



EUROPEAN UNION  
European Structural and Investment Funds  
Operational Programme Research,  
Development and Education



Preparation of the international Ph.D. study programme “Environmental Engineering” CZ.02.2.69/0.0/0.0/16\_018/0002660

# Transport processes in rock and soil

## Lecture 3

Doc. Ing. Milan Hokr, Ph.D.  
Technical University of Liberec

# Topic introduction

- Quantities and equations for variably saturated porous medium
  - General case
- How related to previous?

# Reminder – natural groundwater systems

Hydrogeological  
aquifer / isolator

Infiltration/recharge,  
discharge/drainage

Surface/underground

POJMY: HYDROG. KOLEKTOR  
IZOLATOR

INFILTRACE, DOTACE  
DRENÁŽ

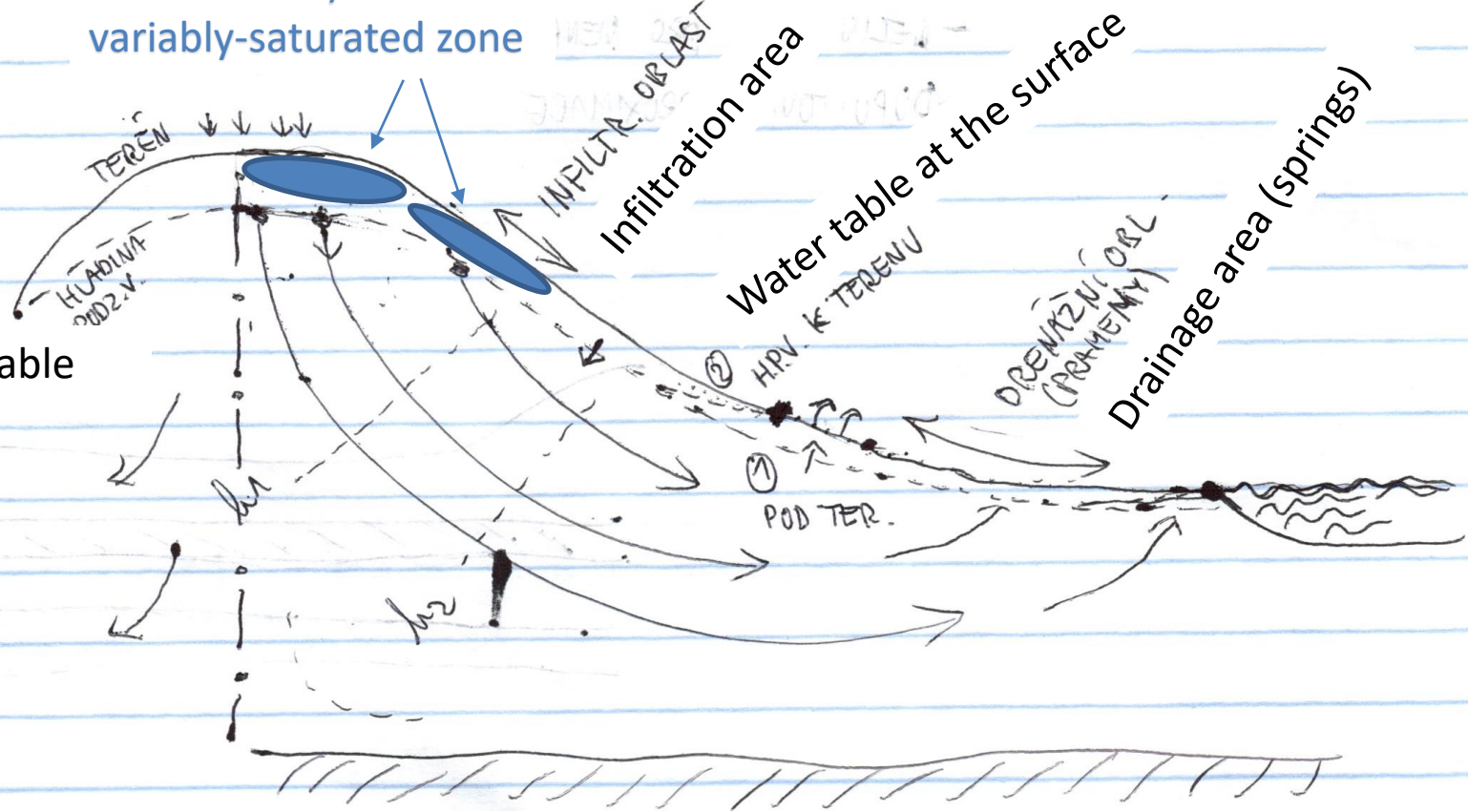
POVRCH → PODZ.  
PODZ. → POVRCH

Unsaturated /  
variably-saturated zone

S VOLNOU/NAPJATOU HLADINOU

$q = -ksh$

Groundwater table



# Balance equation - revisited

System of equations

- Darcy's Law
- Balance equation (mass conservation principle)

quantities

VELICINY  $\vec{q}(\vec{x}, t)$   
 $\vec{h}(\vec{x}, t)$

≡ KONTINUITA

Fully saturated pores

PLNĚ NASYCENÉ (SATUROVANÉ)

ČÁSTEČNĚ NASYCENÉ

(PÓRY: VODA + VZDUCH)

Partially saturated pores

BALANCE



ZMĚNA V OBJEMU ~ TOK PŘES STĚNU (HRANICI)

Balance: change inside the volume vs flux across the boundary

$$\frac{d}{dt} \int_V \rho \cdot m \, dV = - \int_{\partial V} \rho \vec{q} \cdot d\vec{S} + \int_V P \rho \, dV$$

HMOTNOST

mass

OBJEMOVÝ (GAUSS.V.)

Transform to volume integral

ZDROJE/PROPADY

Sources/sinks

Volumetric source

P objemový zdroj  
 $[m^3/m^3/s]$

# Unsaturated/variably-saturated medium

Quantity: „saturation“  
CZ: „nasyčení“

$$S = \frac{\text{Fluid volume}}{\text{Pore volume}}$$

$$0 \leq S \leq 1 \quad \dots \quad S(\vec{x}, t)$$

BILANCE  $\int_V \rho_m \cdot S \, dV$

Water amount  $\left\{ \begin{array}{l} \text{SATURATION } S \\ \text{Volumetric ratio } \theta = \frac{V_{nr}}{V} \end{array} \right.$

$$\frac{d}{dt} (\rho_m S) + \nabla \cdot (\rho \vec{q}) = P \rho$$

Assumption

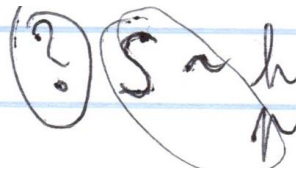
$$\rho = \text{konst} \\ m = \text{konst}$$

$$S = \frac{\theta}{m} = \frac{\theta}{\theta_s}$$

$$1 \leq \theta \leq m$$

$$\rho_m \frac{dS}{dt} + \nabla \cdot (\rho \vec{q}) = P$$

New constitutive relation needed



Capillarity phenomena

$$\rho n \frac{dS}{dz} + \nabla \cdot (\rho \vec{q}) = P$$

(?)  $S \sim \frac{h}{r}$

Capillarity  
... Retention curve

$S(h)$

$$n \left( \frac{dS}{dh} \right) \cdot \frac{dh}{dz} + \nabla \cdot \vec{q} = P$$

$C(h)$

Capacity

Water amount

SATURATION  $S_s$   
Volumetric ratio

$$\theta = \frac{V_{\text{water}}}{V}$$

$$1 \leq \theta \leq n$$

$$S = \frac{\theta}{n} = \frac{\theta}{\theta_s}$$

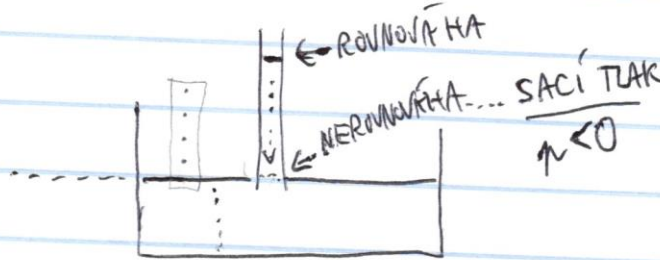
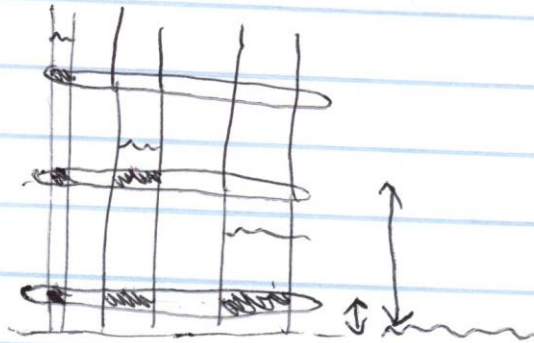
$$S=1 \dots \mu \geq 0$$

$$S < 1 \dots \mu < 0$$

# Retention curve

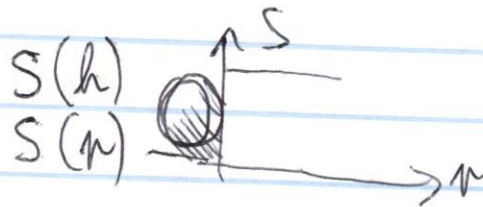
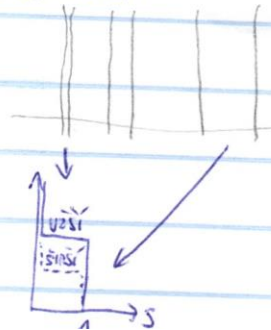
RETENČNÍ KŘIVKA

Equilibrium  
Non-equilibrium



Suction ...  
"negative pressure"

1 KAPILÁRA :

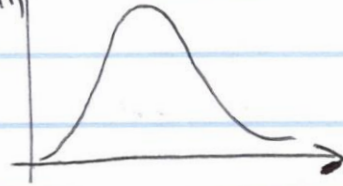


Wider  
Vs  
narrower

# Curve shape related to statistical distribution of pore sizes

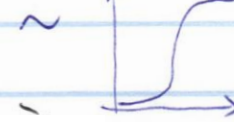
Count / volume

POČET  
(OBJEM)



Dimension

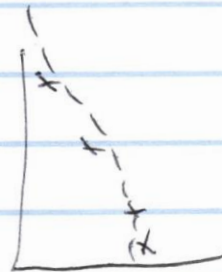
KŘIVKA  
ZRNITOSTI  
(KUMULATIVNÍ)



Granulometry – particle size distribution

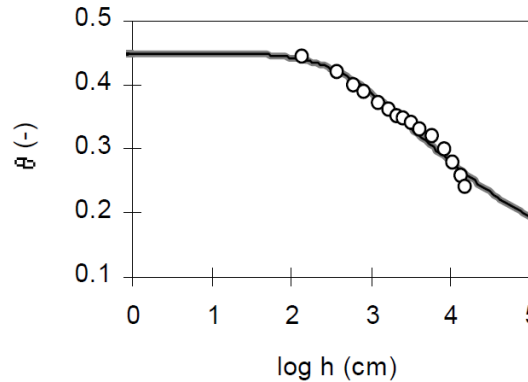
VZOREK  $S(r)$  ... EPIPIRICKÉ

Empirical formulas



VAN-GENUCHTEN

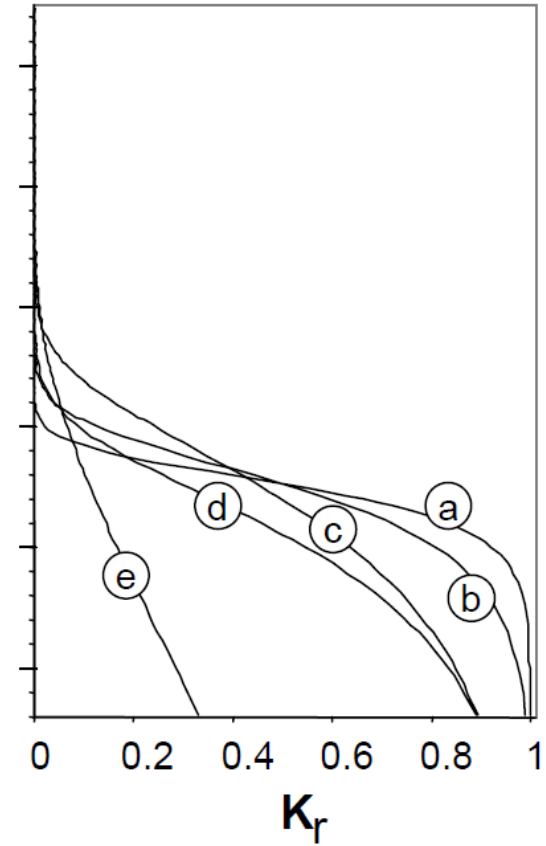
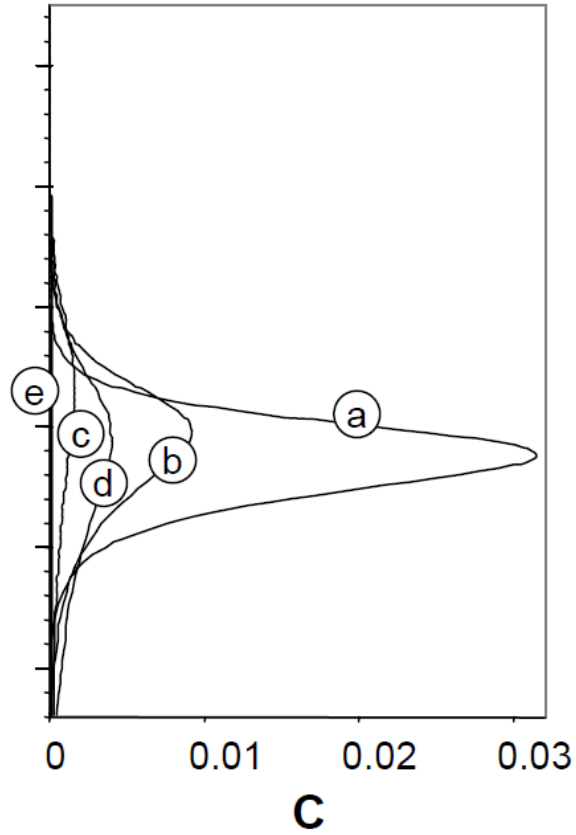
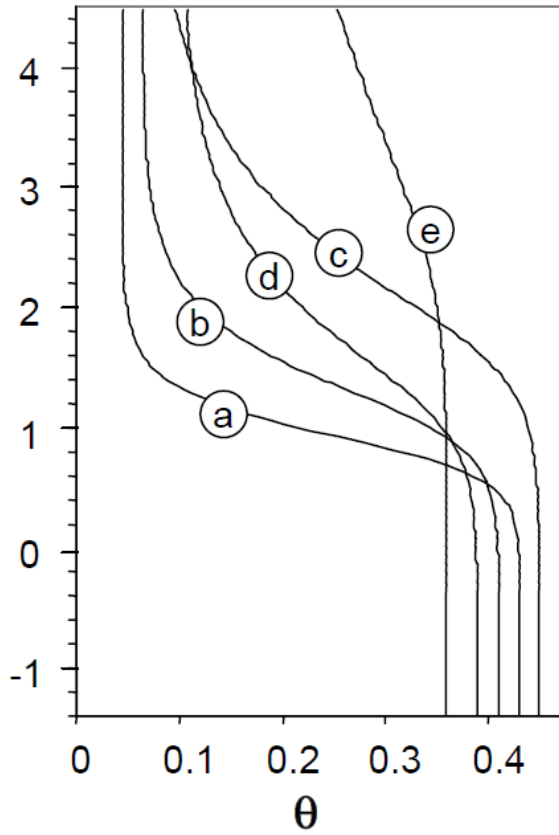
$$S(r) = \frac{1}{\left(1 + \left(\frac{r}{r_0}\right)^m\right)^m}$$



- měřené hodnoty
- původní VGM
- modifikovaný VGM



# Various soils



DARCYHO Z.

$$\vec{q} = -K \nabla h \quad \text{NASYC.}$$



NEENAS.  $\vec{q} = -S \cdot K \cdot \nabla h$  ??

← Changed water volume X  
 ZÁVÍ Dependence on  
 "connectivity" of filled pores

DARCY -  
 -BUCKINGHAMOV  
 ZÁK

$$\vec{q} = -K_r(S) \cdot K_s \nabla h$$

Relative hydr.c.

K for saturated

Empirical

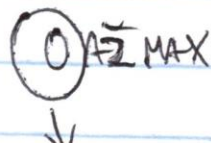
PR. IRMAY  $K_r(S) = S^3$   
 +RETENOVÍ  $K_r(S(h))$

Complete equation

NEZN.  $h(\vec{x}, t)$

$$n C(h) \frac{\partial h}{\partial t} - \nabla \cdot (K_r(S(h)) K_s \nabla h)$$

Strongly nonlinear



"Degenerated (parabolic) equation - parameter value controls the equation type"

# Finished

- Next: solute transport (dissolved mass)