1c

$$\frac{1}{x-2} - \frac{x-2}{x+4} = \frac{6}{\underset{(x+4)(x-2)}{x^2+2x-8}} - 1 \quad / \cdot (x+4)(x-2)$$

Podmínky: $x-2 \neq 0 \wedge x+4 \neq 0 \wedge 1 \cdot x^2 + 2x - 8 \neq 0$

$x \neq 2 \qquad x \neq -4$

$$ax^2 + bx + c \neq 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$$

$$D = b^2 - 4ac = 4 - 4 \cdot 1(-8) = 4 + 32 = 36$$

$$x_{1,2} = \frac{-2 \pm 6}{2}$$

$$\frac{-2+6}{2} = 2$$

$$\frac{-2-6}{2} = -4$$

$$1 \cdot x^2 + \boxed{2}x - 8 = 0$$

součin

$$(x \; +4)(x \; -2) = 0$$

součet

$$(x+4)\cdot(x-2) = 0$$

$$x + 4 = 0 \qquad x - 2 = 0$$

$$x = -4 \qquad x = 2$$

$$8 = 1 \cdot 8$$

$$2 \cdot 4$$

$$1\cdot(x+4) - \overset{(x-2)^2}{\boxed{(x-2)(x-2)}} = 6 - 1\cdot(x+4)(x-2)$$

$$x+4-x^2+2x+2x-4 = 6-(x^2+2x-8)$$

$$\cancel{-x^2}+5x = 6\cancel{-x^2}-2x+8$$

$$5x+2x = 14$$

$$7x = 14 \quad /:7$$

$$x = 2 \qquad ! \quad x \neq 2$$

$$\Downarrow$$

Rešení neexistuje $\qquad K = \emptyset = \{\}$

… prázdná množina

(1g) $\frac{1}{x+4} - \frac{4}{x-4} + \frac{x^2-20}{\underset{(x+4)(x+4)}{x^2-16}} = 0 \quad / \cdot (x+4)(x-4)$

Podmínky: $\underbrace{x+4 \neq 0 \wedge x-4 \neq 0}_{x \neq \pm 4} \wedge x^2-16 \neq 0$

$x^2 \neq 16 \quad / \sqrt{\ }$

$\sqrt{x^2} \neq \sqrt{16}$

$|x| \neq 4$!

$x \neq \pm 4$

$x-4-4(x+4)+x^2-20=0$

$x^2 \boxed{-3} x - 40 = 0$

$(x-8)\cdot(x+5)=0$

$x=8 \qquad x=-5$

$40 = 1 \cdot 40$
$\quad 2 \cdot 20$
$\quad 4 \cdot 10$
$\quad 5 \cdot 8$

$\underline{\underline{K = \{-5; 8\}}}$

2a

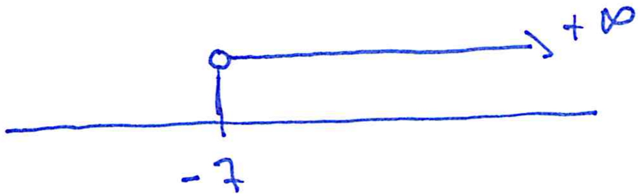
$$\frac{x+3}{2} - \frac{x-2}{3} - 5 < \frac{x-1}{2} \quad /\cdot 6$$

$$3(x+3) - 2(x-2) - 30 < 3(x-1)$$

$$-2x < -3 + 17$$

$$-2x < 14 \quad /:(-2)$$

$$\underline{\underline{x > -7}}$$



$$\underline{\underline{x \in (-7; +\infty)}}$$

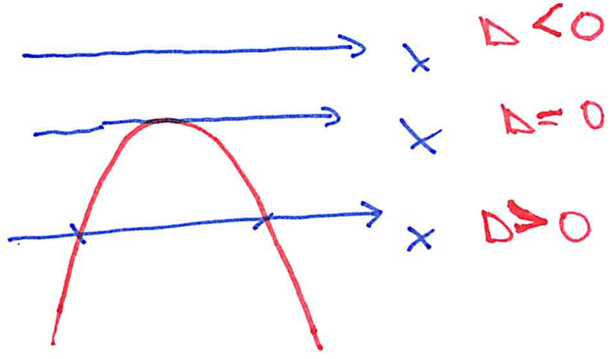
2b

$$-5(1-x)^2 \leq 3x-11$$

$$-5(1-2x+x^2) \leq 3x-11$$

$$-5x^2+10x-5 \leq 3x-11$$

$$-5x^2+7x+6 \boxed{\leq} 0$$



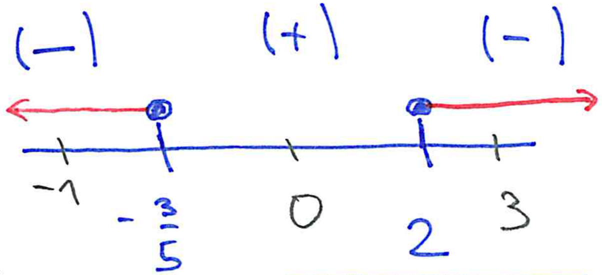
$$D = b^2-4ac = 7^2-4\cdot(-5)\cdot 6 = 49+120 = 169$$

$$\sqrt{D} = \sqrt{169} = 13$$

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{-7 \pm 13}{2\cdot(-5)}$$

$$x_1 = \frac{-20}{-10} = 2$$

$$x_2 = \frac{6}{-10} = -\frac{3}{5}$$



$$x \in \left(-\infty; -\tfrac{3}{5}\right\rangle \cup \left\langle 2; +\infty\right)$$

(2F)

$$\frac{x-1}{x+2} + \frac{x+3}{x-4} \le 2$$

$$\frac{(x-1)(x-4) + (x+3)(x+2) - 2(x+2)(x-4)}{(x+2)(x-4)} \le 0$$

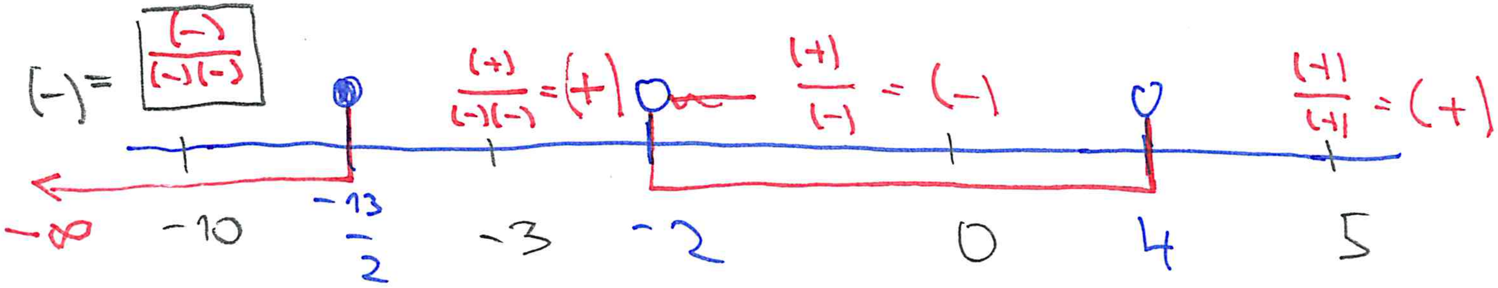
$$\frac{x^2 \cancel{-x-4x} + 4 + x^2 \cancel{+2x+3x} + 6 - 2x^2 + 8x - 4x + 16}{(x+2)(x-4)} \le 0$$

$$\frac{4x + 26}{(x+2)(x-4)} \boxed{\le} 0$$

$4x + 26 \to -\frac{13}{2}$

$x+2 \to -2$

$x-4 \to 4$



$$x \in \left(-\infty; -\frac{13}{2}\right\rangle \cup (-2; 4)$$

3a

$$3^x + 3^{x+1} = 108$$

$$3^x + \boxed{3^x \cdot 3^1} = 108 \quad \longleftrightarrow \quad 3^x + 3 \cdot 3^x = 3^x \cdot \underbrace{[1+3]}_{=4}$$

$$4 \cdot 3^x = 108 \quad /:4$$

$$3^x = 27$$

$$3^x = 3^{\boxed{3}} \quad | \log_3$$

$$x = \boxed{3}$$

3c $4^x - 9\cdot 2^x + 8 = 0$

$(\boxed{2^x})^2 - 9\cdot\boxed{2^x} + 8 = 0$

$4 = 2^2$

$4^x = (2^2)^x = 2^{2x} = (2^x)^2$

substitute : $y = 2^x$

$y^2 - 9y + 8 = 0$

$(y-8)(y-1) = 0$

a) $y_1 = 8$ $\vee$ b) $y_2 = 1$

a) $y_1 = 8$

$2^x = 8$

$2^x = 2^3$

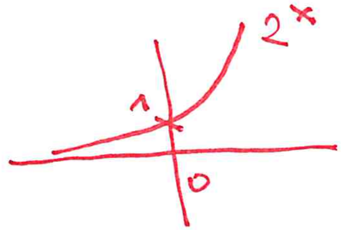
$x = 3$

b) $y_2 = 1$

$2^x = 1$

$2^x = 2^0$

$x = 0$



$K = \{0;3\}$

CV2:

3e

$$\log(7x+6) = 1 + \log(3x-4)$$

Podm.: $7x+6>0$

$3x-4>0$

$$\log(7x+6) = \log 10 + \log(3x-4)$$

$$\log(7x+6) = \log(10\cdot(3x-4)) \quad /10^{\cdots}$$

$$\boxed{\begin{array}{l} \log_a x = y \Leftrightarrow x = a^y \\ \log_{10} z = 1 \Leftrightarrow z = 10^1 \end{array}}$$

$$\log a + \log b = \log(a\cdot b)$$

$$\log a - \log b = \log\left(\frac{a}{b}\right)$$

$$\log a^n = n\cdot\log a$$

$$7x+6 = 10\cdot(3x-4)$$

$$-23x = -46 \quad /:(-23)$$

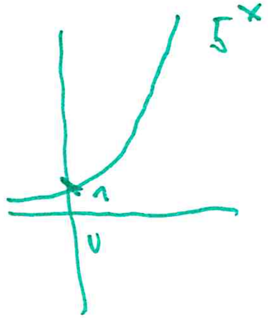
$$\underline{\underline{x = 2}}$$

4b

$$25^x + 2\cdot 5^{x+1} > 11$$

$$(5^2)^x + 2\cdot \boxed{5^x}\cdot 5^1 > 11$$

$$(\boxed{5^x})^2 + 10\cdot \underbrace{\boxed{5^x}}_{=y} > 11$$

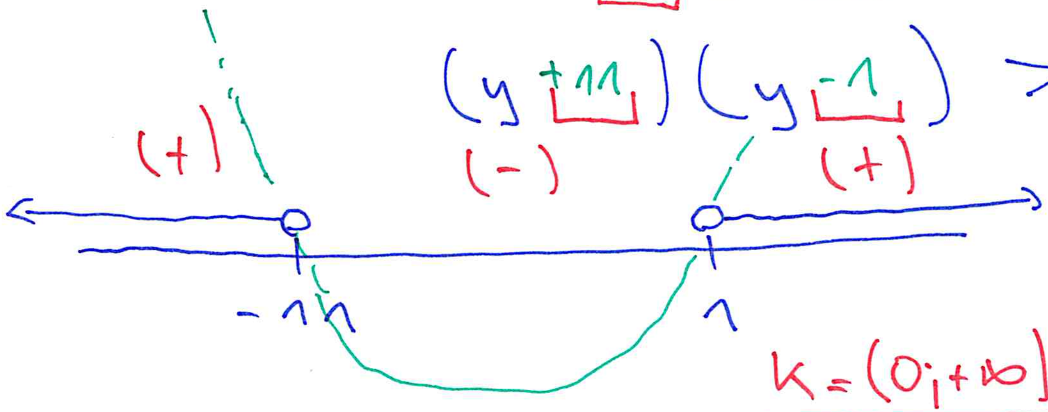


$$y^2 + 10y > 11$$

součet ... součin

$$y^2 + \boxed{10}\,y - 11 > 0$$

$$(y + \boxed{11})(y \boxed{-1}) > 0$$



$$K = (0;+\infty) = \mathbb{R}^+$$

$y \in (-\infty;-11) \cup (1;+\infty)$

a) $y < -11$

b) $y > 1$

a) $5^x < -11$

NEMÁ ŘEŠENÍ

b) $5^x > 1$

$5^x > 5^0$

$$\boxed{x > 0}$$

4c

$$\log_2(x+2) > 3$$

Podm: $x+2>0$

$x>-2$

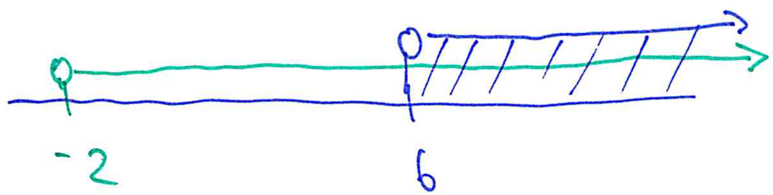
$$\log_2(x+2) > \log_2 8 \quad / \; 2^{\dots}$$

$$x+2 > 8$$

! znaménko se zachová (základ > 1)

$$x > 6$$

$K = (6; +\infty)$

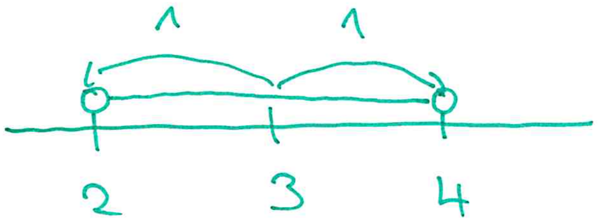


5a $M = \langle 1, 2)$

$D = \mathbb{R}_0^+ = \langle 0; +\infty)$

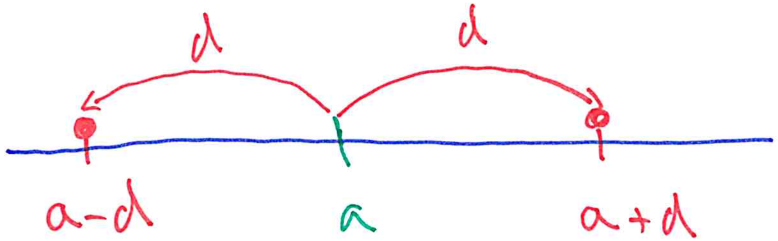
$$N = \{x \in \mathbb{R} : |x-3| < 1\} = (2;4)$$

$|x-3| < 1$

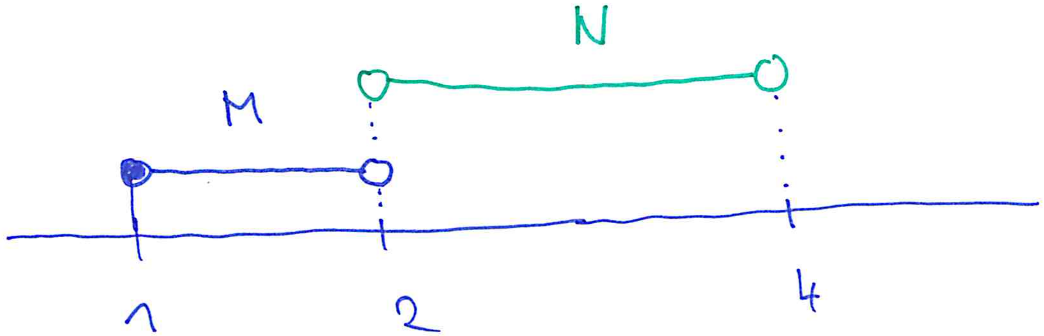


Geometrická vlastnost abs. hodnoty

$|x-a| = d \quad (d \geq 0)$



Sjednocení: $M \cup N = \{x \in \mathbb{R} : x \in M \vee x \in N\}$



$M \cup N = \langle 1;2) \cup (2;4) = \langle 1;4) \setminus \{2\}$

Průnik: $M \cap N = \{x \in \mathbb{R} : x \in M \wedge x \in N\}$

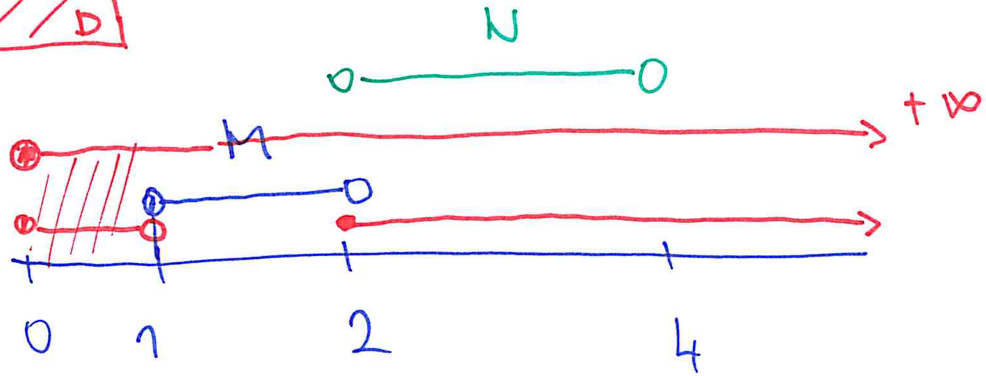
$M \cap N = \emptyset = \{\}$ ... prázdná mn. (M, N ... disjunktní)

Rozdíl: $M \setminus N = \{x \in \mathbb{R} : x \in M \wedge x \notin N\}$

$M \setminus N = \langle 1;2) = M$	$N \setminus M = (2;4) = N$
min ... 1 sup ... 2	min ... neex. sup ... 4
max ... neex ~~max~~ inf ... 1	max ... neex. inf ... 2

Doplněk (v množině)

$$M'_D = \{x \in D : x \notin M\} = D \setminus M$$

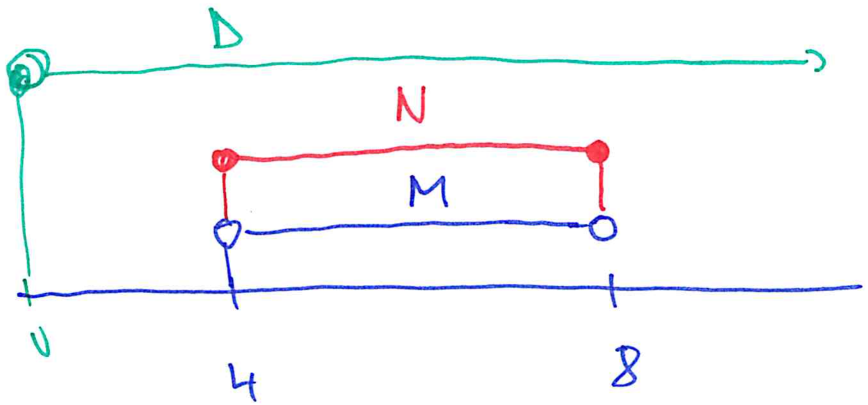


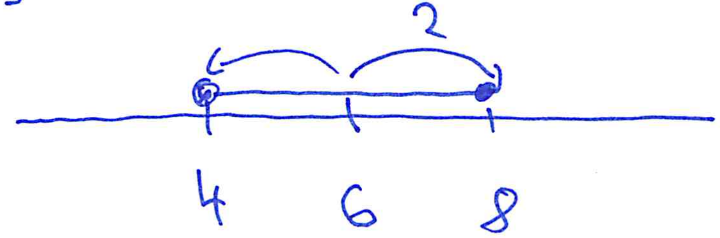
$$M'_D = \langle 0;1) \cup \langle 2;+\infty)$$

$$N'_D = \langle 0;2\rangle \cup \langle 4;+\infty)$$

5b $M = (4;8)$ $D = \mathbb{R}^+ = (0;+\infty)$

$N = \{x \in \mathbb{R} : |x-6| \leq 2\}$





$\boxed{M \subset N}$ podmnožina

$M \cup N = \langle 4;8 \rangle$ $M \cap N = (4;8)$

$M \setminus N = \emptyset = \{\}$ $N \setminus M = \{4;8\}$

$M'_D = (0;4\rangle \cup \langle 8;\infty)$ $N'_D = (0,4) \cup (8,+\infty)$

6 Minimum, Maximum, Supremum, Infimum

$M \cup N = \langle 1;2) \cup (2;4)$

$\parallel$

A

$\min A = 1$

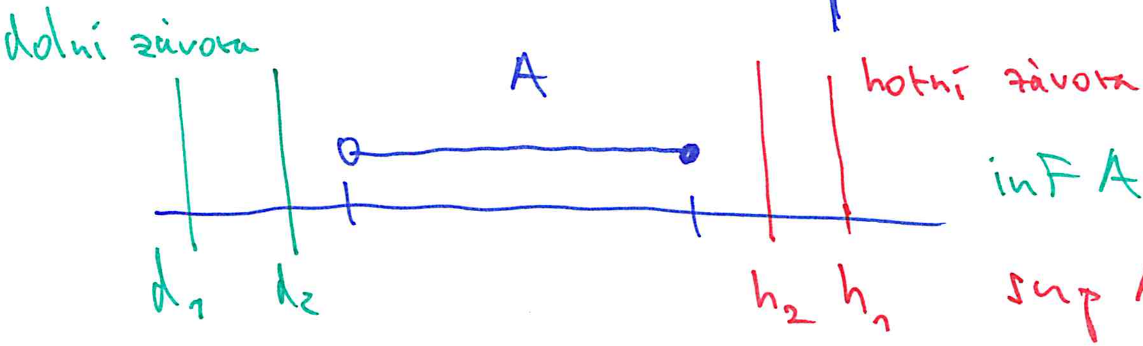
$\max A =$ neex.

$\mathbb{R}^* = \langle -\infty; +\infty \rangle$

... rozšířená reálná osa

$\mathbb{R}^* = (-\infty;+\infty) \cup \{-\infty; +\infty\}$

sup A ... nejmenší horní závora



inf A ... největší dolní závora

$\sup A = 4$ $\quad$ $\inf A = 1$

$\min \emptyset$ = heeft.

$\max \emptyset$ = heeft.

$\sup \emptyset = -\infty$

$\inf \emptyset = \infty$