Substituce (1. druhu)

$$\int F(g(x))\, g'(x) \cdot dx = \left| \begin{array}{l} t = g(x) \\ \dfrac{dt}{dx} = g'(x) \\ \dfrac{dt}{g'(x)} = dx \end{array} \right| = \int f(t)\, \frac{dt}{\cancel{g'(x)}}\, \cancel{g'(x)}$$

$$= F(t) = \left| \begin{array}{l} \text{zpětná substituce} \\ t = g(x) \end{array} \right| = F(g(x)) + C$$

$\boxed{\text{CV 10 / 1a}}$

$$\int \sin^3 x \cdot \cos x \, dx = \left| \begin{array}{l} t = \sin x \\ \dfrac{dt}{dx} = \cos x \\ dx = \dfrac{dt}{\cos x} \end{array} \right| = \int t^3 \cdot \cancel{\cos x} \frac{dt}{\cancel{\cos x}}$$

$$= \int t^3 \, dt = \frac{t^4}{4} = \underline{\frac{\sin^4 x}{4} + C}$$

1b $\int 6x \cdot \sin(3x^2)\,dx = \left|\ t = 3x^2 \right.$

1f $\int 6\, tg(3x)\,dx = \left|\begin{array}{l} t = 3x \\ dt = 3dx \\ dx = \frac{dt}{3} \end{array}\right| = \int 6 \cdot tg(t) \frac{dt}{3}$

$= 2\int tg(t)\,dt = 2\int \frac{\sin t}{\underset{(\cos t)^{-1}}{\cos t}}\,dt = \left|\begin{array}{l} z = \cos t \\ \frac{dz}{dt} = -\sin t \\ dt = \frac{dz}{-\sin t} \end{array}\right|$

$= 2\int \frac{\cancel{\sin t}}{z} \cdot \frac{dz}{-\cancel{\sin t}} = -2\int \frac{1}{z}\,dz = -2\ln|z| = -2\ln|\cos(3x)| + C$

Jednoduchá substituce

$$\int \frac{f'(x)}{f(x)}\,\overset{dt}{dx} = \left|\begin{array}{l} t = f(x) \\ dt = f'(x)\,dx \end{array}\right| = \int \frac{1}{t}\,dt = \ln|t|$$

$$= \underline{\underline{\ln|f(x)| + C}}$$

$$\int \frac{2x}{1+x^2}\,dx = \underline{\underline{\ln|1+x^2| + C}}$$

$$\frac{1}{2}\int \frac{2x}{1+x^2}\,dx = \frac{1}{2}\ln|1+x^2| + C$$

1g $\displaystyle\int \frac{1}{x\sqrt{1-\ln^2 x}}\,dx = \left|\ t = \ln x \right.$

1h $\displaystyle\int \frac{2\operatorname{arctg} x}{1+x^2}\,dx = \left|\ t = \operatorname{arctg} x \right.$

1k $\displaystyle\int \frac{4x}{\sqrt[3]{8-x^2}}\,dx = \left|\begin{array}{l} t = x^2 \\ \dfrac{dt}{dx} = 2x \\ dx = \dfrac{dt}{2x} \end{array}\right| = \int \frac{\overset{2}{\cancel{4x}}}{\sqrt[3]{8-t}} \cdot \frac{dt}{\cancel{2x}}$

1h $\displaystyle\int \frac{1}{x^2-6x+9}\,dx = \int \frac{1}{(x-3)^2}\,dx = \left| \begin{array}{l} t = x-3 \\ dt = dx \end{array} \right|$

$\displaystyle = \int \frac{1}{t^2}\,dt = \int t^{-2}\,dt = \frac{t^{-1}}{-1} = \frac{-1}{t} = \underline{\underline{\frac{-1}{x-3}+C}}$

Lineární substituce (lze provést vždy)

$$\left| \begin{array}{l} t = ax+b \\ dt = a\,dx \\ dx = \dfrac{dt}{a} \end{array} \right|$$

1p

$$\int \sin(2x-5)\,dx = \left| \begin{array}{l} t = 2x-5 \\ dt = 2\,dx \\ dx = \frac{dt}{2} \end{array} \right|$$

$$= \int \sin(t)\cdot\frac{dt}{2} = \frac{1}{2}\int \sin t\,dt =$$

$$= \frac{1}{2}(-\cos t) = -\frac{1}{2}\cos(2x-5) + C$$

CV 11

1a) CV 10

$$\int \frac{1}{\sin^2(3x-7)}\,dx = \left| \begin{array}{l} t = \boxed{3x-7} \\ \frac{dt}{dx} = 3 \\ dx = \frac{dt}{3} \end{array} \right| = \int \frac{1}{\sin^2 t}\,\frac{1}{3}\,dt$$

... lineární substituce

$$= \frac{1}{3}\int \frac{1}{\sin^2 t}\,dt = \frac{1}{3}\left(-\operatorname{cotg} t\right) = -\frac{1}{3}\operatorname{cotg}(3x-7) + C$$

11t $\displaystyle\int \frac{2}{x^2-2x+5}\,dx$

Parciální zlomky

A) příslušné reálným kořenům

$$\frac{A}{(x-\alpha)^k}$$

B) příslušné dvojici komplexně sdružených kořenů

$$\frac{Bx+C}{(x^2+px+q)^l}$$

... nerozložitelný

$D<0$

... ireducibilní

$$\int \frac{2}{x^2-2x+5}\,dx = \int \frac{2}{x^2-\boxed{2}x+1-1+5}\,dx$$

$\Delta < 0$ ... nelze rozložit

$$\left(\frac{2}{2}\right)^2 = 1$$

$$\int \frac{2}{\boxed{x^2-2x+1}+4}\,dx = \int \frac{2}{(x-1)^2+4}\,dx = \frac{1}{\cancel{4}}\int \frac{\cancel{2}}{\boxed{\frac{(x-1)^2}{4}}+\frac{4}{4}}\,dx$$

$$\boxed{\int \frac{1}{t^2+1}\,dt = \operatorname{arctg} t}$$

$$= \frac{1}{2}\int \frac{1}{\left(\frac{x-1}{2}\right)^2+1}\,d\cancel{x} = \left| \begin{array}{l} t = \frac{x-1}{2} = \frac{x}{2}-\frac{1}{2} \\ \frac{dt}{dx} = \frac{1}{2} \\ dx = 2dt \end{array} \right| =$$

$$= \frac{1}{\cancel{2}}\int \frac{1}{t^2+1}\,\cancel{2}dt = \operatorname{arctg} t = \operatorname{arctg}\left(\frac{x-1}{2}\right) + C$$

1v $$\int \frac{1}{\sqrt{-2x-x^2}}\,dx = \int \frac{1}{\sqrt{-(\underbrace{x^2+2x+1}_{(x+1)^2}-1)}}\,dx$$

$$\int \frac{1}{\sqrt{1-t^2}}\,dt = \arcsin(t)$$

$$= \int \frac{1}{\sqrt{1-(x+1)^2}}\,dx = \left| \begin{array}{l} t = x+1 \\ \frac{dt}{dx} = 1 \end{array} \right| = \int \frac{1}{\sqrt{1-t^2}}\,dt =$$

$$= \arcsin(t) = \underline{\underline{\arcsin(x+1) + C}}$$