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# Transport processes in rock and soil

Lecture 3

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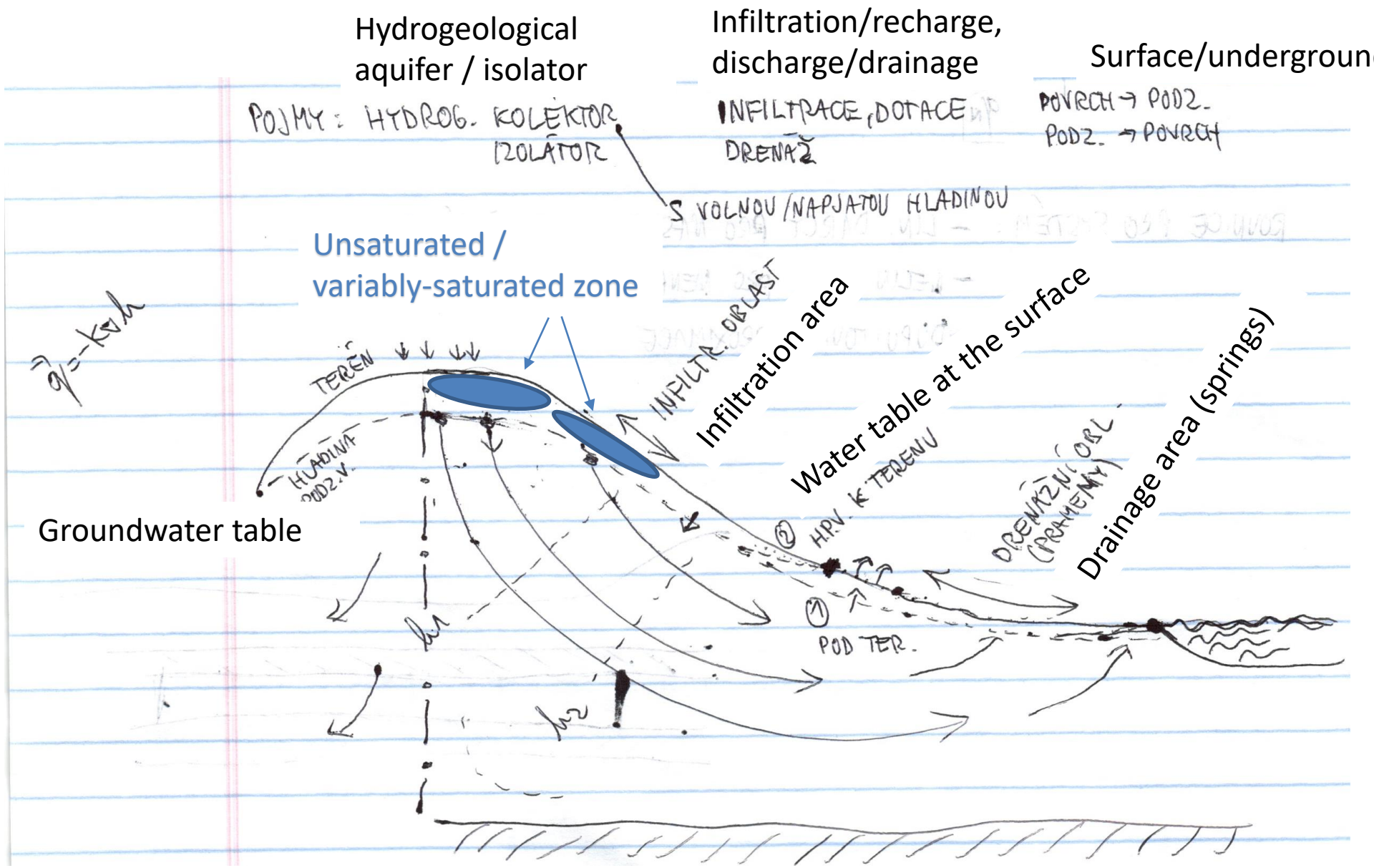
# Topic introduction

- Quantities and equations for variably saturated porous medium
  - General case

- How related to previous?

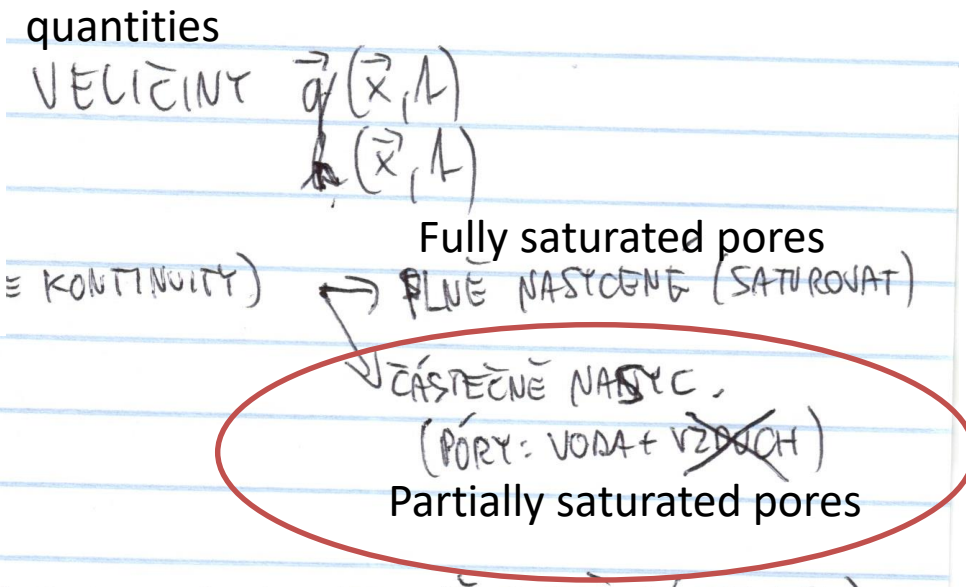
# Reminder – natural groundwater systems



# Balance equation - revisited

System of equations
- Darcy's Law
- Balance equation (mass conservation principle)

quantities




Fully saturated pores

Partially saturated pores

Balance: change inside the volume vs flux across the boundary

Volumetric source

Sources/sinks

mass

Transform to volume integral

# Unsaturated/variably-saturated medium

Quantity: „saturation“

CZ: „nasycení“

$$S = \frac{\text{Fluid volume}}{\text{Pore volume}}$$

$0 \leq S \leq 1 \quad \dots \quad S(\vec{x}, t)$

BILANCE $\int_V \rho_n \cdot S \, dV$

$\downarrow$

$\frac{d}{dt}(\rho_n S) + \nabla \cdot (\rho \vec{q}) = P_\rho$

$\rho_n \frac{dS}{dt} + \nabla \cdot (\rho \vec{q}) = P$

Water amount ← SATURACE $S$

Volumetric ratio $\theta = \frac{V_w}{V}$

$1 \leq \theta \leq n$

$S = \frac{\theta}{n} = \frac{\theta}{\theta_s}$

Assumption $\rho = \text{konst}$, $n = \text{konst}$

New constitutive relation needed

(?) $S \sim h$

Capillarity phenomena

$$n\frac{dS}{dt} + \nabla\cdot(\vec{q}) = P$$

? $S \sim h$

Capillarity

… Retention curve

$S(h)$

$$n\left(\frac{dS}{dh}\cdot\frac{dh}{dt}\right) + \nabla\cdot\vec{q} = P$$

$C(h)$

Capacity

Water amount

- SATURACE $S$
- Volumetric ratio $\theta = \frac{V_w}{V}$

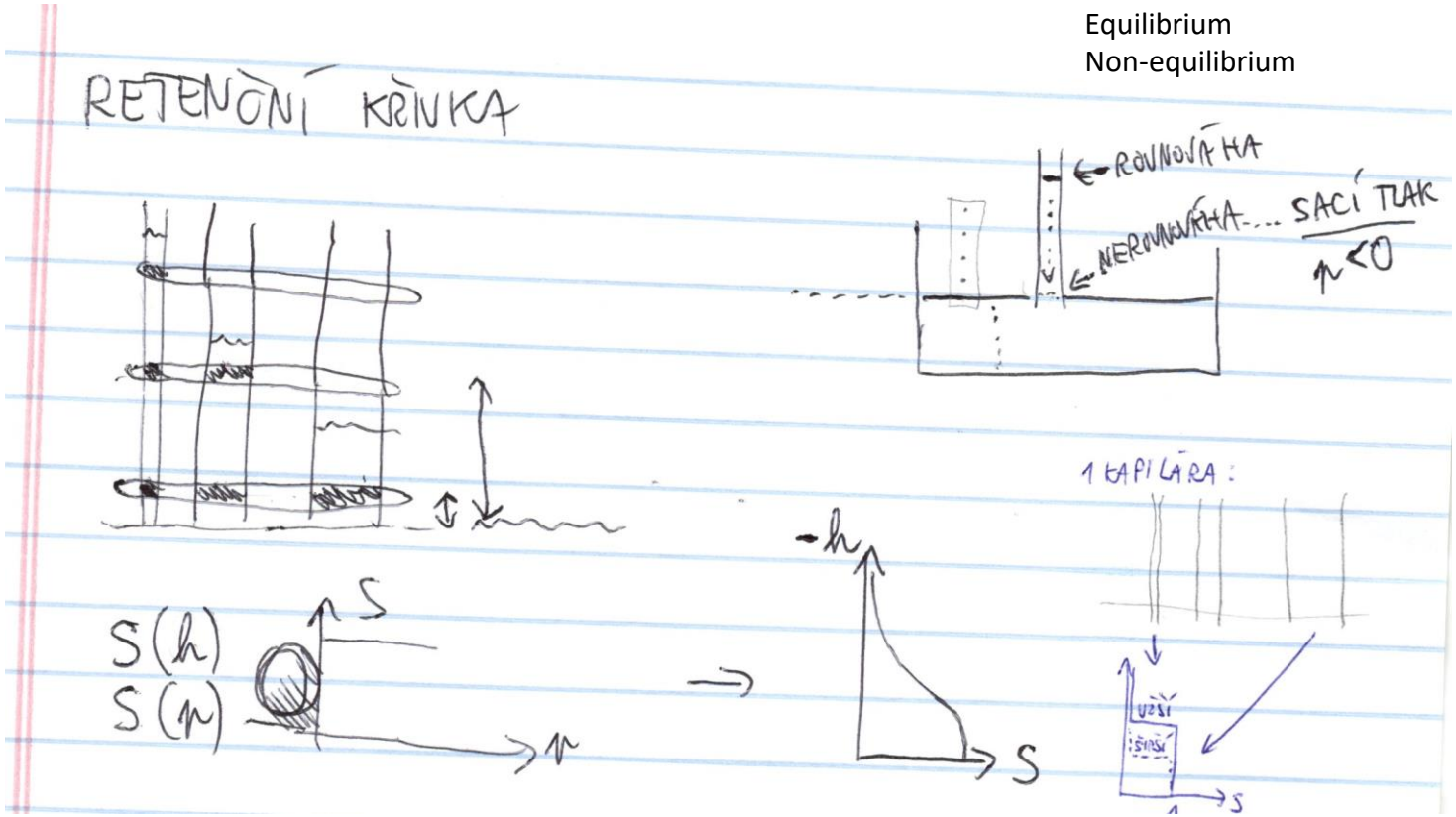
$1 \le \theta \le n$

$$S = \frac{\theta}{n} = \frac{\theta}{\theta_s}$$

$S = 1 \dots p \ge 0$

$S < 1 \dots p < 0$

# Retention curve



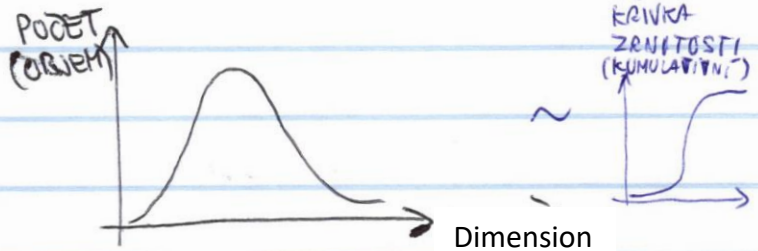

Equilibrium
Non-equilibrium

Suction …
“negative
pressure”

Wider
Vs
narrower

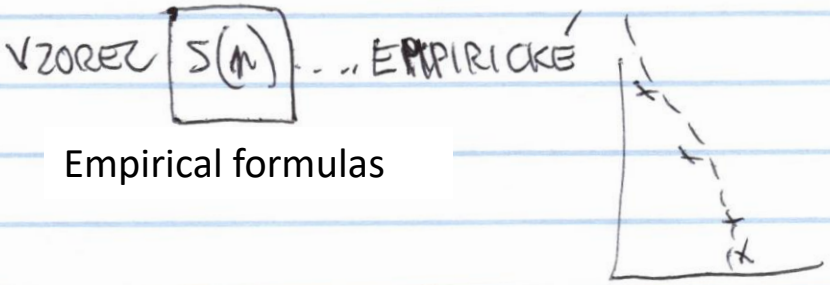
# Curve shape related to statistical distribution of pore sizes

Count / volume




Dimension

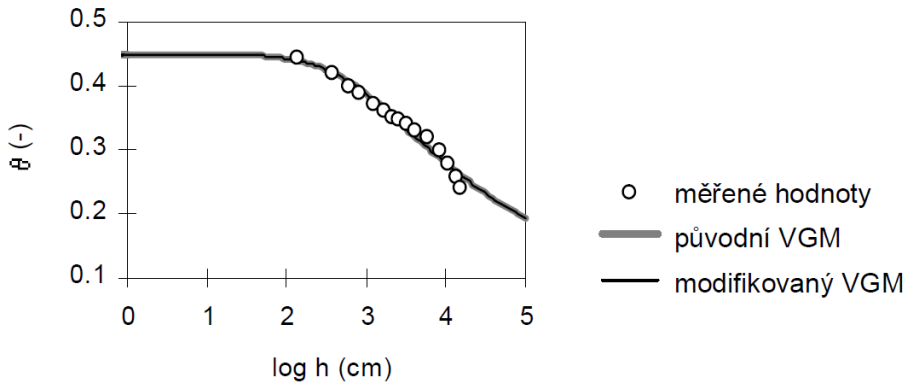
Granulometry – particle size distribution



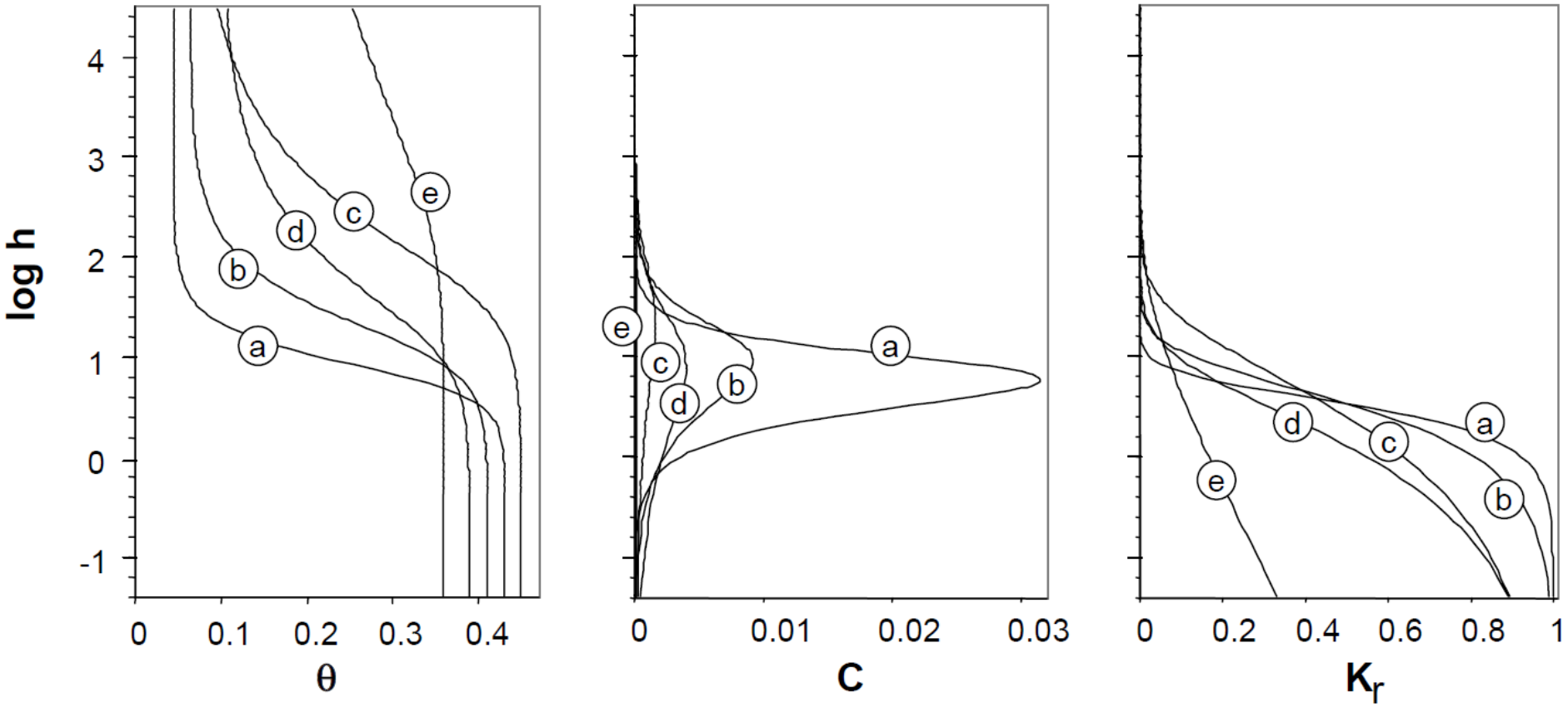

Empirical formulas

VAN-GENUCHTEN

$$S(h) = \frac{1}{\left(1 + \left(\frac{h}{h_0}\right)^n\right)^m}$$



# Various soils



DARCYHO Z.

$\vec{q} = -K \nabla h$ NASYC.

NENAS. $\vec{q} = -S \cdot K \nabla h$ ?? ← Changed water volume
X
ZÁVI Dependence on
"connectivity" of filled pores

DARCY – BUCKINGHAMŮV ZÁKON

$$\vec{q} = -K_r(S) \cdot K_s \nabla h$$

K for saturated

Relative hydr.c.

Empirical

PŘ. IRMAY $K_r(S) = S^3$

+RETENČNÍ $K_r(S(h))$

Complete equation

NEZN. $h(\vec{x}, t)$

$$n\, C(h) \frac{\partial h}{\partial t} - \nabla \cdot \left( K_r(S(h))\, K_s \nabla h \right)$$

Strongly nonlinear

0 AŽ MAX

"Degenerated (parabolic) equation – parameter value controls the equation type

# Finished

- Next: solute transport (dissolved mass)