FAKULTA TEXTILNÍ TUL

Calculations
input of calculations and results

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EXAMPLE 1

- **Calculate the trichromatic coordinates *x*, *y*, *z* in the CIE XYZ for a color sample with tristimulus values *X*, *Y*, *Z*.**

$$X = 20 \qquad x = ?$$
$$Y = 40 \qquad y = ?$$
$$Z = 10 \qquad z = ?$$

EXAMPLE 1

$$X = 20 \qquad x = ?$$
$$Y = 40 \qquad y = ?$$
$$Z = 10 \qquad z = ?$$

$$x = \frac{X}{X + Y + Z}$$

$$y = \frac{Y}{X + Y + Z}$$

$$z = \frac{Z}{X + Y + Z}$$

$$x + y + z = 1$$

EXAMPLE 1

$$X = 20 \qquad x = ?$$
$$Y = 40 \qquad y = ?$$
$$Z = 10 \qquad z = ?$$

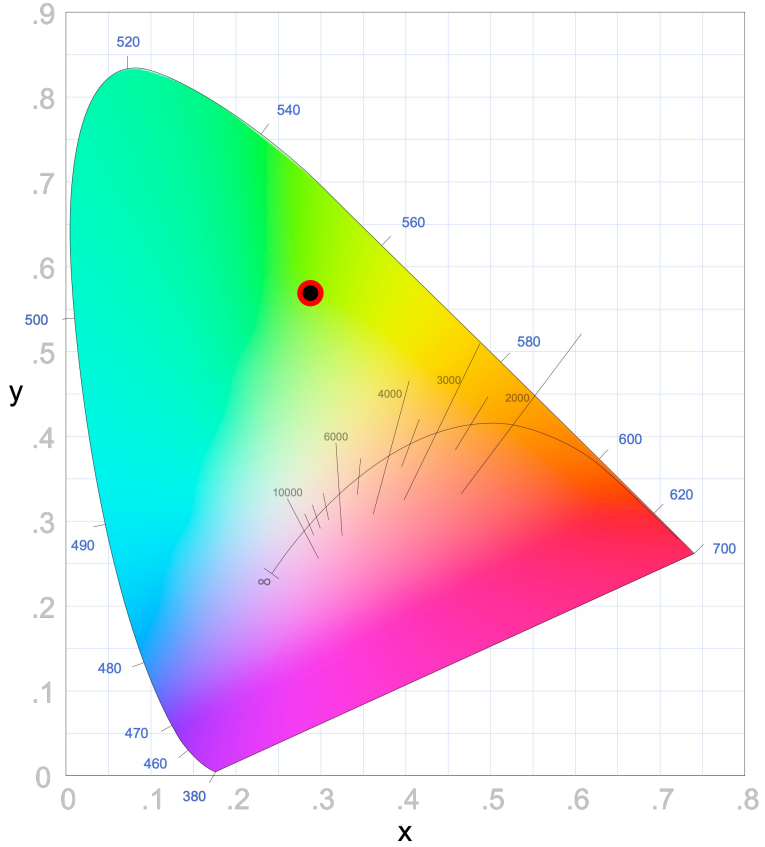
$$x = \frac{X}{X + Y + Z} = \frac{20}{20 + 40 + 10} = \frac{20}{70} = 0,2857$$

$$y = \frac{Y}{X + Y + Z} = \frac{40}{20 + 40 + 10} = \frac{40}{70} = 0,5714$$

$$z = \frac{Z}{X + Y + Z} = \frac{10}{20 + 40 + 10} = \frac{10}{70} = 0,1429$$

$$x + y + z = 0,2857 + 0,5714 + 0,1429 = 1$$

EXAMPLE 1



$$\mathbf{x = 0,2857}$$

$$\boldsymbol{y = 0,5714}$$

EXAMPLE 2

- **Calculate the rectangular coordinates *L*u*v** in the CIELUV color space for the sample with the tristimulus values *X,Y,Z*.**

$X = 25$	$X_0 = 94{,}81$	$L^* = ?$
$Y = 30$	$Y_0 = 100$	$u^* = ?$
$Z = 18$	$Z_0 = 107{,}32$	$v^* = ?$

EXAMPLE 2

$X = 25$	$X_0 = 94{,}81$	$L^* = ?$
$Y = 30$	$Y_0 = 100$	$u^* = ?$
$Z = 18$	$Z_0 = 107{,}32$	$v^* = ?$

$$u' = \frac{4X}{X+15Y+3Z}$$

$$v' = \frac{9Y}{X+15Y+3Z}$$

$$u_0 = \frac{4X_0}{X_0+15Y_0+3Z_0}$$

$$v_0 = \frac{9Y_0}{X_0+15Y_0+3Z_0}$$

EXAMPLE 2

$X = 25$	$X_0 = 94{,}81$	$L^* = ?$
$Y = 30$	$Y_0 = 100$	$u^* = ?$
$Z = 18$	$Z_0 = 107{,}32$	$v^* = ?$

$$u' = \frac{4X}{X + 15Y + 3Z} = \frac{4 \cdot 25}{25 + 15 \cdot 30 + 3 \cdot 18} = \frac{100}{529} = 0{,}1890$$

$$v' = \frac{9Y}{X + 15Y + 3Z} = \frac{9 \cdot 30}{25 + 15 \cdot 30 + 3 \cdot 18} = \frac{270}{529} = 0{,}5104$$

$$u_0 = \frac{4X_0}{X_0 + 15Y_0 + 3Z_0} = \frac{4 \cdot 94{,}81}{94{,}81 + 15 \cdot 100 + 3 \cdot 107{,}32} = \frac{379{,}24}{1916{,}77} = 0{,}1979$$

$$v_0 = \frac{9Y_0}{X_0 + 15Y_0 + 3Z_0} = \frac{9 \cdot 100}{94{,}81 + 15 \cdot 100 + 3 \cdot 107{,}32} = \frac{900}{1916{,}77} = 0{,}4695$$

EXAMPLE 2

$X = 25$	$X_0 = 94{,}81$	$L^* = ?$
$Y = 30$	$Y_0 = 100$	$u^* = ?$
$Z = 18$	$Z_0 = 107{,}32$	$v^* = ?$

$$L^* = 116\sqrt[3]{\frac{Y}{Y_0}} - 16$$

$$u^* = 13L^*(u' - u_0)$$

$$v^* = 13L^*(v' - v_0)$$

EXAMPLE 2

$X = 25$	$X_0 = 94{,}81$	$L^* = ?$
$Y = 30$	$Y_0 = 100$	$u^* = ?$
$Z = 18$	$Z_0 = 107{,}32$	$v^* = ?$

$$L^* = 116\sqrt[3]{\frac{Y}{Y_0}} - 16 = 116 \cdot \sqrt[3]{\frac{30}{100}} - 16 = \mathbf{61{,}6542}$$

$$u^* = 13L^*(u' - u_0) = 13 \cdot 61{,}6542 \cdot (0{,}1890 - 0{,}1979) = \mathbf{-7{,}0675}$$

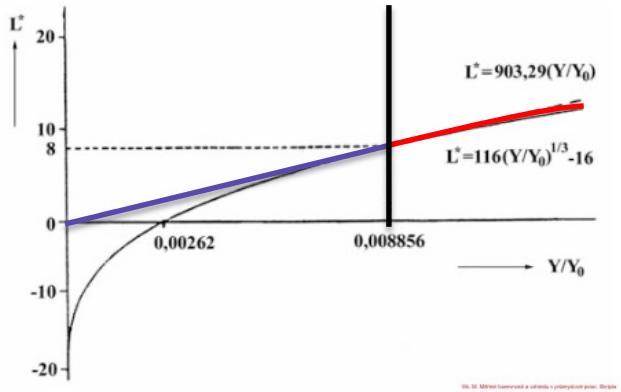
$$v^* = 13L^*(v' - v_0) = 13 \cdot 61{,}6542 \cdot (0{,}5104 - 0{,}4695) = \mathbf{32{,}7471}$$

EXAMPLE 3

- **Calculate the rectangular coordinates *L*a*b** in the CIELAB color space for the sample with the tristimulus values *X,Y,Z*.**

$X = 34$	$X_0 = 94{,}81$	$L^* = ?$
$Y = 50$	$Y_0 = 100$	$a^* = ?$
$Z = 30$	$Z_0 = 107{,}32$	$b^* = ?$

EXAMPLE 3



Tree-power transformation

$$\frac{Y}{Y_0} > 0,008856 \qquad Y^* = \left(\frac{Y}{Y_0}\right)^{1/3}$$

Linear transformation

$$\frac{Y}{Y_0} \leq 0,008856 \qquad Y^* = 7,787\frac{Y}{Y_0} + 0,138$$

EXAMPLE 3

Tree-power transformation

$$\frac{X}{X_0} > 0,008856 \qquad X^* = \left(\frac{X}{X_0}\right)^{1/3}$$

$$\frac{Y}{Y_0} > 0,008856 \qquad Y^* = \left(\frac{Y}{Y_0}\right)^{1/3}$$

$$\frac{Z}{Z_0} > 0,008856 \qquad Z^* = \left(\frac{Z}{Z_0}\right)^{1/3}$$

Linear transformation

$$\frac{X}{X_0} \leq 0,008856 \qquad X^* = 7,787\frac{X}{X_0} + 0,138$$

$$\frac{Y}{Y_0} \leq 0,008856 \qquad Y^* = 7,787\frac{Y}{Y_0} + 0,138$$

$$\frac{Z}{Z_0} \leq 0,008856 \qquad Z^* = 7,787\frac{Z}{Z_0} + 0,138$$

EXAMPLE 3

$$L^* = 116\sqrt[3]{\frac{Y}{Y_0}} - 16$$

$$a^* = 500 \cdot (X^* - Y^*)$$

$$b^* = 200 \cdot (Y^* - Z^*)$$

EXAMPLE 3

$$L^* = 116 \cdot \sqrt[3]{\frac{Y}{Y_0}} - 16 = 116 \cdot \sqrt[3]{\frac{50}{100}} - 16 = \mathbf{76,0693}$$

$$a^* = 500 \cdot (X^* - Y^*) = 500 \cdot (0{,}7105 - 0{,}7939) = \mathbf{-41,6187}$$

$$b^* = 200 \cdot (Y^* - Z^*) = 200 \cdot (0{,}7939 - 0{,}6539) = \mathbf{27,9695}$$

EXAMPLE 4

- **Calculate the color difference ΔE^* between sample (batch) and standard using coordinates *L*a*b** from CIELAB color space.**
- **Calculate the difference in lightness ΔL^*, in choma ΔC^* and in hue ΔH^* in cylindrical space CIELCH color space and all differences express verbally.**

$$L^*_{st} = 36,43 \qquad L^*_b = 39,80 \qquad \Delta L^* = ?$$
$$a^*_{st} = 48,65 \qquad a^*_b = 48,64 \qquad \Delta C^* = ?$$
$$b^*_{st} = 24,62 \qquad b^*_b = 22,21 \qquad \Delta H^* = ?$$
$$\Delta E^* = ?$$
$$\Delta a^* = ?$$
$$\Delta b^* = ?$$

EXAMPLE 4

$$L^*_{st} = 36,43 \qquad L^*_b = 39,80$$
$$a^*_{st} = 48,65 \qquad a^*_b = 48,64$$
$$b^*_{st} = 24,62 \qquad b^*_b = 22,21$$

$$\Delta E^* = \sqrt{(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2}$$

$$\Delta L^* = L^*_{batch} - L^*_{standard}$$

$$\Delta a^* = a^*_{batch} - a^*_{standard}$$

$$\Delta b^* = b^*_{batch} - b^*_{standard}$$

EXAMPLE 4

$$L^*_{st} = 36,43 \qquad L^*_{vz} = 39,80$$
$$a^*_{st} = 48,65 \qquad a^*_{vz} = 48,64$$
$$b^*_{st} = 24,62 \qquad b^*_{vz} = 22,21$$

$$\Delta L^* = L^*_{batch} - L^*_{standard} = 39{,}80 - 36{,}43 = \mathbf{3,37}$$

$$\Delta a^* = a^*_{batch} - a^*_{standard} = 48{,}64 - 48{,}65 = \mathbf{-0,01}$$

$$\Delta b^* = b^*_{batch} - b^*_{standard} = 22{,}21 - 24{,}62 = \mathbf{-2,41}$$

$$\Delta E^* = \sqrt{(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2} = \sqrt{(3{,}37)^2 + (-0{,}01)^2 + (-2{,}41)^2}$$
$$= \sqrt{17{,}1651} = \mathbf{4,1431}$$

EXAMPLE 4

$$L^*_{st} = 36,43 \qquad L^*_b = 39,80$$
$$a^*_{st} = 48,65 \qquad a^*_b = 48,64$$
$$b^*_{st} = 24,62 \qquad b^*_b = 22,21$$

$$C^* = \sqrt{(a^*)^2 + (b^*)^2}$$

$$C^*_{batch} = \sqrt{\left(a^*_{batch}\right)^2 + \left(b^*_{batch}\right)^2} = \sqrt{(48{,}64)^2 + (22{,}21)^2} = 53,4709$$

$$C^*_{standard} = \sqrt{\left(a^*_{standard}\right)^2 + \left(b^*_{standard}\right)^2} = \sqrt{(48{,}65)^2 + (24{,}62)^2} = 54,5249$$

$$\Delta C^* = C^*_{batch} - C^*_{standard} = 53{,}4709 - 54{,}5249 = -1{,}0541$$

EXAMPLE 4

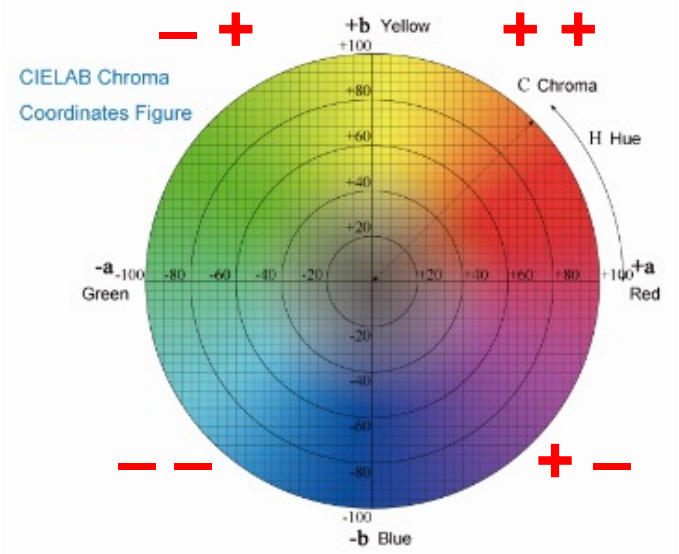
$$h_{ab} = arctan\left(\frac{b^*}{a^*}\right)$$

$$h_{ab,b} = arctan\left(\frac{22{,}21}{48{,}64}\right) = \mathbf{0{,}4283}$$

$$h_{ab,st} = arctan\left(\frac{24{,}62}{48{,}65}\right) = \mathbf{0{,}4685}$$

$$\Delta \boldsymbol{h_{ab}} = \boldsymbol{h_{ab,b}} - \boldsymbol{h_{ab,st}} = 0{,}4283 - 0{,}4685 = \mathbf{-0{,}0401}$$

EXAMPLE 4



Angle	a*	b*	add
0 – 90 °	+	+	nic
90 – 180 °	–	+	π
180 – 270 °	–	–	π
270 - 360 °	+	–	2π

EXAMPLE 4

$$\Delta H^* = 2 \cdot \sqrt{C_b^* C_{st}^*}\, sin\left(\frac{\Delta h_{ab}}{2}\right)$$

$$\Delta H^* = 2 \cdot \sqrt{53{,}4709 \cdot 54{,}5249}\, sin\left(\frac{-0{,}0401}{2}\right) = -2{,}1673$$

$$\Delta H^* = \sqrt{(\Delta E)^2 - (\Delta C^*)^2 - (\Delta L^*)^2}$$

$$= \sqrt{(4{,}1431)^2 + (-1{,}0541)^2 + 3{,}37^2} = \sqrt{4{,}6972} = 2{,}1673$$

EXAMPLE 4

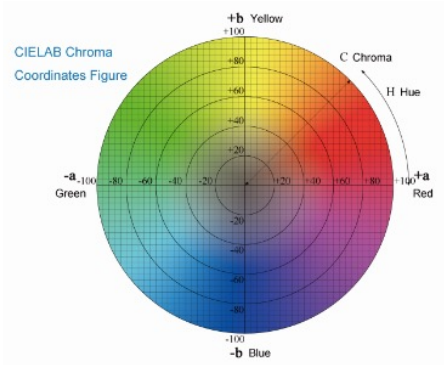
$$h_{ab,b} = arctan\left(\frac{22{,}21}{48{,}64}\right) = 0{,}4283 \cdot \frac{180}{\pi} = \mathbf{24,5424}$$

$$h_{ab,st} = arctan\left(\frac{24{,}62}{48{,}65}\right) = 0{,}4685 \cdot \frac{180}{\pi} = \mathbf{26,8423}$$

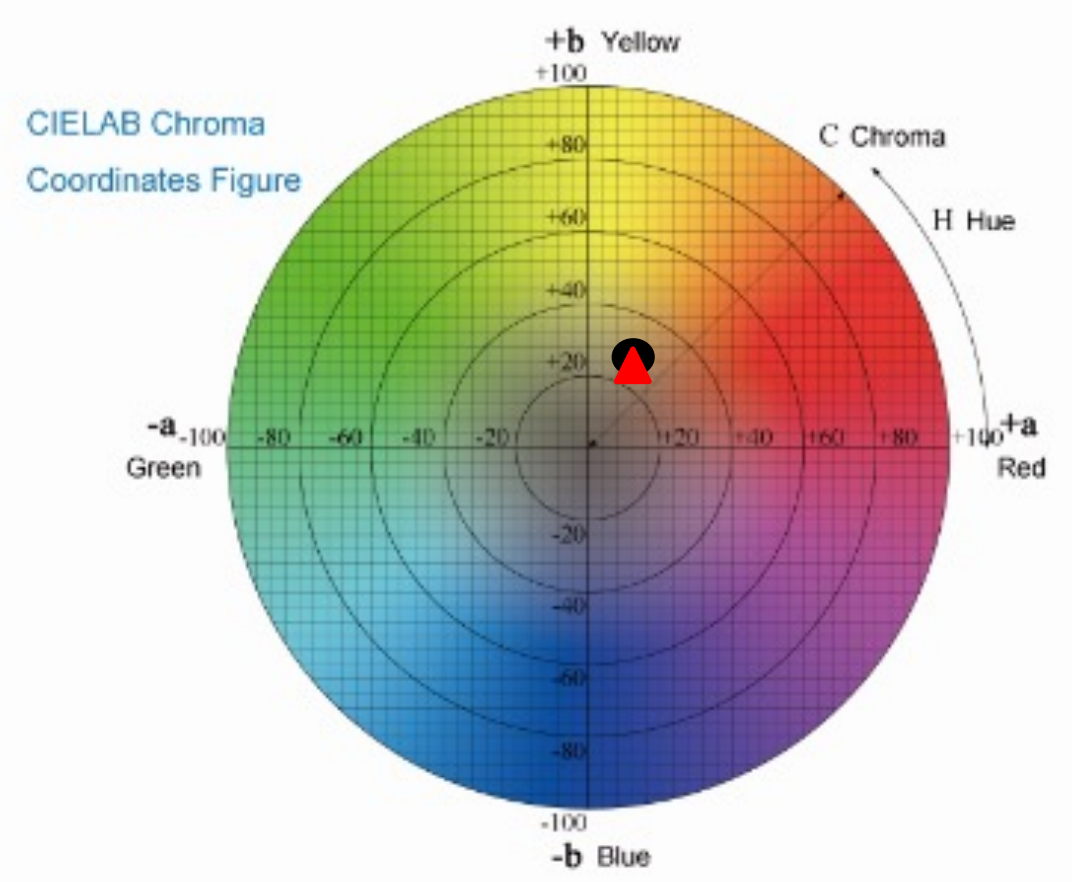
$$\boldsymbol{\Delta h_{ab} = h_{ab,b} - h_{ab,st} = -2,2999}$$

EXAMPLE 4

	verbal expression of differences
$+\Delta a^*$	redder
$-\Delta a^*$	greener
$+\Delta b^*$	yellowish
$-\Delta b^*$	bluer
$+\Delta L^*$	lighter
$-\Delta L^*$	darker
$+\Delta C^*$	more chromatic, cleaner
$-\Delta C^*$	less chromatic
$+\Delta H^*$	change in counterclockwise direction
$-\Delta H^*$	change in clockwise direction



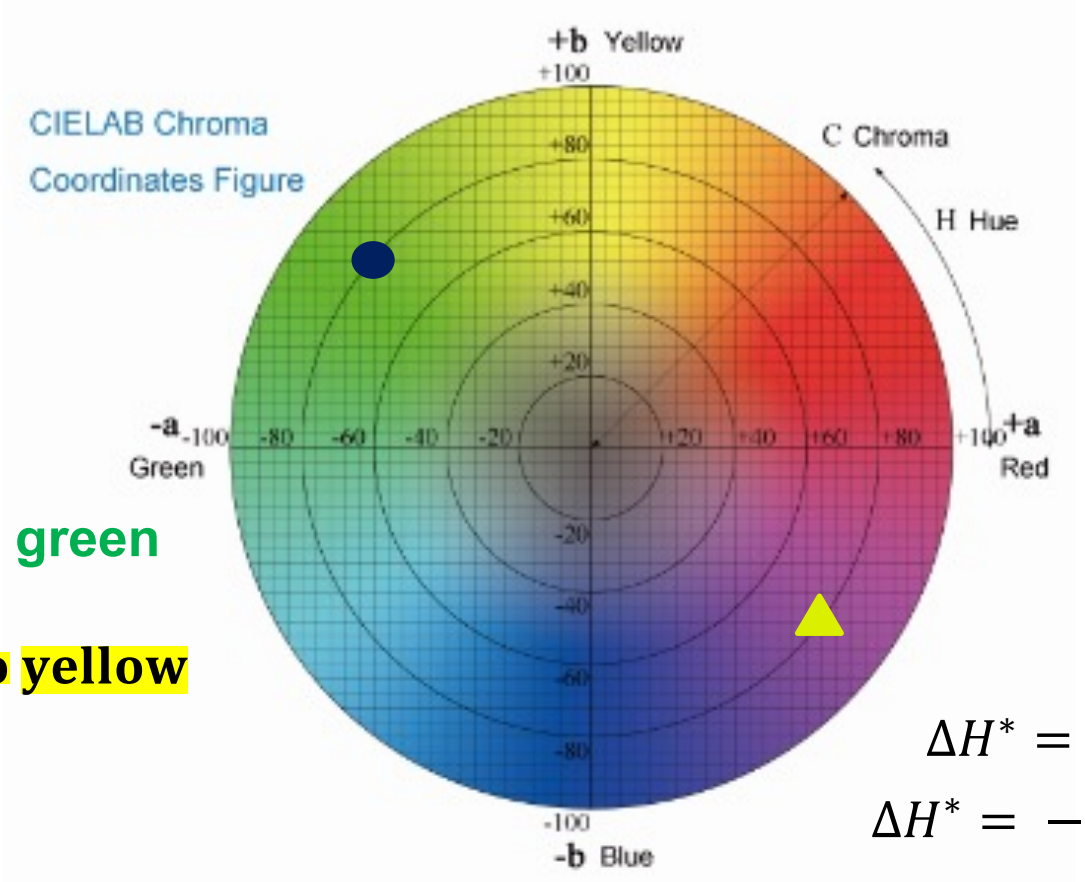
EXAMPLE 4



standard

sample (batch)

example



$\Delta H^* = +0{,}5$ → **to green**

$\Delta H^* = -0{,}5$ → **to yellow**

$\Delta H^* = +0{,}5$ → **to red**

$\Delta H^* = -0{,}5$ → **to blue**

EXAMPLE 5

- **Calculate the CIE whiteness W_{CIE} a CIE tint T_{CIE} (tint) for sample with tristimulus values.**

$X = 90$ $X_o = 94{,}81$ **x=?**

$Y = 94$ $Y_o = 100$ **y=?**

$Z = 105$ $Z_o = 107{,}32$ **W_{CIE}=?**

T_{CIE}=?

EXAMPLE 5

$$W_{CIE} = Y + 800 \cdot (x_0 - x) + 1700 \cdot (y_0 - y)$$

$$T_{CIE} = 900 \cdot (x_0 - x) - 650 \cdot (y_0 - y)$$

$$x = \frac{X}{X + Y + Z}$$

$$y = \frac{Y}{X + Y + Z}$$

EXAMPLE 5

$$x = \frac{\mathbf{X}}{\mathbf{X}+\mathbf{Y}+\mathbf{Z}} = \frac{90}{90+94+105} = \mathbf{0,3114}$$

$$y = \frac{\mathbf{Y}}{\mathbf{X}+\mathbf{Y}+\mathbf{Z}} = \frac{94}{90+94+105} = \mathbf{0,3253}$$

$$x_0 = \frac{\mathbf{X_0}}{\mathbf{X_0}+\mathbf{Y_0}+\mathbf{Z_0}} = \frac{94{,}81}{94{,}81+100+107{,}32} = \mathbf{0,3138}$$

$$y_0 = \frac{\mathbf{X_0}}{\mathbf{X_0}+\mathbf{Y_0}+\mathbf{Z_0}} = \frac{100}{94{,}81+100+107{,}32} = \mathbf{0,3310}$$

EXAMPLE 5

$$\boldsymbol{W_{CIE} = Y + 800 \cdot (x_0 - x) + 1700 \cdot (y_0 - y)}$$

$$= 94 + 800 \cdot (0{,}3138 - 0{,}3114) + 1700 \cdot (0{,}3310 - 0{,}3253)$$

$$= \mathbf{105{,}6398}$$

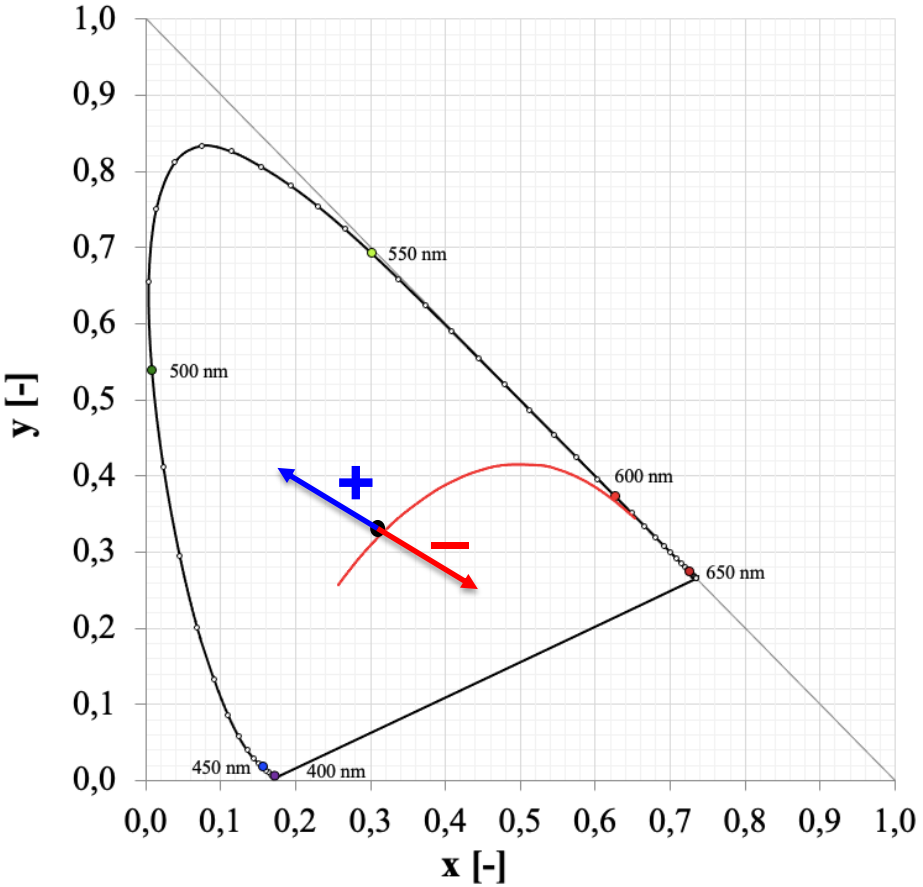
$$\boldsymbol{T_{CIE} = 900 \cdot (x_0 - x) - 650 \cdot (y_0 - y)}$$

$$= 900 \cdot (0{,}3138 - 0{,}3114) - 650 \cdot (0{,}3310 - 0{,}3253)$$

$$= \mathbf{-1{,}5725}$$

EXAMPLE 5

$$T_{CIE} = -1,5725$$



EXAMPLE 6

- **Calculate the difference between the values of the Kubelka–Munk function for the standard and sample (batch).**

$$R_{st} = 60\ \%$$

$$R_b = 40\ \%$$

$$K / S_{st} = ?$$

$$K / S_b = ?$$

$$\Delta K / S = ?$$

EXAMPLE 6

$$\frac{K}{S_{st}} = \frac{(1-\beta_{st})^2}{2\beta_{st}}$$

$$\frac{K}{S_{b}} = \frac{(1-\beta_{b})^2}{2\beta_{b}}$$

$$\Delta\frac{K}{S} = \frac{K}{S_{b}} - \frac{K}{S_{st}}$$

EXAMPLE 6

$$\frac{K}{S_{st}} = \frac{(1-\beta_{st})^2}{2\cdot\beta_{st}} = \frac{(1-0{,}6)^2}{2\cdot 0{,}6} = \mathbf{0,1333}$$

$$\frac{K}{S_{b}} = \frac{(1-\beta_{b})^2}{2\cdot\beta_{b}} = \frac{(1-0{,}4)^2}{2\cdot 0{,}4} = \mathbf{0,4500}$$

$$\Delta\frac{K}{S} = \frac{K}{S_{b}} - \frac{K}{S_{st}} = 0{,}45 - 0{,}1333 = \mathbf{0,3167}$$

EXAMPLE 7

- **Calculate the excitation (coordinate) purity p_E for hue with the coordinates given in the table together with the white point of CIE D65 (W) and the corresponding purest color of this hue (F_1).**

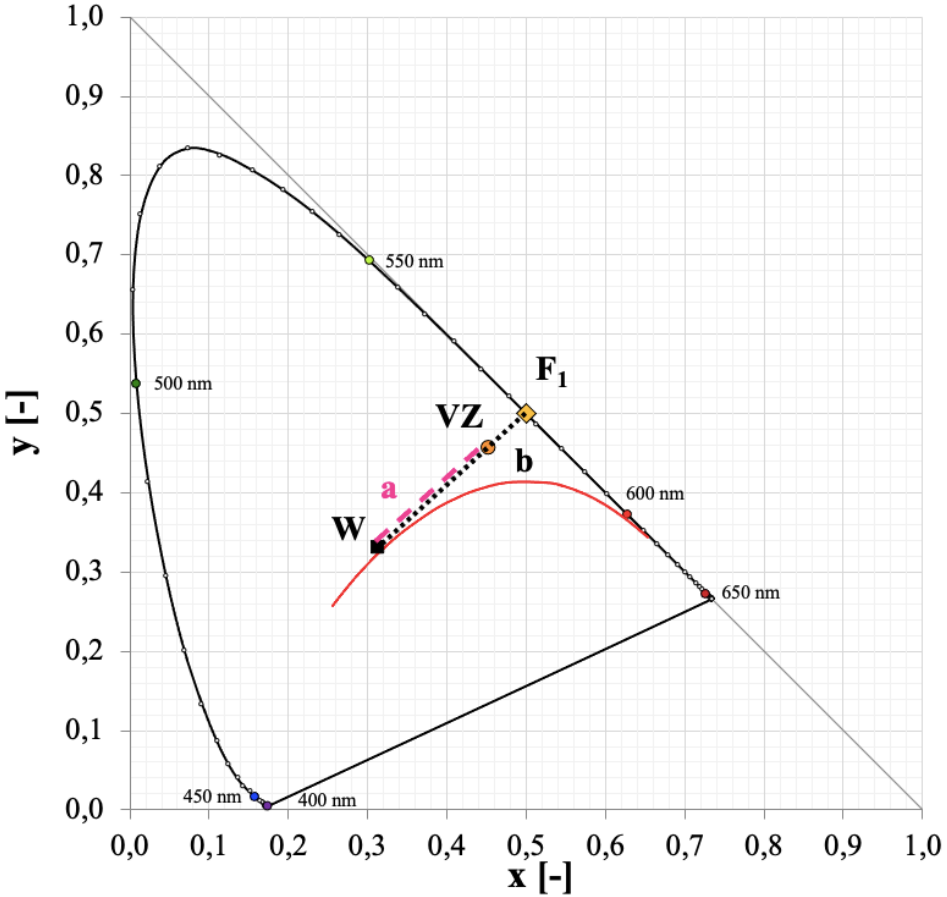
Sample VZ	$x = 0{,}4523$	$y = 0{,}4567$
Pure color F_1	$x_{F1} = 0{,}5$	$y_{F1} = 0{,}5$
White point W	$x_0 = 0{,}3138$	$y_0 = 0{,}3310$

EXAMPLE 7

$$p_{Ey} = \frac{y - y_0}{y_{F1} - y_0} = \frac{a}{b}$$

$$p_{Ex} = \frac{x - x_0}{x_{F1} - x_0} = \frac{a}{b}$$

$$p_{Ex} = p_{Ey}$$



EXAMPLE 7

$$p_{Ey} = \frac{y - y_0}{y_{F1} - y_0} = \frac{0{,}4567 - 0{,}3310}{0{,}5 - 0{,}3310} = \mathbf{0,7438}$$

$$p_{Ex} = \frac{x - x_0}{x_{F1} - x_0} = \frac{0{,}4523 - 0{,}3138}{0{,}5 - 0{,}3138} = \mathbf{0,7438}$$

EXAMPLE 8

- **Calculate the maximum wavelength λ_{max} (nm) of the maximum spectral density of the emission from an absolute blackbody at 7500 °C from Wien's displacement law. The value of Wien's displacement constant is b=0,002896 mK.**

$$\lambda_{max} = \frac{b}{T}$$

EXAMPLE 8

$$\boldsymbol{7500\,°C} \square \boldsymbol{7500 + 273,15 = 7773,15\,K}$$

$$\boldsymbol{\lambda_{max}} = \frac{\boldsymbol{b}}{\boldsymbol{T}} = \frac{0{,}002896}{7773{,}15} = 3{,}7256 \cdot 10^{-7} m$$

EXAMPLE 8

$$\boldsymbol{7500\ °C \rightarrow 7500 + 273,15 = 7773,15\ K}$$

$$\boldsymbol{\lambda_{max}} = \frac{\boldsymbol{b}}{\boldsymbol{T}} = \frac{0{,}002896}{7773{,}15} = 3{,}7256 \cdot 10^{-7} m$$

$$\boldsymbol{\lambda_{max}} = 3{,}7256 \cdot 10^{-7} m \rightarrow 3{,}7256 \cdot 10^{-7} \cdot 10^{+9} = 372{,}56\ nm$$

$$\boldsymbol{\lambda_{max} = 373\ nm}$$

FAKULTA TEXTILNÍ TUL

Thank you for attention

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