Matematika I (KMD/MA1) - cvičení 5

FAKULTA STROJNÍ (akad. rok 2019/2020 a vyšší)

Příklad 1. Vypočtěte f' (výsledek upravte) a určete D_f a $D_{f'}$, jsou-li funkce f zadány předpisem:

a) $f(x) = \frac{5}{3}x - 2 + \frac{2}{3x^2}$ $\left[\frac{5x^3-4}{3x^3},\ x \neq 0\right]$

b) $f(x) = 6\sqrt[3]{x} - 4\sqrt[4]{x} + \frac{5}{3\sqrt[3]{x}}$ $\left[\frac{2}{\sqrt[3]{x^2}} - \frac{1}{\sqrt[4]{x^3}} - \frac{5}{9\sqrt[3]{x^4}},\ x > 0\right]$

c) $f(x) = \frac{5}{3\sqrt[4]{x^3}}$ $\left[\frac{-5}{4\sqrt[4]{x^7}},\ x \neq 0\right]$

d) $f(x) = 3x^3 - 2\sqrt{x} + \frac{1}{x^3}$ $\left[9x^2 - \frac{1}{\sqrt{x}} - \frac{3}{x^4},\ x > 0\right]$

e) $f(x) = 3\sqrt[3]{x^2} - \frac{1}{3}\operatorname{cotg} x$ $\left[\frac{2}{\sqrt[3]{x}} + \frac{1}{3\sin^2 x},\ \sin x \neq 0\right]$

f) $f(x) = 2x^3 + 5\sin x$ $\left[6x^2 + 5\cos,\ x \in I\!R\right]$

g) $f(x) = (x^2-1)(x^3-5)$ $\left[5x^4 - 3x^2 - 10x,\ x \in I\!R\right]$

h) $f(x) = x^2 \operatorname{tg} x$ $\left[\frac{x(x+\sin(2x))}{\cos^2 x},\ \cos \neq 0\right]$

i) $f(x) = (x^2+1)\ln x$ $\left[2x\ln x + x + \frac{1}{x},\ x > 0\right]$

j) $f(x) = x^2 \operatorname{cotg} x$ $\left[2x\operatorname{cotg} x - \frac{x^2}{\sin^2 x},\ \sin x \neq 0\right]$

k) $f(x) = \sqrt{x}\cos x$ $\left[\frac{\cos x}{2\sqrt{x}} - \sqrt{x}\sin x,\ x > 0\right]$

l) $f(x) = \sin x \cos x$ $\left[\cos(2x),\ x \in I\!R\right]$

m) $f(x) = \sqrt[3]{x^2}\operatorname{arctg} x$ $\left[\frac{2\operatorname{arctg} x}{3\sqrt[3]{x}} + \frac{\sqrt[3]{x^2}}{1+x^2},\ x \neq 0\right]$

n) $f(x) = x^3\sqrt{x}e^x$ $\left[e^x\left(\frac{7}{2}\sqrt{x^5} + \sqrt{x^7}\right),\ x \geq 0\right]$

o) $f(x) = xe^x\cos x$ $\left[e^x(\cos x + x(\cos x - \sin x)),\ x \in I\!R\right]$

p) $f(x) = (x^5 - x)(x^6+1)x^3$ $\left[14x^{13} - 10x^9 + 8x^7 - 4x^3,\ x \in I\!R\right]$

q) $f(x) = x\ln x + e^x \sin x$ $\left[\ln x + 1 + e^x(\sin x + \cos x),\ x > 0\right]$

r) $f(x) = \frac{5x}{9} - 5$ $\left[\frac{5}{9},\ x \in I\!R\right]$

s) $f(x) = \frac{6}{7x} - \frac{x}{2}$ $\left[-\frac{6}{7x^2} - \frac{1}{2},\ x \neq 0\right]$

Příklad 2. Vypočtěte f' (výsledek upravte) a určete D_f a $D_{f'}$, jsou-li funkce f zadány předpisem:

a) $f(x) = \dfrac{x}{x+1}$ $\left[\dfrac{1}{(1+x)^2},\ x \neq -1\right]$

b) $f(x) = \dfrac{\cos x}{1-\sin x}$ $\left[\dfrac{1}{1-\sin x}\right]$

c) $f(x) = \dfrac{e^x}{\sin x}$ $\left[\dfrac{e^x(\sin x - \cos x)}{\sin^2 x}\right]$

d) $f(x) = \dfrac{1+x-x^2}{1-x+x^2}$ $\left[\dfrac{2(1-2x)}{(1-x+x^2)^2},\ x \in \mathbb{R}\right]$

e) $f(x) = \dfrac{x+\sqrt[3]{x}}{x-\sqrt[3]{x}}$ $\left[\dfrac{-4\sqrt[3]{x}}{3(x-\sqrt[3]{x})^2}\right]$

f) $f(x) = \dfrac{\operatorname{arctg} x}{\log x}$ $\left[\dfrac{x\ln(10)\log x - (1+x^2)\operatorname{arctg} x}{(x+x^3)\ln(10)\log^2 x}\right]$

g) $f(x) = \dfrac{xe^x}{1+x^2}$ $\left[\dfrac{e^x(1+x-x^2+x^3)}{(1+x^2)^2}\right]$

h) $f(x) = \dfrac{(x^2+1)\operatorname{arctg} x}{\ln x}$ $\left[\dfrac{x(2x\operatorname{arctg} x + 1)\ln x - (x^2+1)\operatorname{arctg} x}{x\ln^2 x}\right]$

i) $f(x) = \dfrac{x^2\ln x}{x+1}$ $\left[\dfrac{(2\ln x+1)(x^2+x) - x^2\ln x}{(x+1)^2}\right]$

j) $f(x) = 2e^{3x}$ $\left[6e^{3x}\right]$

k) $f(x) = 3\ln(5x)$ $\left[\dfrac{3}{x}\right]$

l) $f(x) = \ln(x^2-1)$ $\left[\dfrac{2x}{x^2-1}\right]$

m) $f(x) = \arcsin\left(\dfrac{x-2}{2}\right)$ $\left[\dfrac{1}{\sqrt{4x-x^2}}\right]$

n) $f(x) = \operatorname{arctg}\left(\dfrac{1+x}{1-x}\right)$ $\left[\dfrac{1}{1+x^2}\right]$

o) $f(x) = \dfrac{1}{(x^3-1)^2}$ $\left[\dfrac{-6x^2}{(x^3-1)^3}\right]$

p) $f(x) = \dfrac{\operatorname{tg}^2 x}{2} + \ln(\cos x)$ $\left[\operatorname{tg}^3 x\right]$

q) $f(x) = \ln(4-x^2) + \arcsin\left(\dfrac{x-2}{2}\right)$ $\left[\dfrac{2x}{x^2-4} + \dfrac{1}{\sqrt{4x-x^2}}\right]$

r) $f(x) = \ln(1+\cos x)$ $\left[\dfrac{-\sin x}{1+\cos x}\right]$

s) $f(x) = \operatorname{arctg}\sqrt{6x-1}$ $\left[\dfrac{1}{2x\sqrt{6x-1}}\right]$

t) $f(x) = (x-2)\sqrt{1+e^x} - \ln\left(\dfrac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}\right)$ $\left[\dfrac{xe^x}{2\sqrt{1+e^x}}\right]$

u) $f(x) = \sqrt{\dfrac{1-e^x}{1+e^x}}$ $\left[\dfrac{-e^x}{(1+e^x)\sqrt{1-e^{2x}}}\right]$

v) $f(x) = \ln\left(e^x + \sqrt{1+e^{2x}}\right)$ $\left[\dfrac{e^x}{\sqrt{1+e^{2x}}}\right]$

w) $f(x) = \left(\dfrac{1}{1-x}\right)^x$ $\left[\left(\dfrac{1}{1-x}\right)^x\left(\dfrac{x}{1-x} - \ln(1-x)\right)\right]$

x) $f(x) = (x^2+1)^{\operatorname{arctg} x}$ $\left[(x^2+1)^{\operatorname{arctg} x - 1}\left(2x\operatorname{arctg} x + \ln(x^2+1)\right)\right]$

y) $f(x) = x^{\sin x}$ $\left[x^{\sin x}\left(\cos x\ln x + \dfrac{\sin x}{x}\right)\right]$