CV 12

Určitý integrál

Newton - Leibnitz

$$\int_a^b f(x)\,dx = \Big[F(x)\Big]_{x=a}^{b} = F(b) - F(a)$$

$F(x)$.. primitivní fce k $f(x)$

$$F(x) = \int f(x)\,dx \quad (\text{bez konstanty } c)$$

1b $\int_0^{\pi} 5 \cdot \sin(4x)\, dx$

1. způsob:

$$\int 5 \cdot \sin(4x)\, dx = \left| \begin{array}{l} t = 4x \\ dt = 4\,dx \\ dx = \frac{dt}{4} \end{array} \right| = \int 5 \cdot \sin(t)\, \frac{dt}{4} = \frac{5}{4} \int \sin t\, dt$$

$$= \frac{5}{4}(-\cos t) = -\frac{5}{4} \cos(4x) + C$$

N-L:

$$\int_0^{\pi} 5 \cdot \sin(4x)\, dx = \left[-\frac{5}{4} \cos(4x) \right]_{x=0}^{\pi} = -\frac{5}{4}\cos(4\pi) - \left(-\frac{5}{4}\cos 0\right) =$$

$$= -\frac{5}{4} + \frac{5}{4} = \underline{\underline{0}}$$

2. způsob:

$$\int_0^{\pi} 5 \cdot \sin(4x)\, dx = \left| \begin{array}{l} \boxed{t = 4x} \\ \frac{dt}{dx} = 4 \\ dx = \frac{dt}{4} \\ \hline x = 0 \longrightarrow t = 0 \\ x = \pi \longrightarrow t = 4\pi \end{array} \right| = \int_0^{4\pi} 5 \cdot \sin t \, \frac{dt}{4}$$

! přepočítat meze →

$$= \frac{5}{4} \int_0^{4\pi} \sin t \, dt = \frac{5}{4} \cdot \Big[-\cos t \Big]_{t=0}^{4\pi} = \frac{5}{4} \big(-\cos(4\pi) + \cos 0 \big)$$

$$= \frac{5}{4} (-1 + 1) = 0$$

1.1 $\displaystyle\int_{-2}^{2} \frac{x^2}{x^2+1}\,dx$

$$\int \frac{x^2}{x^2+1}\,dx = \int \frac{\boxed{x^2+1} - 1}{\boxed{x^2+1}}\,dx = \int 1 - \frac{1}{x^2+1}\,dx$$

$$= x - \operatorname{arctg} x + C$$

N-L :

$$\int_{-2}^{2} \frac{x^2}{x^2+1}\,dx = \Big[x - \operatorname{arctg} x\Big]_{x=-2}^{2} = 2 - \operatorname{arctg} 2 - \big(-2 - \operatorname{arctg}(-2)\big)$$

$$= 4 - \operatorname{arctg} 2 + \underbrace{\operatorname{arctg}(-2)}_{\text{lichá fce}:\ F(-x) = -F(x)} = 4 - \operatorname{arctg} 2 - \operatorname{arctg} 2 = \underline{\underline{4 - 2\operatorname{arctg} 2}}$$

(10)

$$\int_0^1 \frac{x}{\sqrt{4-x^2}}\,dx = \left| \begin{array}{l} \boxed{t = 4-x^2} \\ \frac{dt}{dx} = -2x \\ dx = \frac{dt}{-2x} \\ x=0 \to t=4 \text{ ... dolni} \\ x=1 \to t=3 \text{ ... horni} \end{array} \right| = \int_4^3 \frac{\cancel{x}\ 1}{\sqrt{t}} \frac{dt}{-2\cancel{x}}$$

≥ 0

$$= -\frac{1}{2}\int_4^3 \frac{1}{\sqrt{t}}\,dt = \frac{1}{2}\int_3^4 t^{-\frac{1}{2}}\,dt = \frac{1}{\cancel{2}}\left[\frac{t^{1/2}}{\cancel{1/2}}\right]_{t=3}^4$$

$$= \left[\sqrt{t}\right]_{t=3}^4 = \sqrt{4} - \sqrt{3} = \underline{\underline{2-\sqrt{3}}} > 0$$

1) j) $\int_0^3 x \cdot e^{-\frac{x}{2}} \, dx = \left| \begin{array}{l} \boxed{t = -\frac{x}{2}} \\ dt = -\frac{1}{2} dx \\ dx = -2dt \\ \hline x = 0 \to t = 0 \\ x = 3 \to t = -\frac{3}{2} \end{array} \right| = \int_0^{-\frac{3}{2}} \underbrace{x}_{-2t} \cdot e^{t} (-2dt)$

$= \int_0^{-\frac{3}{2}} -2t \cdot e^{t} (-2\, dt) = \int_0^{-\frac{3}{2}} 4t \cdot e^{t} \, dt = \left| \begin{array}{ll} u = 4t & v' = e^{t} \\ u' = 4 & v = \int e^{t} dt = e^{t} \end{array} \right|$

$= \left[4 \cdot t \cdot e^{t} \right]_{t=0}^{-\frac{3}{2}} - \int_0^{-\frac{3}{2}} 4 \cdot e^{t} \, dt$

$$= 4\left(-\frac{3}{2}\right) \cdot e^{-\frac{3}{2}} - 4 \cdot 0 \cdot e^{0} - \left[4 \cdot e^{t}\right]_{t=0}^{-\frac{3}{2}} =$$

$$= -6 \cdot e^{-\frac{3}{2}} - 0 - \left(4 \cdot e^{-3/2} - 4 \cdot e^{0}\right) =$$

$$= -6 \cdot e^{-3/2} - 4 e^{-3/2} + 4$$

$$= \underline{\underline{4 - 10 e^{-3/2}}}$$