$\boxed{1d/CV4}$

$$\lim_{x\to 0} \frac{\sqrt{x+1}-1}{x} \left|\frac{0}{0}\right| \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} =$$

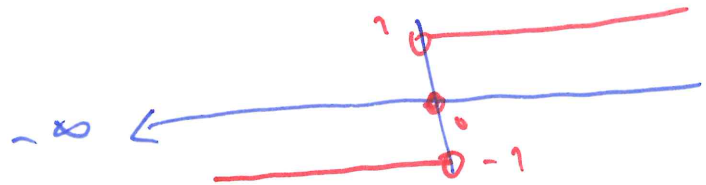
$$= \lim_{x\to 0} \frac{\cancel{(x+1)-1}\;1}{\cancel{x}\left[\sqrt{x+1}+1\right]} = \frac{1}{1+1} = \underline{\underline{\frac{1}{2}}}$$

2a

$$\lim_{x\to-\infty} \frac{\sqrt{x^2+1}}{x} \left(\frac{\infty}{-\infty}\right) = \lim_{x\to-\infty} \frac{|x|\sqrt{1+1/x^2}}{x} \qquad 1/x^2 \to 0$$

$$\boxed{\sqrt{x^2+1} = |x|\cdot\sqrt{1+\frac{1}{x^2}}} \qquad = \lim_{x\to-\infty} \frac{|x|}{x}$$

$$\boxed{|x| = x\cdot \operatorname{sgn}(x)} \qquad = \lim_{x\to-\infty} \frac{\cancel{x}\cdot \operatorname{sgn}(x)}{\cancel{x}} = -1$$



(2c)

$$\lim_{x\to-\infty} \frac{\sqrt{x^2+3x}}{\sqrt[3]{x^3-2x^2}}$$

$\sqrt{\infty-\infty}$, není def.

$$= \lim_{x\to-\infty} \frac{|x|\cdot\sqrt{1+\frac{3}{x}}}{\cancel{x}\cdot\sqrt[3]{1-\frac{2}{x}}} = \lim_{x\to-\infty}\frac{|x|}{x} = \underline{\underline{-1}}$$

$\frac{3}{x} \to 0$, $\frac{2}{x} \to 0$

Pamatuj:

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \to 0} \frac{\operatorname{tg} x}{x} = 1$$

$$\lim_{x \to 0} \frac{\arcsin x}{x} = 1$$

$$\lim_{x \to 0} \frac{\operatorname{arctg} x}{x} = 1$$

Modifikace

$$\lim_{f(x) \to 0} \frac{\sin(f(x))}{f(x)} = 1$$

$$\lim_{x \to 0} \frac{\sin(\overset{z}{5x})}{\underset{z}{5x}} = 1$$

$$\lim_{z \to 0} \frac{\sin z}{z} = 1$$

3b

$$\lim_{x \to 0} \frac{\sin x - x}{\sin x + x} \quad \left(\frac{0}{0}\right) \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$$

$$= \lim_{x \to 0} \frac{\underbrace{\frac{\sin x}{x}}_{\to 1} - \underbrace{\frac{x}{x}}_{=1}}{\underbrace{\frac{\sin x}{x}}_{\to 1} + \underbrace{\frac{x}{x}}_{=1}} = \frac{1 - 1}{1 + 1} = 0$$

3d

$$\lim_{x \to 0} \frac{\sin(7x) \cdot 7x \cdot 4x}{\sin(4x) \cdot 7x \cdot 4x} = \lim_{x \to 0} \frac{7x}{4x} = \frac{7}{4}$$

4a $\lim\limits_{x \to 0} \dfrac{\boxed{\sqrt[3]{8+x}} - 2}{x} \quad \left|\dfrac{0}{0}\right|$

substituce: $z = \sqrt[3]{8+x} \quad /^3$

$z^3 = 8 + x$

$z^3 - 8 = x$

$x \to 0$

$z = \sqrt[3]{8+0} = 2$!

$z \to 2$

$$\lim_{z \to 2} \frac{z-2}{z^3 - \underbrace{8}_{=2^3}} \left|\frac{0}{0}\right| = \lim_{z \to 2} \frac{\cancel{z-2}^{\,1}}{\cancel{(z-2)}_{1}(z^2+2z+4)}$$

$$= \frac{1}{2^2 + 2 \cdot 2 + 4} = \frac{1}{12}$$

4c $\lim\limits_{x \to 0} \dfrac{x}{\sqrt[3]{1+x}-1}$ $\left|\dfrac{0}{0}\right|$ $\dfrac{\sqrt[3]{1+x}+1}{\sqrt[3]{1+x}+1}$ ... takto nelze rozšířit !

$\left(\sqrt[3]{1+x}\right)^2$

substituce:

$z = \sqrt[3]{1+x}$

$z^3 = 1+x$

$z^3 - 1 = x$

$x \to 0$

$z = \sqrt[3]{1+0} = \sqrt[3]{1} = 1$

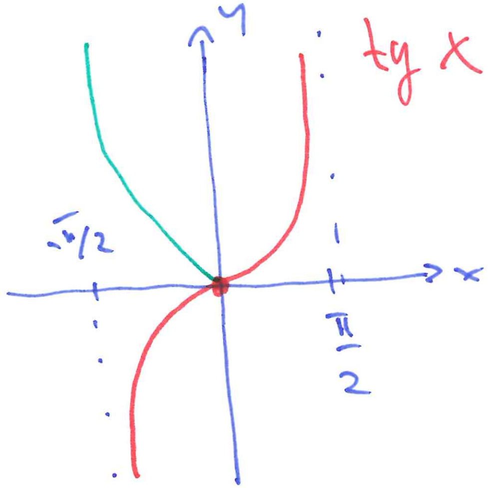
$z \to 1$

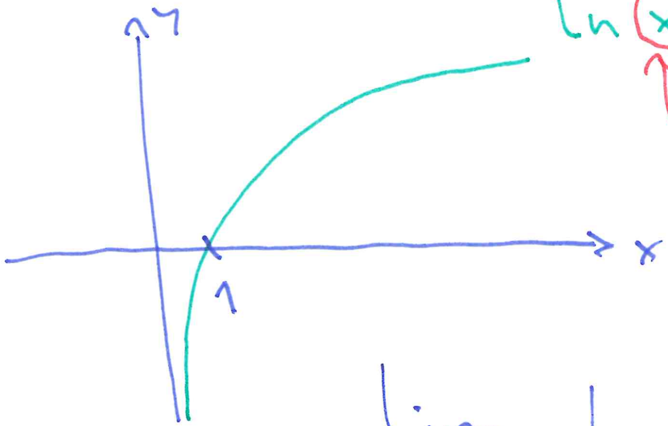
$$\lim_{z \to 1} \frac{z^3-1}{z-1} = \lim_{z \to 1} \frac{\cancel{(z-1)}(z^2+z+1)}{\cancel{z-1}} = \underline{\underline{3}}$$

CV6

5a/CV4

$$\lim_{x \to 0} \ln(|\mathrm{tg}\, x|) = (\ln(0))$$ není definováno





$\ln(x)$

$$\boxed{\lim_{x \to 0} \mathrm{tg}\, x}$$

$$\boxed{\cancel{1} \to 0+}$$

$$\lim_{x \to 0} \ln|\mathrm{tg}\, x| = -\infty$$

5b

$$\lim_{x \to 0} \cos\left(\boxed{x^2 \cdot \sin\left(\frac{1}{x}\right)}\right) = \cos 0 = \underline{\underline{1}}$$

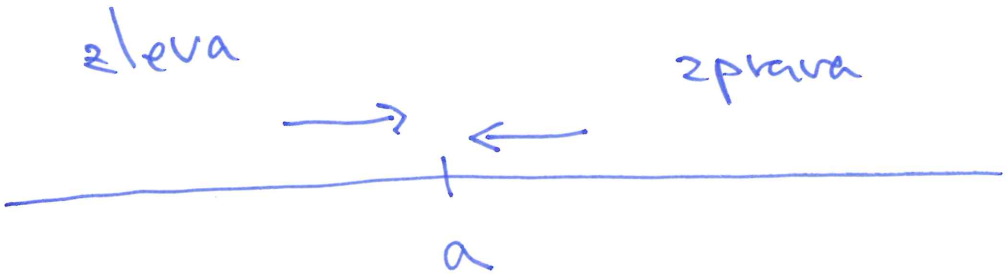
$\frac{1}{0}$ není def.

$$\lim_{x \to 0} \underbrace{x^2}_{\to\, 0} \cdot \underbrace{\sin\left(\frac{1}{x}\right)}_{\text{mezi } -1 \text{ a } 1} = 0$$

omezená

$$\boxed{-1 \leq \sin\left(\frac{1}{x}\right) \leq 1 \quad \forall x \in \mathbb{R}}$$

Jednostranné limity



$x \to a^-$ $\qquad$ $x \to a^+$

$$\frac{1}{0^+} = +\infty$$

0^- ... největší záporné číslo

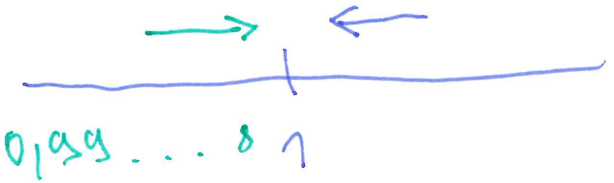
0^+ .... nejmenší kladné číslo

$$\frac{1}{0^-} = -\infty$$

6a

$$\lim_{x \to 1+} \frac{1}{1-x} = \left(\frac{1}{0}\right) \text{ není definované} = \frac{1}{0-} = -\infty$$

1,000....1



b

$$\lim_{x \to 1-} \frac{1}{1-x} = \frac{1}{0+} = +\infty$$

Jednostranné limity jsou různé

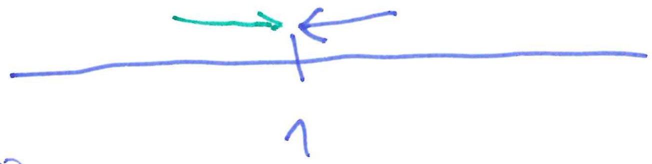
$\Downarrow$

oboustranná limita neexistuje

6c

$$\lim_{x \to 1+} \frac{x^2-1}{x-1} = \left|\frac{0}{0}\right| \text{ není def.}$$

$$\lim_{x \to 1+} \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}\; 1} = 2$$



d

$$\lim_{x \to 1-} \frac{x^2-1}{x-1} = \lim_{x \to 1-} \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}\; 1} = 2$$

Existuje $\lim_{x \to 1} \frac{x^2-1}{x-1} = 2$

7a $\lim\limits_{x \to 0} \dfrac{x^2-1}{x^2} = \left(\dfrac{-1}{0}\right)$ není def. $= -\infty$

Pamatuj:

$\dfrac{\text{konečné nenulové číslo (i } \pm\infty)}{0} \Rightarrow$ limitu rozepíši na 2 jednostranné limity

$\lim\limits_{x \to 0+} \dfrac{x^2-1}{x^2} = \dfrac{-1}{(0+)^2} = \dfrac{-1}{0+} = -\infty$

$\lim\limits_{x \to 0-} \dfrac{x^2-1}{x^2} = \dfrac{-1}{(0-)^2} = \dfrac{-1}{0+} = -\infty$

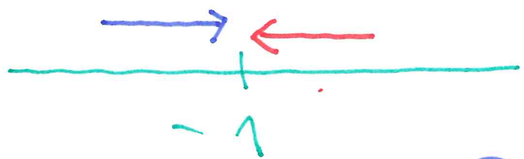
stejné ⇓ ex. lim.

7b

$$\lim_{x \to -1} \frac{2x}{x+1} = \left(\frac{-2}{0}\right) \text{ není def. } \dots\text{ neex.}$$

$$\frac{+\infty}{0+} = \infty \cdot \frac{1}{0+} = \infty \cdot \infty = \infty$$

$$\lim_{x \to -1+} \frac{2x}{x+1} = \frac{-2}{0+} = -\infty$$



$$\lim_{x \to -1-} \frac{2x}{x+1} = \frac{-2}{0-} = +\infty$$

různé

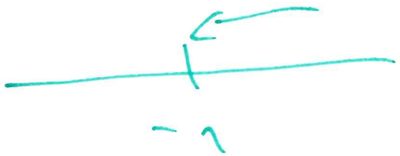
$\Downarrow$

neexistuje limita

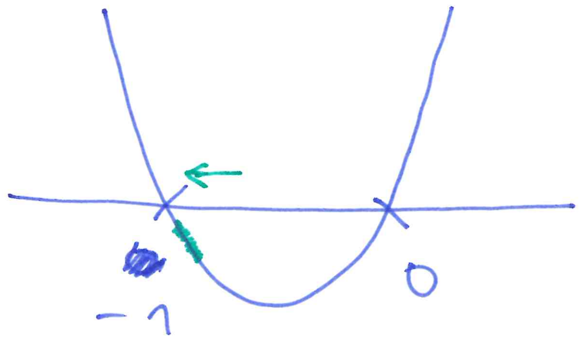
Složitější příklad jmenovatele:

1. zp.

$$\lim_{x \to -1+} \frac{1}{x^2 + x} = \frac{1}{\underbrace{\underbrace{x}_{(-1+)} \underbrace{(x+1)}_{0+}}_{0-}}$$



2. zp.



$$x^2 + x = (-1+)^2 + (-1+) = \boxed{1- \quad + \quad (-1+)} \; \dots$$

nevede k cíly