Open and short circuit

- Open circuit – resistor $R \to \infty\ \Omega$
- Short circuit – resistor $\mathrm{R} \to 0\ \Omega$

Maximum current of real EMF $I_z = U_e / r$



Terminal voltage

$$U = \mathscr{E} - Ir.$$

Maximum power

$R = r$

Measurement of electrical voltage and current

- Voltage – voltmeter, basic range + series resistors
- Current – ammeter, basic range + shunt resistors connected in parallel

Current conductivity in metals

Free and mobile charge carriers available

Charge carriers drift at relatively low velocity within metal

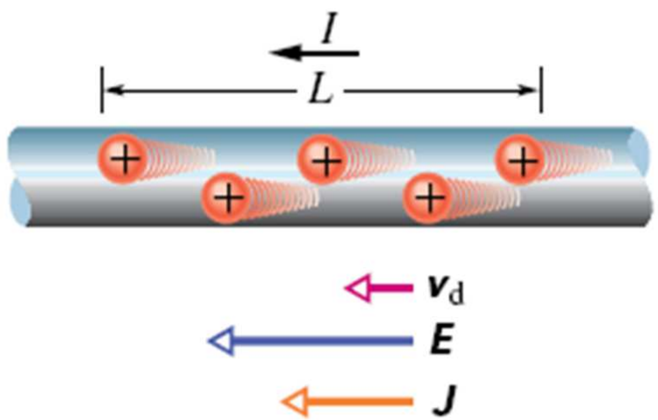
Low electric resistivity

Current density

$$I = \int \boldsymbol{J} \cdot d\boldsymbol{S}$$

Relationship between current density and drift velocity

Current by the charges mobility



$$\boldsymbol{J} = (ne)\boldsymbol{v}_d$$

$$Q = (nSL)e$$

$$t = \frac{L}{v_d}$$

$$I = \frac{Q}{t} = \frac{nSLe}{L/v_d} = nSev_d$$

$$v_d = \frac{I}{nSe} = \frac{J}{ne}$$

Current conductivity in liquids

Molecules are dissociated into ions – electrolyte

Electrolysis – dissociation of electrolyte into ions by electrical current

Positive ions attracted to the negative electrode (cathode), negative ions attracted to the positive electrode (anode)

Dynamic equilibrium between EMF and external voltage

Faraday's law of electrolysis

1. Mass of substance originated at the electrode is proportional to the electric charge transported within electrolyte.

$$m = kQ$$

2. Electrochemical equivalent is proportional to the molar mass and valence of ions.

$$k = \frac{1}{F}\frac{M_m}{v} \qquad F = N_A e = 9.648 \cdot 10^4\, Cmol^{-1}$$

Current conductivity in gasses

Lack of charge carriers in gasses - ionization:

- Heating
- Radiation (UV, X-Ray, light, …)
- Radioactivity
- Bombardment by electrons and ions
- Electric, magnetic field

Discharge in gas

Avalanche-like current increase during discharge in gas.

Ionization work = energy needed for the free charge carriers creation (minimum for valency electrons)

Secondary ionization by collisions of electrons and ions – ignition voltage, discharge continuous above ignition voltage

Types of discharge

- Glowing discharge (low pressure)
- Corona discharge (normal pressure)
- Spark discharge (low current density, high voltage)
- Arch discharge (high current density, low voltage)

Current conductivity in vacuum

- Photoemission – charge carriers generated from the metal surface by radiation impact (e.g. photoelectric effect for UV radiation)
- Thermoemission – kinetic collisions of electrones at the surface of hot electrode (cathode)

Literature

Pictures used from the books:

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Velká ilustrovaná encyklopedie, Fyzika, Chemie, Biologie, Fragment, Havlíčkův Brod 2000

and material data from tables:

BROŽ, J., ROSKOVEC, V., VALOUCH, M.: Fyzikální a matematické tabulky, SNTL Praha 1980