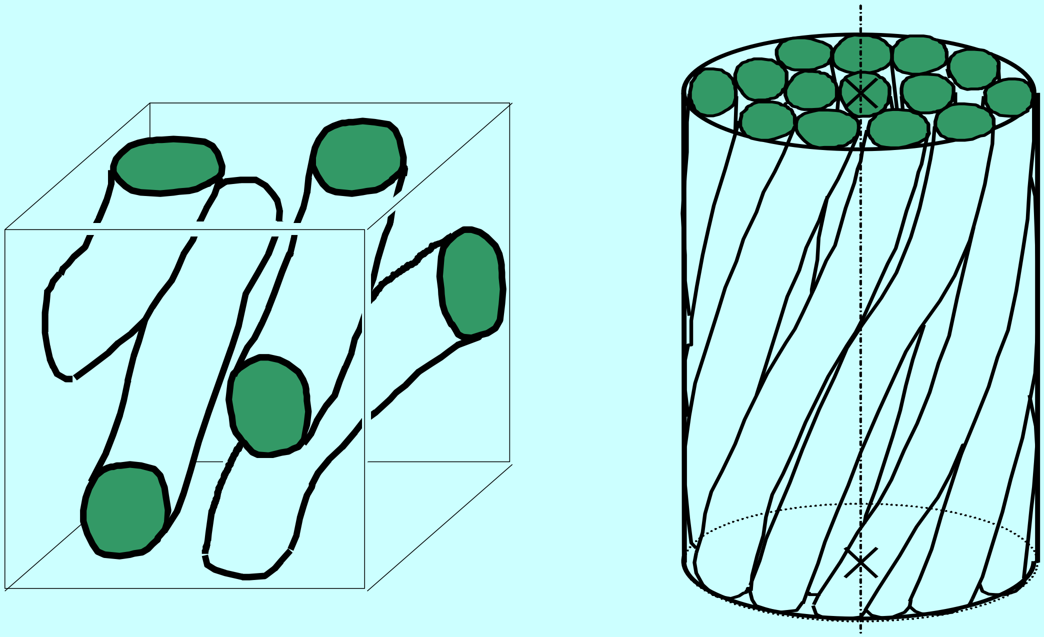
FIBERS AND YARNS: TERMS, DEFINITIONS AND RELATIONS



TERMS, DEFINITIONS AND RELATIONS

Staple fibers are the fundamental units of yarn. So, the structural theory of yarn must include the required parameters of fibers and their relationships. Some of them are introduced here.

One of most frequently used fiber parameters is the fiber fineness – the ratio between fiber mass and fiber length. The main physical unit of fiber fineness is 1Mtex, which is equal to 1kg/1m. But this unit is not very practical. A more useful unit is one-millionth of 1Mtex, i. e. 1tex and especially for fibers, 10 times smaller value, so-called "decitex" – dtex, is used.

Let us introduce the convention: All derived equations correspond to the international standard unit system.

FIBERS

Fiber length (mean)… l

Fiber mass… m

Fiber volume… V

Fiber surface area… A

Fiber cross section (green) area… s

Fiber perimeter… p

Fiber density… ρ

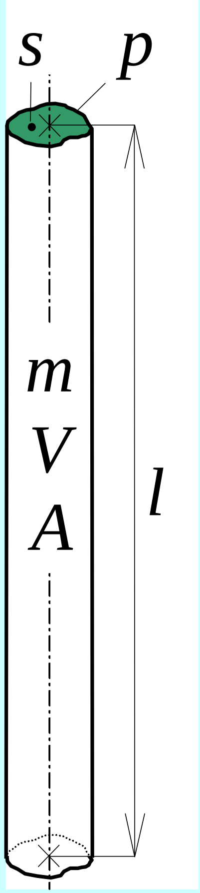
$$\rho = \frac{m}{V}$$

Fiber fineness… t

$$t = \frac{m}{l}$$

unit (SI): $1\text{kg}/1\text{m} = 1\text{Mtex}$

usually: $1\text{tex} = 10^{-6}\,\text{Mtex}$



Fineness of different types of fibers

Fibrous material	*Fineness*
Micro-fibers	< 1 dtex
Cotton and compatible chemical fibers	about 1,6 dtex
Wool and compatible chemical fibers	about 3,5 dtex
Coarse (carpet) fibers	> 7 dtex

Example: We consider a cotton fiber of 1.7 dtex fineness and 28 mm length. The fiber mass is 0.00476 mg. 1 kg of these cotton fibers has a total length of 5882 km. An ordinary shirt of 200 g contains fibers of total length 1176 km.

TERMS, DEFINITIONS AND RELATIONS

The shown equations are valid for fiber fineness.
From geometrical standpoint, the fiber "fineness" is characterized by the ratio V/l, but the standard fineness is moreover influenced by fiber density. Therefore, it is not correct to compare the finenesses of fibers having different densities by the standard fineness; it is better to use the ratio t/ρ. Fineness and density define the cross-sectional area. Cross-sectional area enables to evaluate the equivalent fiber diameter. For cylindrical fibers, the derivation is trivial. For non-cylindrical fibers, the same equation represents the diameter of a ring having the same cross-sectional area.

It is valid

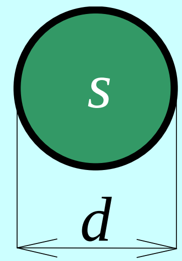
$$t = \frac{V\rho}{l},\quad t = \left(\frac{V}{l}\right)\rho,\quad V = \frac{tl}{\rho},\quad \frac{V}{l} = \frac{t}{\rho}$$

Cross-sectional area… s

- from geometry: $s = V/l = t/\rho$

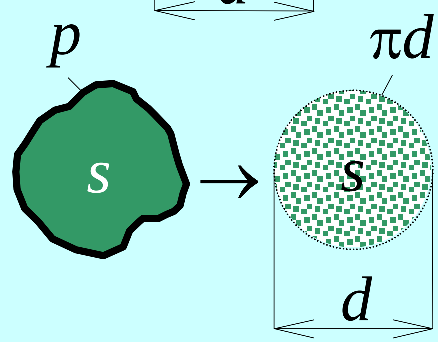
$$t = s\rho$$

Equivalent fiber diameter… d



$$s = \pi d^2/4,$$

$$d = \sqrt{4s/\pi} = \sqrt{4t/(\pi\rho)}$$



The perimeter of a non-cylindrical fiber is always higher than the perimeter of a ring having the same cross-sectional area. In this context, the fiber shape factor, according to Malinowska, is illustrated in the table.
If we know the (typical) shape factor of our fiber in advance, we can calculate its perimeter from the equation as shown here.

Fiber shape factor… q

$$q=\frac{p}{\pi d}-1\geq 0$$ (Malinowska)

Shape of fiber cross-section	$q\,[1]$
Circle – ideal ○	0
Circle - real	0 to 0,07
Triangle - ideal △	0,29
Triangle - real	0,09 to 0,12
Mature cotton	0,20 to 0,35
Irregular saw	> 0,60

Fiber perimeter… p

$$p=\pi d\left(1+q\right)$$

TERMS, DEFINITIONS AND RELATIONS

A lot of important properties of textiles (especially transport of moisture) depends on the total area of fibrous surface. Therefore, it is very useful to know the surface area per unit mass of fibrous material. The derivation of the so-called specific surface area is shown here.

(Note: We can neglect the topmost and bottommost cross sectional areas of fibers, because as fiber is very long, these two areas are relatively very small – e.g. 0.025%.)

The common value of specific surface area is often surprisingly high. It is illustrated in the given example.

Fiber surface... A

$$A = pl = \pi d\left(1+q\right)l$$

Fiber mass... m

$$m = V\rho = \left(\pi d^2/4\right)l\rho$$

Specific surface area...a

$$a = \frac{A}{m} = \frac{\pi d\left(1+q\right)l}{\left(\pi d^2/4\right)l\rho} = \frac{4\left(1+q\right)}{\rho d}$$

Example: Cotton shirt, ρ= 1520kgm^{-3}, q=0.28. The specific surface a=300.5 m^{2}kg^{-1}. The weight of shirt is 0.2 kg. So, the total surface area of this shirt is 60.1 m^2.

TERMS, DEFINITIONS AND RELATIONS

The physical definition of mechanical (normal) stress is the ratio of force and the corresponding area (Pa, MPa). On the other hand, by the term stress in textiles, we mean the ratio of force and the corresponding fiber fineness (N/tex, cN/dtex, also "breaking length km"). The relationship between these two definitions is given on the right-hand side.

The given example illustrates that

1. It is not correct to compare stress or strength of fibers having different densities by using the textile units (e.g. N/tex, breaking length km).
2. Some textile fibers are more tenacious than the ordinary steel.

Mechanical stress…σ^*

$$\sigma^* = F/s$$

where F…tensile force

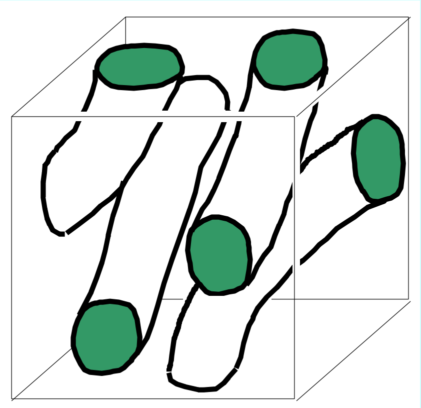
Stress in textiles…σ

$$\sigma = F/t = F/(s\rho), \quad \sigma = \frac{\sigma^*}{\rho} \quad \sigma^* = \sigma\rho$$

Example: Cotton fiber, tensile strength σ=0.32 Ntex^{-1}, ρ= =1520 kgm^{-3}; $\Rightarrow$ σ^*=487 MPa. PET fiber, tensile strength σ= =0.43 Ntex^{-1} (133% of co.), ρ=1360 kgm^{-3} $\Rightarrow$ σ^*=585 MPa (120% of co.) Ordinary steel, σ^*=500 MPa.

Inside of the textile fibrous assembly, or inside of some spatial part of them, there lies fiber volume V. Total volume of this body is called V_c.
The compactness of this body is characterized by the ratio between these 2 volumes and is known as fiber packing density.
(Note: Alternatively, this value is called as packing ratio or volume fraction.)
Evidently, the fiber packing density value must lie in the interval from 0 to 1.

FIBROUS ASSEMBLY



Volume of fibers (only)…V

Total volume (incl. air)…V_c

It is valid

$$V \le V_c$$

Compactness of fibrous assemblies – **fiber packing density**…μ

1. Definition: $\mu = \dfrac{V}{V_c}, \ \mu \in \langle 0, 1 \rangle$

TERMS, DEFINITIONS AND RELATIONS

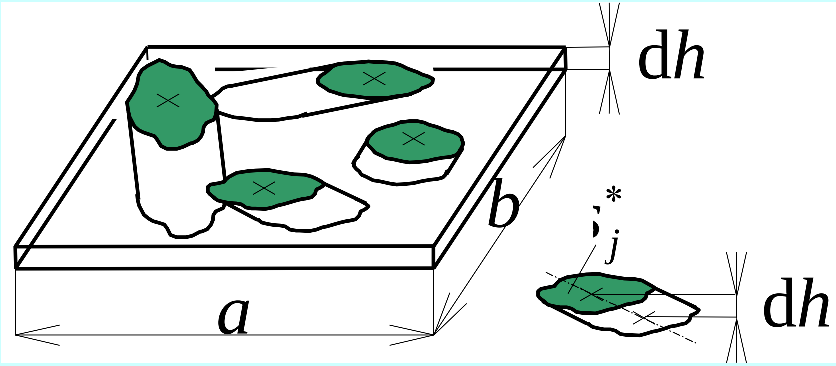
The table on the right hand side illustrates the approximated values of fiber packing density for different materials.
(Note: The packing density values around 0.4 to 0.5 are specific just for the twisted staple yarns and practically no other fibrous structures have this value. Therefore, the yarns represent very specific type of fibrous assembly.)

Group	*Fibrous assemblies*	μ [1]
Linear textiles	Monofilament	1
	Limit structure*)	0,907
	Hard twisted silk	0,75 to 0,85
	Wet spun linen yarn	about 0,65
	Combed cotton yarn	0,5 to 0,6
	Carded cotton yarn	0,38 to 0,55
	Worsted yarn	0,38 to 0,50
	Woolen yarn	0,35 to 0,45
	Cotton roving	0,10 to 0,20
	Sliver	about 0,03
Other textiles	Woven fabric	0,15 to 0,30
	Knitted fabric	0,10 to 0,20
	Cotton wool**)	0,02 to 0,04
	Leather (textiles)**)	0,005 to 0,02
Other materials	Earthenware**)	0,20 to 0,23
	Wood**)	0,3 to 0,7
	Animal leather**)	0,33 to 0,66

*) Theoretical value, **) Rough values

Let us imagine a very (elementary) flat box as shown here. The lengths of fiber sections inside this box are elementary short; hence they can be interpreted as straight abscissas. It is now possible to derive the total volume and the volume of fibers.

2. Areal interpretation



$\mathrm{d}V_c = ab\,\mathrm{d}h = S_c\,\mathrm{d}h,$ where $ab = S_c$

Volume of j-th ($j = 1, 2, \cdots, N$) fiber element is $s_j^*\,\mathrm{d}h$

Volume of all fiber elements

$$\mathrm{d}V = \sum_{j=1}^{N} \left(s_j^*\,\mathrm{d}h\right) = \mathrm{d}h \sum_{j=1}^{N} s_j^* = S\,\mathrm{d}h$$

where S is the total sectional (green) area of all elements

TERMS, DEFINITIONS AND RELATIONS

Now the fiber packing density of our flat box is the ratio between two areas.
(Note: Fiber packing density in yarn is often expressed as the ratio between two cross-sectional areas.)
Next interpretation is based on the densities. The mass of fibrous assembly is given by the mass of fibrous material only. Using total volume of fibrous assembly, we can define the density of fibrous assembly. Using only volume of fibers, we can define the density of fibers.
Fiber packing density is the ratio between these two densities now.

Area packing density

$$\mu = \frac{\mathrm{d}V}{\mathrm{d}V_\mathrm{c}} = \frac{S\,\mathrm{d}h}{S_\mathrm{c}\,\mathrm{d}h} \qquad \mu = \frac{S}{S_\mathrm{c}}$$

3. Mass interpretation

Density of fibrous assembly ρ^*

$$\rho^* = m/V_\mathrm{c} \qquad \left(V_\mathrm{c} = m/\rho^*\right)$$

Fiber density

$$\rho = m/V, \quad \left(V = m/\rho\right)$$

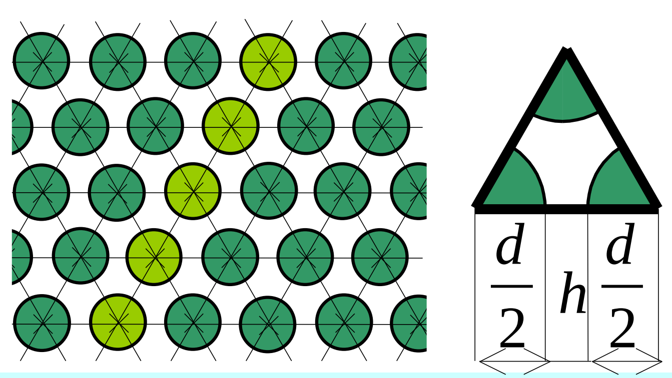
Packing density factor

$$\mu = \frac{V}{V_\mathrm{c}} = \frac{m/\rho}{m/\rho^*}, \qquad \mu = \frac{\rho^*}{\rho}$$

TERMS, DEFINITIONS AND RELATIONS

Sometimes an idealized fiber structure is used. Let us imagine parallel cylindrical fibers in regular, the so-called “hexagonal” form, as shown here. The shown triangle creates the structural unit. So, the fiber packing density can be derived from this one.

Idealized fibrous assembly



$$S_c = (d+h)\,(d+h)\cos 30^\circ/2 = \left(\sqrt{3}/4\right)(d+h)^2$$

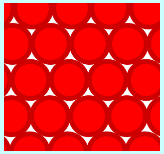
$$S = \left(\pi d^2/4\right)/2 = \pi d^2/8$$

$$\mu = \frac{S}{S_c} = \frac{\pi d^2/8}{\left(\sqrt{3}/4\right)(d+h)^2} = \left[\pi/\left(2\sqrt{3}\right)\right]/(1+h/d)^2$$

TERMS, DEFINITIONS AND RELATIONS

A special case of the idealized structure is the so-called "limit structure", where the distance h=0. (See red illustration.) The corresponding value of fiber packing density is shown on the right-hand side.
An important characteristic, especially for yarn, is the coefficient k_n.
A general section plane α intersects the fibrous assembly and it results a section of area S_c (light green). Inside this plane, there are N cut fibers of total sectional area S. So, the mean sectional area per 1 fiber is S/N. The coefficient k_n is defined by the given equation.
(Note: Usually k_n<1. Only for parallel fiber bundle, k_n=1.)

Limit structure: h=0

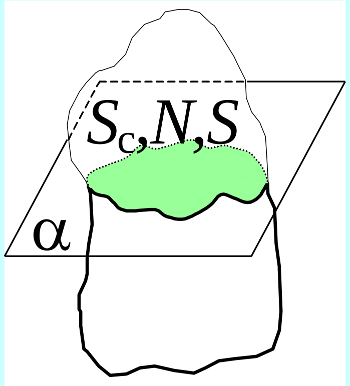


$$\mu = \left[\pi \big/ \left(2\sqrt{3}\right)\right] \Big/ \left(1 + \overset{=0}{h} \big/ d\right)^2$$

$$\mu = \pi \big/ \left(2\sqrt{3}\right) \cong 0.903$$

Coefficient k_n

α…section plane
S_c…section area
N…fiber number
S…area of fibers
$\overline{s^*} = S/N$…mean fiber section



Definition: $k_n = s \big/ \overline{s^*}$

TERMS, DEFINITIONS AND RELATIONS

Twisted staple yarn is a special type of fibrous assembly. For its description we use quantities, as shown.
Yarn fineness ("yarn count") is one of most important standard values, but it is not enough well.
The ratio fiber volume per unit length characterizes the yarn "size" geometrically well. But, yarn fineness is moreover influenced by fiber mass density. Therefore, it is not possible to compare the yarns from different fibers using yarn "count" (e.g. cotton and PES yarns). In this case, we can compare only ratios yarn count to fiber mass density, as shown here.

YARNS

Fiber parameters:

t...fiber fineness

ρ...fiber density

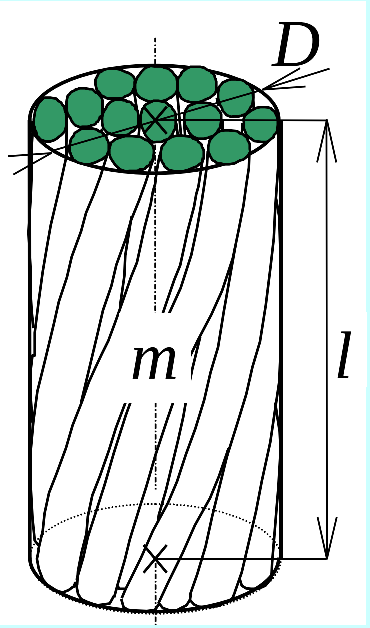
Yarn parameters:

T...yarn fineness ("yarn count")

Z...yarn twist

D...yarn diameter

n...number of fibers in yarn

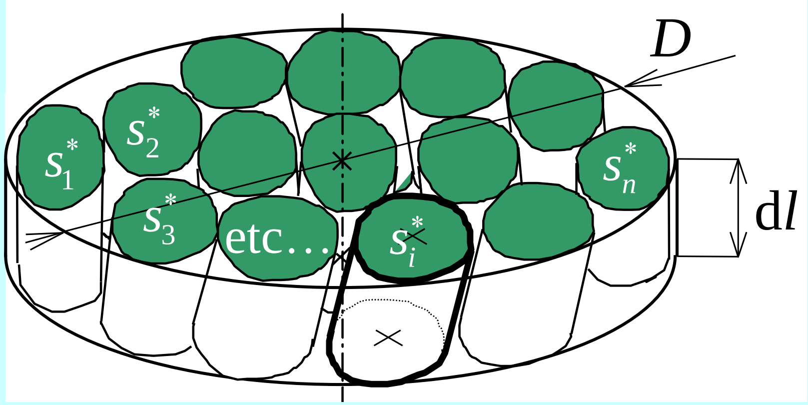


Yarn fineness

$$T = \frac{m}{l} = \frac{V\rho}{l}, \quad T = \frac{V}{l}\rho \qquad \frac{V}{l} = \frac{T}{\rho}$$

Let us imagine an elementary short part of yarn. We call the sum of the areas of all fiber sections in yarn cross-section as substance cross-sectional area of yarn (Johansen). It is shown that the substance cross-sectional area of yarn is equal to the ratio yarn count to fiber mass density, or fiber volume per unit length of yarn.

Substance cross-section



Definition: $S=\sum_{i=1}^{n} s_i^*$ (green)

$$\mathrm{d}V=\sum_{i=1}^{n} s_i^* \,\mathrm{d}l=\mathrm{d}l\sum_{i=1}^{n} s_i^*=\mathrm{d}l\, S$$

$$\mathrm{d}m=\mathrm{d}V\,\rho=\mathrm{d}l\, S\rho$$

$$T=\mathrm{d}m/\mathrm{d}l=\mathrm{d}l\, S\rho/\mathrm{d}l=S\rho$$

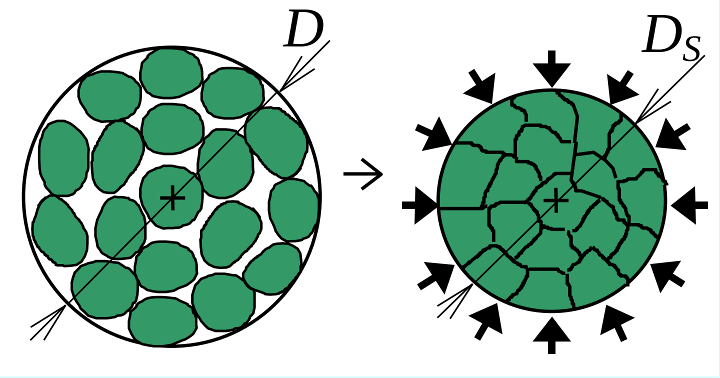
Subst. cross-section $S=\dfrac{T}{\rho}=\dfrac{V}{l}$

Let us imagine – hypothetically – that we compress the yarn (from some plastic fibers) so that we will remove all air from the inside of yarn. Then the yarn cross-sectional area will be equal to the substance cross sectional area and hence the compressed yarn will have the substance diameter. (Note: The diameter of real yarn is every time higher than the substance diameter.)

The ratio of yarn fineness (count) to fiber fineness is another important quantity and is called relative fineness of yarn.

(Note: We must note that this value is not identical with number of fibers in yarn cross-section. This will be discussed later.)

Substance diameter



$$S=\frac{\pi D_S^2}{4},\qquad D_S=\sqrt{\frac{4S}{\pi}}=\sqrt{\frac{4T}{\pi\rho}}$$

Relativ fineness

$$\tau=T/t \qquad \tau=S\rho/(s\rho), \qquad \tau=S/s$$

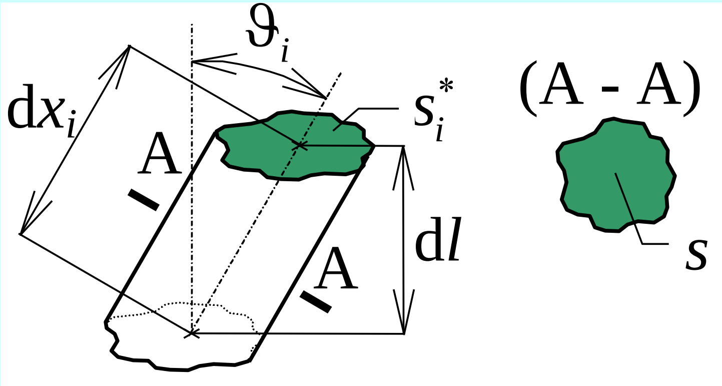
$$\tau=\left(\pi D_S^2/4\right)/\left(\pi d^2/4\right) \qquad \tau=\left(D_S/d\right)^2$$

τ is not identical to n !

TERMS, DEFINITIONS AND RELATIONS

Let us analyze an element of one fiber from the elementary short part of a yarn. The fiber sectional area is the function of fiber slope, as shown here.
Using this equation, the substance cross-sectional area and the mean fiber sectional area can be expressed.

Coefficient k_n



Geometry – angle: $\cos\vartheta_i = \mathrm{d}l/\mathrm{d}x_i$

– volume $s_i^* \,\mathrm{d}l = s\,\mathrm{d}x_i$

$$\Rightarrow \quad s_i^* = s\,\mathrm{d}x_i/\mathrm{d}l\,, \quad s_i^* = s/\cos\vartheta_i$$

$$S = \sum_{i=1}^{n} s_i^* = \sum_{i=1}^{n} \frac{s}{\cos\vartheta_i} = s\sum_{i=1}^{n}\frac{1}{\cos\vartheta_i}$$

$$\overline{s^*} = \frac{S}{n} \qquad \overline{s^*} = \frac{s}{n}\sum_{i=1}^{n}\frac{1}{\cos\vartheta_i}$$

TERMS, DEFINITIONS AND RELATIONS

The coefficient k_n is the ratio between the fiber cross-sectional area and the mean fiber sectional area. It is given by the equation on the right hand side.
It is shown that k_n is the weighted harmonic mean value of cosines of slope angles.
Only, if each angle is equal to 0 (so that cos 0 = 1), the coefficient k_n=1. This configuration corres-ponds to the parallel fiber bundle.

$$k_n = \frac{s}{\overline{s^*}} = \frac{s}{\frac{s}{n}\sum_{i=1}^{n}\frac{1}{\cos\vartheta_i}}$$

Coefficient:

$$k_n = \frac{1}{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{\cos\vartheta_i}}$$

It is also valid

$$\frac{1}{k_n} = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{\cos\vartheta_i}$$

k_n is the harmonic mean value of reciprocal values of $\cos\vartheta_i$

k_n=1 for parallel fiber bundle only

TERMS, DEFINITIONS AND RELATIONS

The coefficient k_n is related to the relative fineness and number of fibers, as shown here.
(Note: Beyond the scope of parallel fiber bundle, the number of fibers in yarn cross section is always smaller than the relative yarn fineness.)
The given equation also enables to determine the coefficient k_n experimentally.
The fiber packing density in yarn can be derived based on its definition and equations as discussed before.

Number of fibers in yarn

$$n = \frac{S}{\bar{\bar{s}}^*} = \underbrace{\left(\frac{S}{\bar{s}}\right)}_{=\tau}\underbrace{\left(\frac{\bar{s}}{\bar{\bar{s}}^*}\right)}_{=k_n}, \quad \boxed{n = \tau k_n}$$

Note: Experim. evaluation of k_n based on the eqn. $k_n = \dfrac{n}{\tau} = \dfrac{nt}{T}$

Packing density of yarn

$$(V/l = T/\rho = S) \quad V = Tl/\rho = Sl$$

$$V_c = \pi D^2 l/4$$

Packing density

$$\mu = \frac{V}{V_c} = \frac{Sl}{\pi D^2 l/4}, \quad \boxed{\mu = \frac{4S}{\pi D^2} = \frac{4T}{\pi D^2 \rho}}$$

TERMS, DEFINITIONS AND RELATIONS

Fiber packing density of yarn is also possible to derive alternatively by using the substance diameter.
If we know the value of packing density, it will be possible to evaluate yarn diameter by the equation as shown here.
The quantities Ks and K are called as diameter multipliers.
(Note: Quantities, related to the substance cross section, we will call "areal". Above all, they are important theoretically. Quantities, related to the yarn fineness (count) are common quantities as used in praxis.)
(Note: K_s and/or K are constant, only when the packing density is constant.)

Alternatively: $\left(S = \pi D_S^2/4\right)$

$$\mu = \frac{4S}{\pi D^2} = \frac{4\left(\pi D_S^2/4\right)}{\pi D^2} \qquad \mu = \left(\frac{D_S}{D}\right)^2$$

Yarn diameter

$$D = \sqrt{\frac{4S}{\pi\mu}}, \qquad D = \underbrace{\left(2/\sqrt{\pi\mu}\right)}_{=K_S}\sqrt{S}$$

$$D = \sqrt{\frac{4T}{\pi\mu\rho}}, \qquad D = \underbrace{\left(2/\sqrt{\pi\mu\rho}\right)}_{=K}\sqrt{T}$$

Diameter multipliers:

K_S ... areal multiplier, [1]
K ... common multiplier

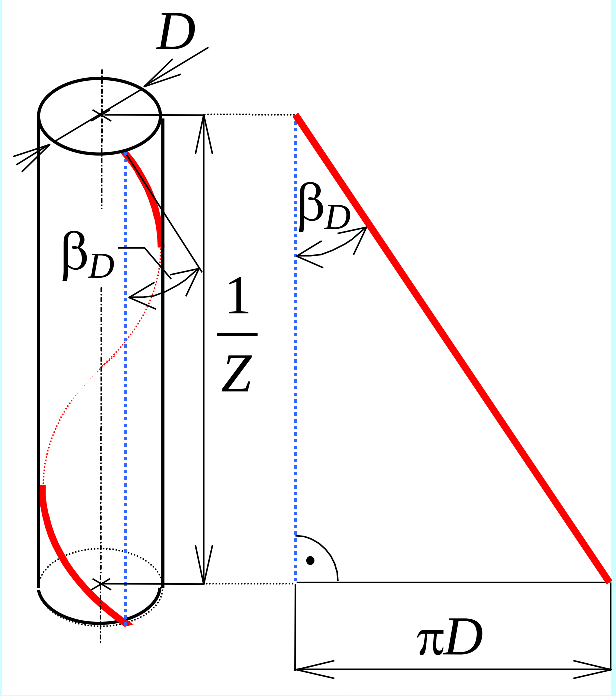
Some of the yarn characteristics are based on the yarn twist. Because the dimension of twist is [m^{-1}] and the dimension of yarn diameter is [m], the product of DZ is dimensionless and therefore, it is an interesting value. Usually we multiply this value by π and call it as twist intensity.
Let us assume that the peripheral (red) fiber of yarn follows the helical path. In this case, the twist intensity is also the tangent of angle β_D of this fiber.

Twist intensity

Definition: $\kappa = \pi D Z$, [1]

Geometrical interpretation:
(helical fibers on yarn surface)

$$\operatorname{tg}\beta_D = \frac{\pi D}{1/Z} = \pi D Z$$



$$\kappa = \tan\beta_D$$

TERMS, DEFINITIONS AND RELATIONS

The so called “twist coefficients” (twist multipliers) create another group of twist characteristics.
We introduce 2 types of coefficients:
First type is related to the yarn substance cross-sectional area S. These coefficients are called as “areal”; above all, they are important theoretically.
Second type is related to the yarn fineness (count). These coefficients are commonly used in praxis.
General coefficients use a general value of twist exponent q. Koechlin’s coefficients use the twist exponent 0.5. These can be also written in an alternative form as shown here.

Twist coefficients (def.)

1. General (q… twist expon.)

$$\alpha_S^* = ZS^q$$ … areal

$$\alpha^* = ZT^q$$ … common

2. Koechlin’s type

$$\alpha_S = Z\sqrt{S}$$ … areal, [1]

$$\alpha_S = Z\sqrt{S} = \frac{\kappa}{\pi D}\sqrt{\pi D_S^2/4} =$$

$$= \frac{\kappa}{2\sqrt{\pi}}\sqrt{\left(D_S/D\right)^2}$$

$$\alpha_S = \frac{\kappa\sqrt{\mu}}{2\sqrt{\pi}}$$

TERMS, DEFINITIONS AND RELATIONS

(Note: The common version of Koechlin's coefficient is usually used in the spinning mills.)
(Note: The use of theoretical relations related to the Koechlin's coefficients in the yarn models will be discussed in one of our lectures.)
The term <u>Phrix coefficients</u> (marked by a) is well known among the textile researchers in Czech Republic. They use the twist exponent 2/3. The common version of Phrix coefficient is used in praxis according to the Czech Standards.

$\alpha = Z\sqrt{T}$ ... common

$$\alpha = Z\sqrt{T} = \frac{\kappa}{\pi D}\sqrt{S\rho} =$$

$$= \frac{\kappa}{\pi D}\sqrt{\left(\pi D_S^2/4\right)\rho} =$$

$$= \frac{\kappa}{2\sqrt{\pi}}\sqrt{\left(D_S/D\right)^2 \rho}$$

$$\alpha = \frac{\kappa\sqrt{\mu\rho}}{2\sqrt{\pi}}$$

3. Phrix type (CZ standard)

$a_S = ZS^{2/3}$ ... areal

$a = ZT^{2/3}$ ... common

TERMS, DEFINITIONS AND RELATIONS

The dimensionless quantities are always very interesting, because they characterize the internal relationships in material body, processes. etc.
The list of 6 derived dimensionless quantities of yarn is shown on the right hand side. The knowledge of 4 of them can substitute the knowledge of 4 input characteristics of yarn.

Dimensionless quantities

1) relative fineness τ,
2) coefficient k_n,
3) fiber packing density μ,
4) areal diameter multiplier K_s,
5) twist intensity κ,
6) Koechlin's areal twist coefficient α_S

In place of the yarn parameters T, Z, D and n it is possible to use 4 of dimensionless parameters – e.g. τ, κ, μ, and k_n.