Electromagnetic spectrum. Quantum optics

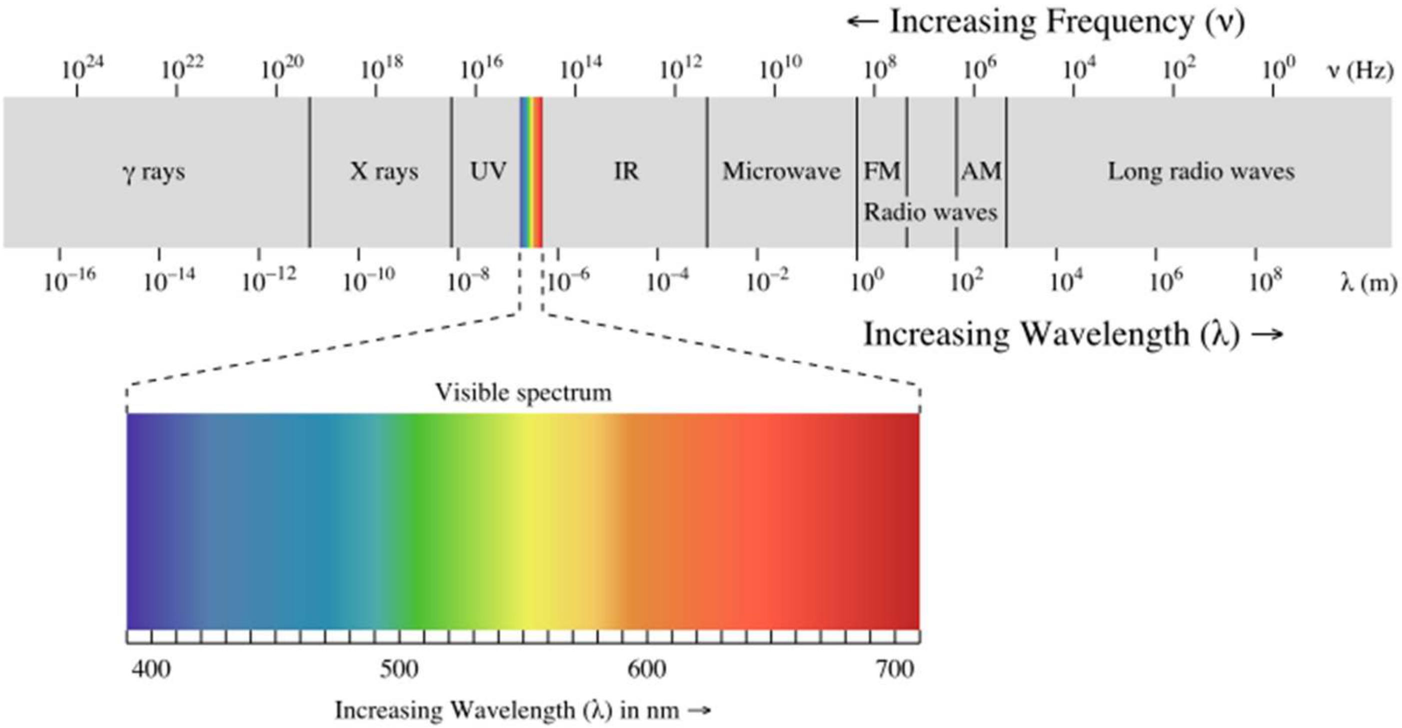
Electromagnetic spectrum, absorption of radiation, light sources, ultraviolet and infrared radiation, X-Ray, gamma radiation. Fundamental of quantum optics. Planck's blackbody radiation law, photoelectric effect, reverse photoelectric effect.

Physical optics

Energy transfer at light propagation, processes of emission and absorption of radiation, spectral characteristics of radiation

- Photometry
- absorption and emission of light
- spectral characteristics of radiation
- various wavelengths of radiation – UV, IR, X-Ray, gamma

Electromagnetic spectrum



Electromagnetic spectrum

	f [Hz]	λ[m]	E[eV]
Radio waves	$3000\text{-}3\cdot10^{8}$	$10^{5}\text{-}1$	$10^{-11}\text{-}10^{-6}$
Microwave	$3\cdot10^{11}$	10^{-3}	$10^{-6}\text{-}10^{-3}$
Infrared	$4\cdot10^{14}$	$750\cdot10^{-9}$	$10^{-3}\text{-}1.6$
Light	$7.5\cdot10^{14}$	$400\cdot10^{-9}$	1.6 - 3
UV	$6\cdot10^{15}$	$50\cdot10^{-9}$	3 - 25
X-ray	10^{22}	10^{-14}	$50 - 10^{8}$
γ-radiation	$> 10^{18}$	$< 10^{-10}$	>10000

Quantum optics

Quantum properties of light, its emission by photons etc.

- Blackbody radiation law
- Photoelectric effect
- Reverse photoelectric effect – X-Ray radiation

Blackbody

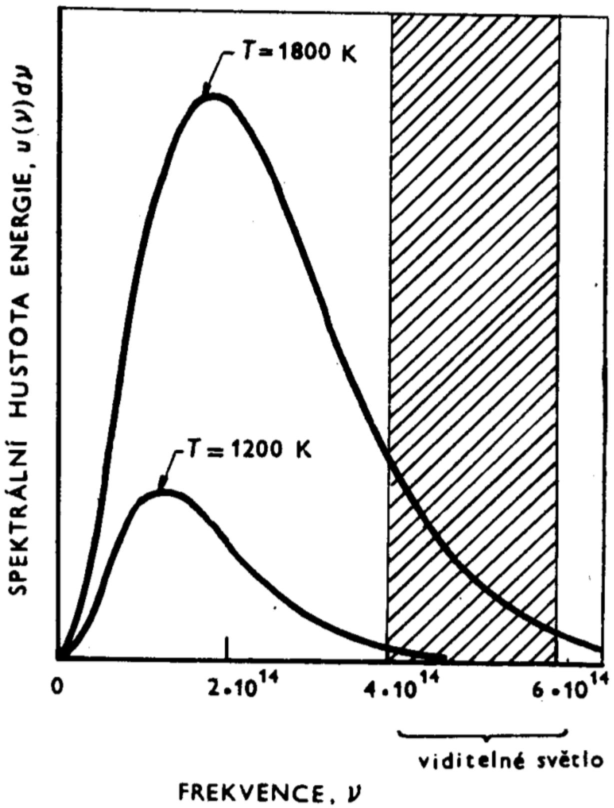
Body which emits all absorbed energy into radiation

Sun is blackbody (6000K) – not necessarily black colour!

Blackbody is characterized by equilibrium of absorbed and emitted energy, but spectra of absorbed and emitted energy may differ!

Blackbody radiation

Planck – quantum hypothesis of radiation emission in photons with quantized energies *hf*, 2*hf*, 3*hf*, …



Classical Rayleigh-Jeans radiation law

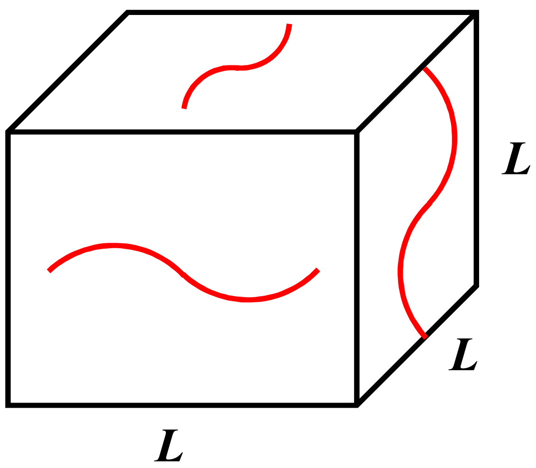
$$u(f)df = \frac{8\pi k_B T}{c^3} f^2 df$$

suggested „ultraviolet catastrophy“

$u(f)\mathrm{d}f \rightarrow \infty$ pro $f \rightarrow \infty$

Rayleigh-Jeans radiation law

Classical wave calculations of power density

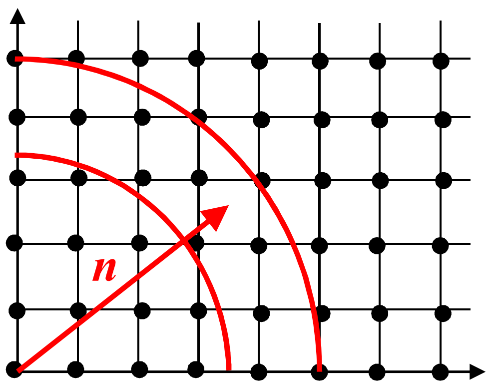


Standing waves in axes directions

$$L = n_x \frac{\lambda_x}{2}, L = n_y \frac{\lambda_y}{2}, L = n_z \frac{\lambda_z}{2}$$

Standing wave in arbitrary direction

$$n^2 = n_x^2 + n_y^2 + n_z^2 = \left(\frac{2L}{\lambda_x}\right)^2 + \left(\frac{2L}{\lambda_y}\right)^2 + \left(\frac{2L}{\lambda_z}\right)^2 = \left(\frac{2L}{\lambda}\right)^2$$



Total number of standing waves in a cavity

= number of grid points inside the sphere

$$n = \frac{2L}{\lambda}, dn = -\frac{2L}{\lambda^2} d\lambda$$

$$-n(\lambda)d\lambda = -\frac{1}{8} 4\pi n^2 dn = \frac{4\pi L^3}{\lambda^4} d\lambda$$

Rayleigh – Jean radiation law

Two possible transversal polarization directions of photons within narrow wavelength range

$$n(\lambda)d\lambda = \frac{8\pi L^3}{\lambda^4} d\lambda$$

Energy density $u(\lambda)d\lambda = 2\frac{1}{2}k_B T \frac{1}{L^3} n(\lambda)d\lambda = \frac{8\pi k_B T}{\lambda^4} d\lambda$

Expressed using frequency $\lambda = c/f$

$$u(\lambda)d\lambda = \frac{8\pi k_B T}{c^3} f^2 df$$

Planck's blackbody radiation law

UV catastrophe solved by M.Planck – quantum emission hypothesis – photons emitted with energies *hf*, 2*hf*, 3*hf*, …

Mean photon energy

$$\overline{E} = \frac{\sum_{n=0}^{\infty} nhf e^{-nhf/k_B T}}{\sum_{n=0}^{\infty} e^{-nhf/k_B T}}$$

$$\sum_{n=0}^{\infty} e^{-nhf/k_B T} = \frac{1}{1 - e^{-hf/k_B T}}$$

$$\sum_{n=1}^{\infty} nhf e^{-nhf/k_B T} = -hf \frac{d}{dx}\left[\sum_{n=1}^{\infty} e^{-nx}\right] = -hf \frac{d}{dx}\left[\frac{e^{-x}}{1 - e^{-x}}\right]$$

$$x = hf / k_B T$$

Planck's blackbody radiation law

Mean energy

$$\overline{E} = \frac{hf}{e^{hf/k_B T} - 1}$$

Energy density

$$u(f)df = \frac{8\pi f^2}{c^3} \frac{hf}{e^{hf/k_B T} - 1} df$$

$$u(\lambda)d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/k_B T\lambda} - 1} d\lambda$$

Blackbody

Wien's law

$$\lambda_{\max} = \frac{b}{T}, \quad b = 2{,}896 \cdot 10^{-3}\, mK$$

Wavelength of maximum emitted energy changes inversely proportional to the temperature

$$\frac{du(\lambda)}{d\lambda} = 0 \rightarrow \frac{hc}{k_B T \lambda_{\max}} = 4.965$$

Example: $\lambda_{\max} = 800nm, \quad T = 3620K$

Blackbody

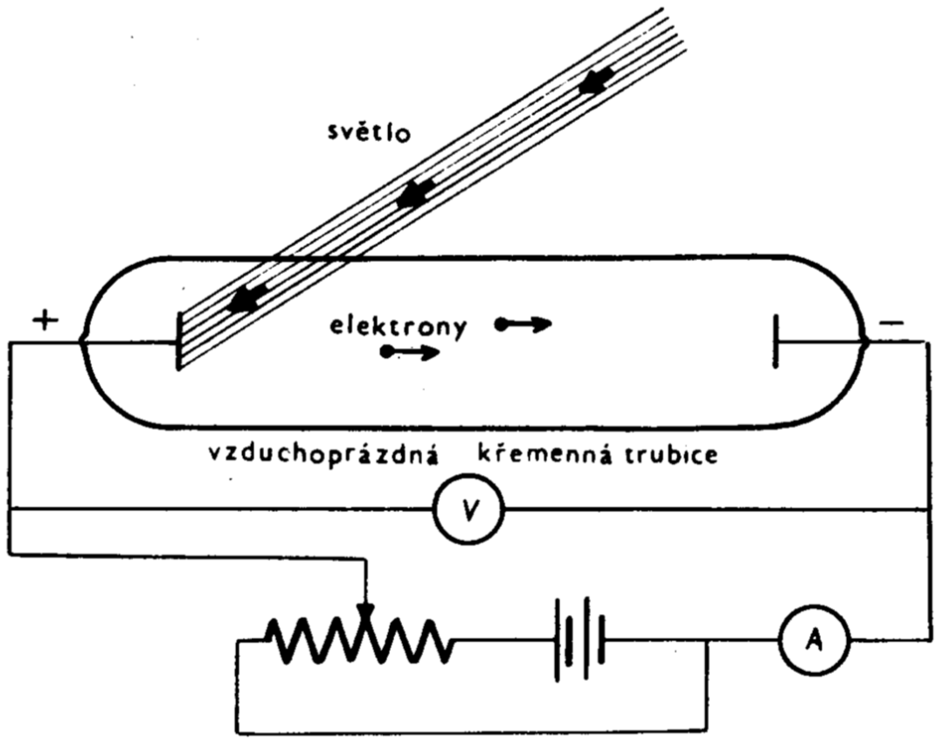
Stefan-Boltzmann's equation – the rate at which an object radiates energy (within all wavelengths) has been found to be proportional to the fourth power of temperature.

$$u = \int_0^{+\infty} u(f)df = \frac{8\pi^5 k_B^4}{15c^3h^2}T^4$$

$$M_e = \sigma T^4, \quad \sigma = 5{,}67 \cdot 10^{-8} Wm^{-2}K^{-4}$$

Photoelectric effect

Light absorption by metals tends to the electron emission from the surface



accelerated electrons
jump over potential barrier and cause electric current in circuit

Photoelectric effect

Expected

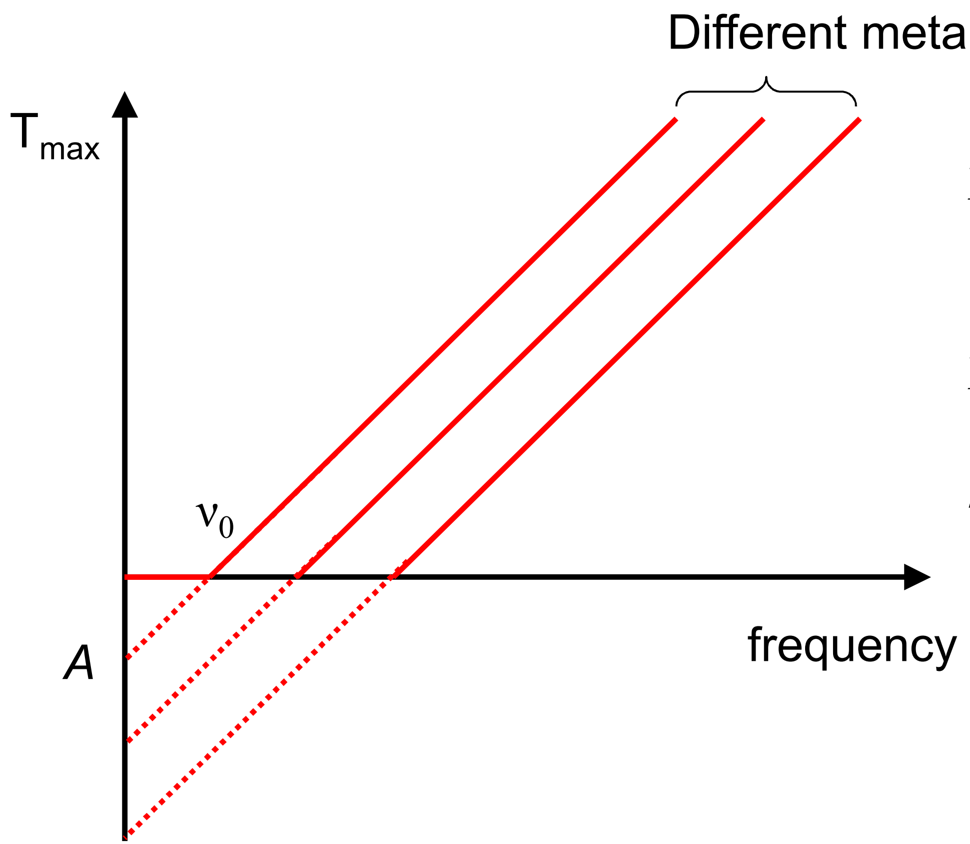
- Electrons should be emitted at any light intensity, independently from light wavelength
- Energy of emitted electrons depends on light intensity

Observed

- Energy spectrum of photoelectrons is independent from light intensity
- Average photoelectron energy is independent from light intensity
- Photoelectron energy depends on light frequency
- Maximum photoelectron energy increases linearly with light frequency

Photoelectric effect

Planck's constant *h*



$$T_{\max} = eV^0$$

$$T_{\max} = h\nu - h\nu_0$$

$$h = 6.63 \cdot 10^{-34}\, Js$$

Photoelectric effect

Theory by A.Einstein – Nobel prize 1921

$$h\nu = T_{\max} + h\nu_0$$

Work function $A = h\nu_0$

M.Planck (1900) – radiation emission is quantized by energy $h\nu$ and its multiples (no conflict with wave theory)

A.Einstein – radiation absorption is also quantized by the same quanta as emitted (conflict with wave theory)

Thermoemission

Electrons are emitted from metal surface by thermal energy

Electron is ejected if its energy is higher than work function

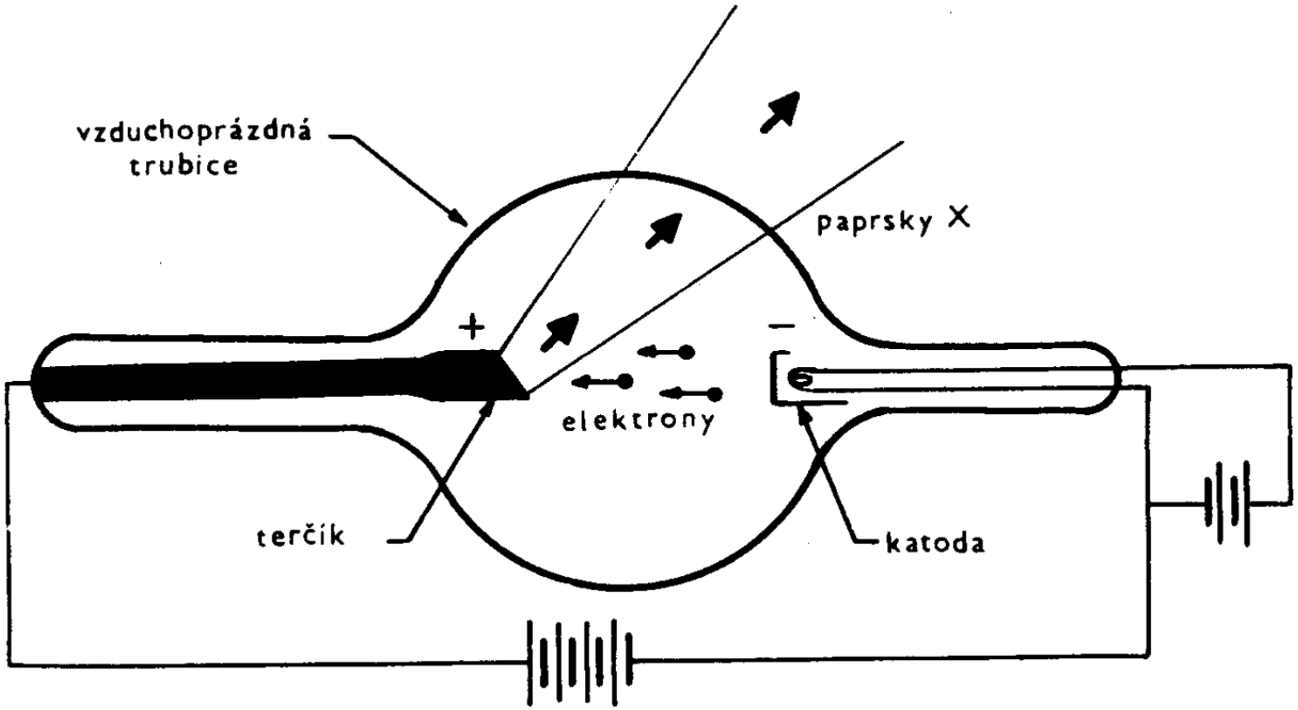
Vacuum tube emission of electrons

Example: Work function for potassium is 2eV, radiation of 350nm wavelength emits photoelectrons with energies

$$T_{\max} = h\nu - A = 1.6eV$$

X-Ray
(Brehmsstrahlung)

Reverse photoelectric effect, W.C.Röntgen (1895), 1st Nobel prize 1901



X-Ray

Rays are not inclined in electric or magnetic field → particles are not electrically charged, i.e. possibly photons of electromagnetic spectrum

No ray refraction observed

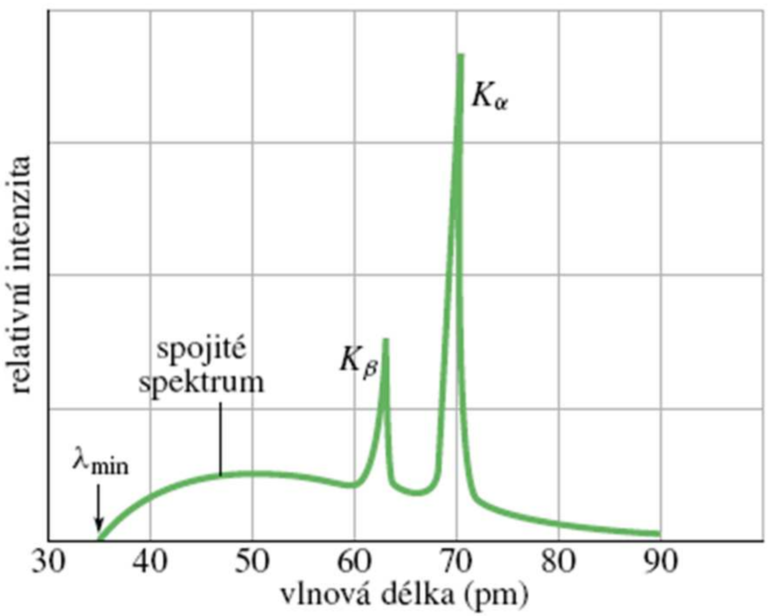
Radiation intensity increases with number of electrons, penetration depth increases with electron speed (acceleration voltage for electrons)

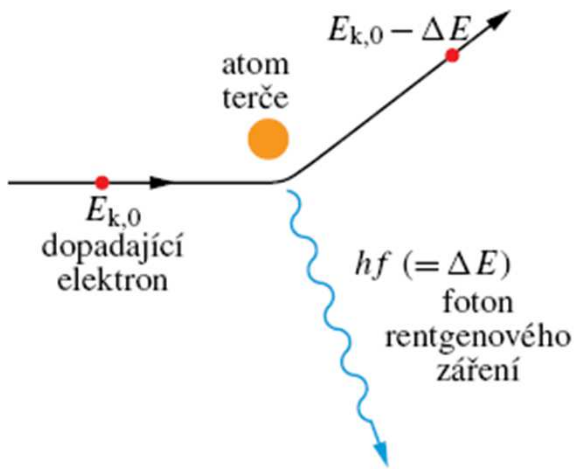
X-Ray

Continuous spectrum and characteristic X-rays

Example: Molybdenum target

$$\lambda_{\min} = \frac{hc}{E_{k,0}}$$



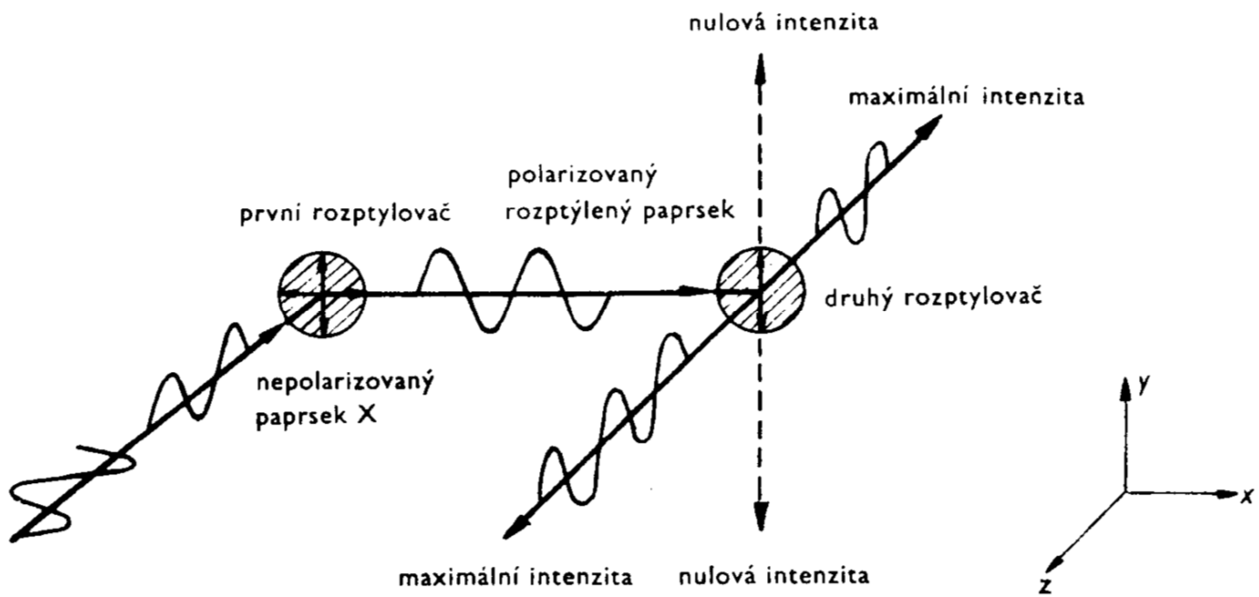


Barkla’s experiment

Proof of X-Ray polarization, i.e. electromagnetic nature of X-Ray

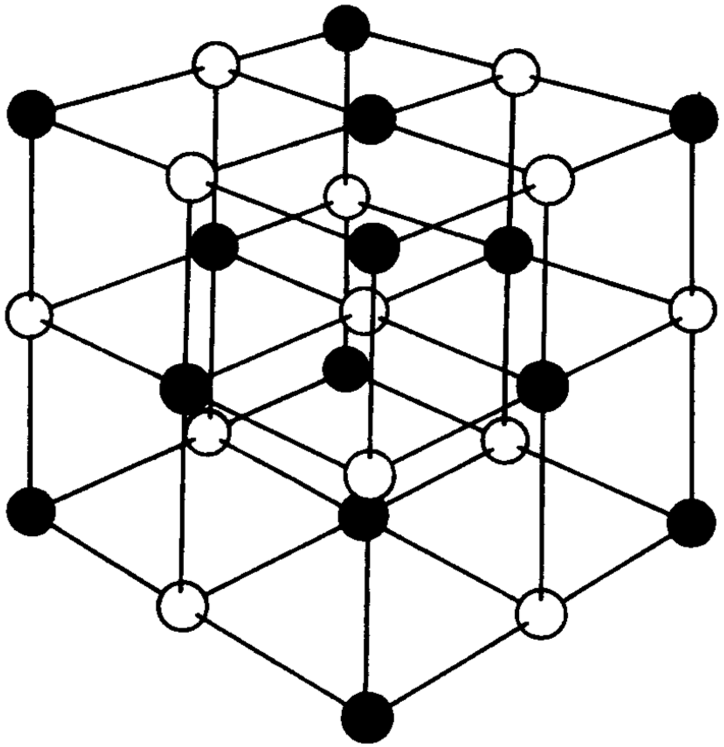
Experiment – Ch.G.Barkla 1906, double scattering of X-Ray

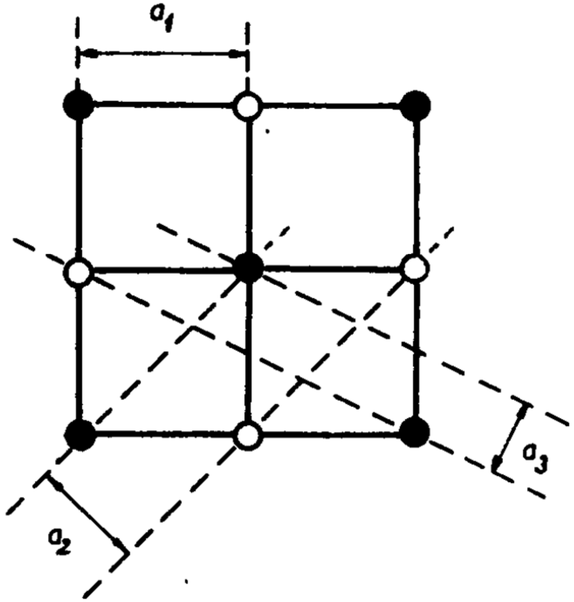
Nobel prize 1917



Diffraction of X-Ray

Max von Laue, 1912 – diffraction on crystal lattice, interatomic distances correspond to X-Ray wavelengths

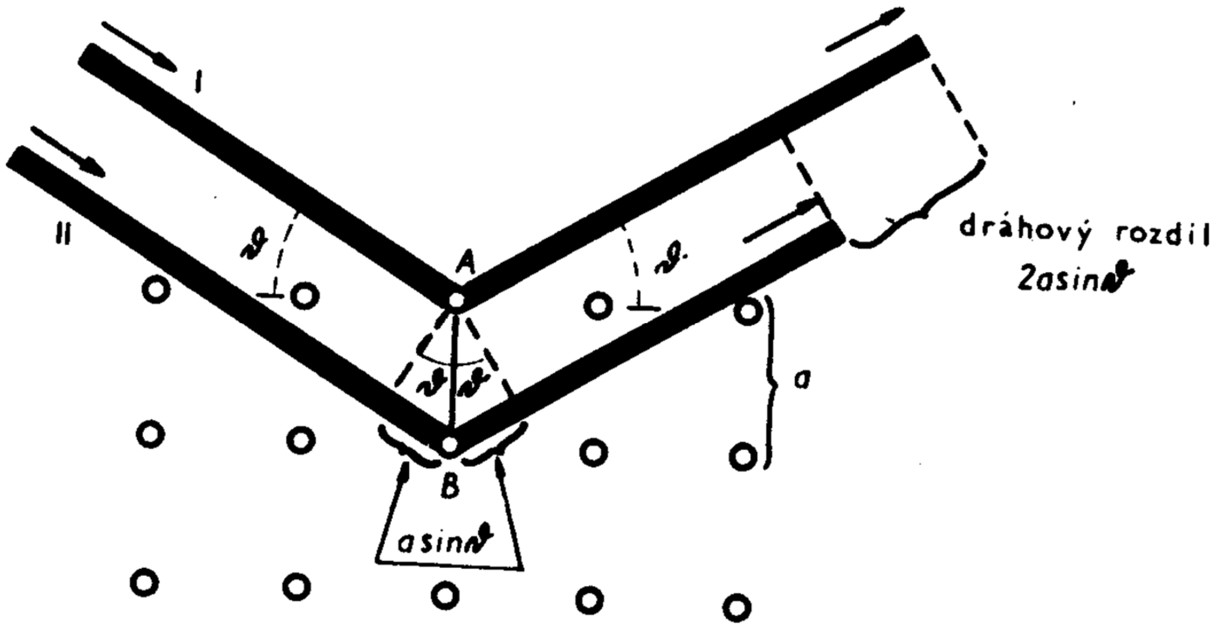




Diffraction of X-Ray

X-Ray radiation stimulates charged ions in lattice, atoms emit electromagnetic radiation, interference by regularly positioned atoms in crystallographic lattice

1913, W.L.Bragg



Bragg's condition

$$2a\sin\vartheta = n\lambda,$$

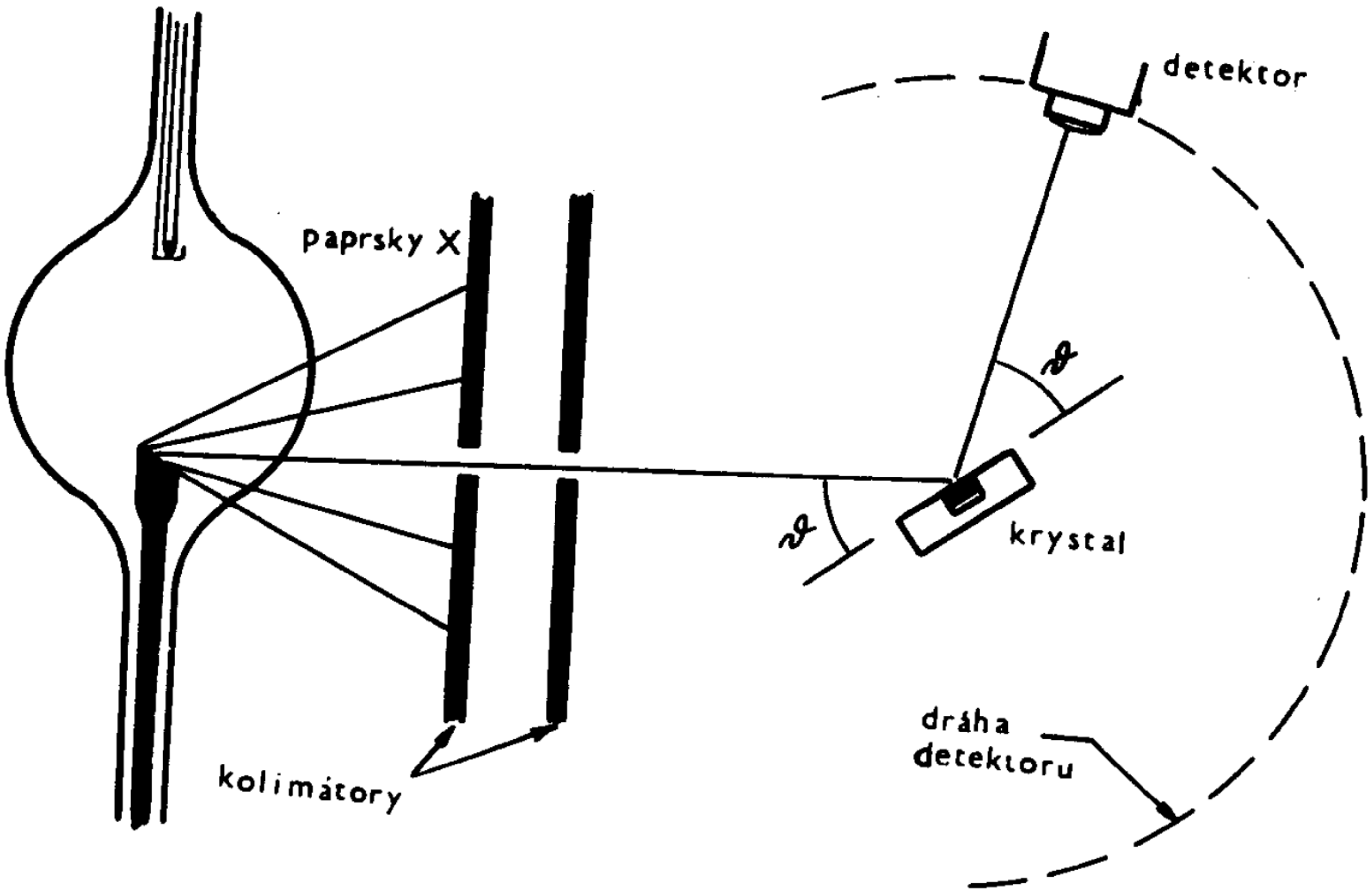
$$n = 1,2,3,...$$

X-Ray diffraction

- Monochromatic radiation, rotation of crystal to fulfill Bragg's condition, e.g. for powder samples

- „white“ X-Ray radiation, fixed crystal position, Bragg's condition is fulfilled in different directions for different wavelengths – Laue-gramm

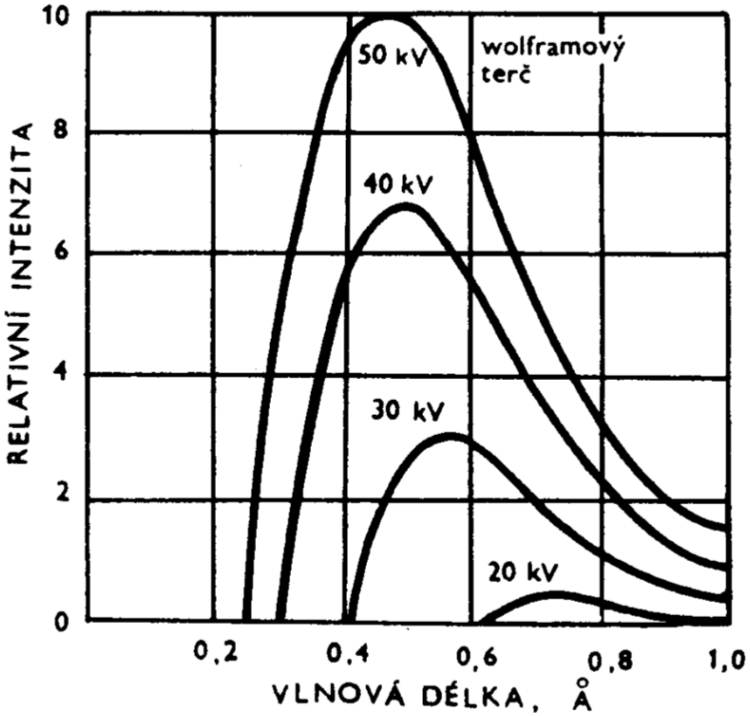
X-Ray spectrometer



X-Ray spectrum - continuous

Decelerated electrons emit elmg. radiation – continuous spectrum

X-Ray intensity increases for higher acceleration voltage, intensity maximum shifts to shorter wavelengths



minimum wavelength exists, which depends on acceleration voltage

$$eU = \frac{hc}{\lambda_{\min}} = h\nu_{\max}$$

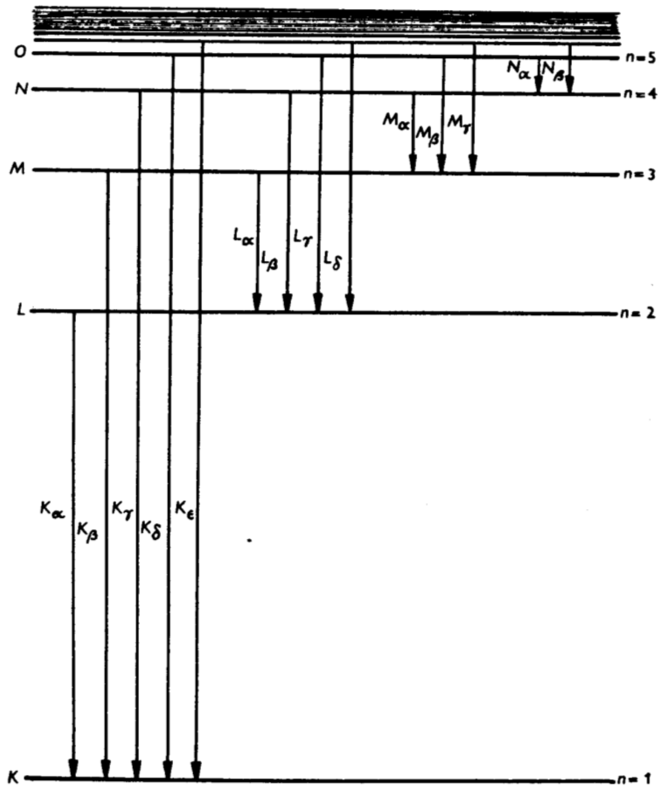
Duane – Hunt's law

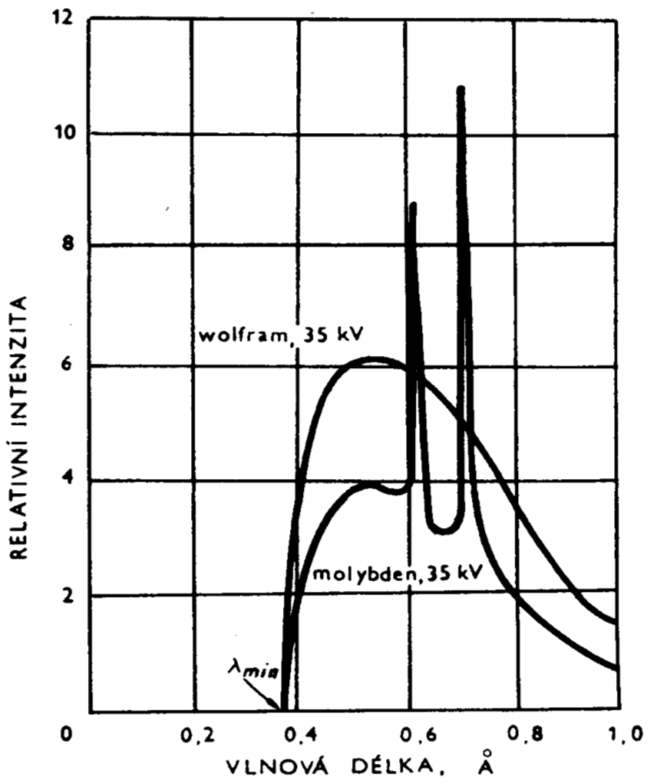
$$\lambda_{\min} = \frac{hc}{eU} = \frac{1.24 \cdot 10^{-6} Vm}{U}$$

Example: U=50kV, $\lambda_{\min}$=0.25·10^{-10}m

X-Ray spectrum - characteristic

Energy of absorbed electron stimulates inner energy electrons of target material (e.g. molybdenum, tungsten)





Literature

Pictures used from the books:

HALLIDAY, D., RESNICK, R., WALKER, J.: Fyzika (část 4 – Elektromagnetické vlny – Optika – Relativita, část 5 – Moderní fyzika), Vutium, Brno 2000

A.Beiser: Úvod do moderní fyziky, Academia Praha 1975