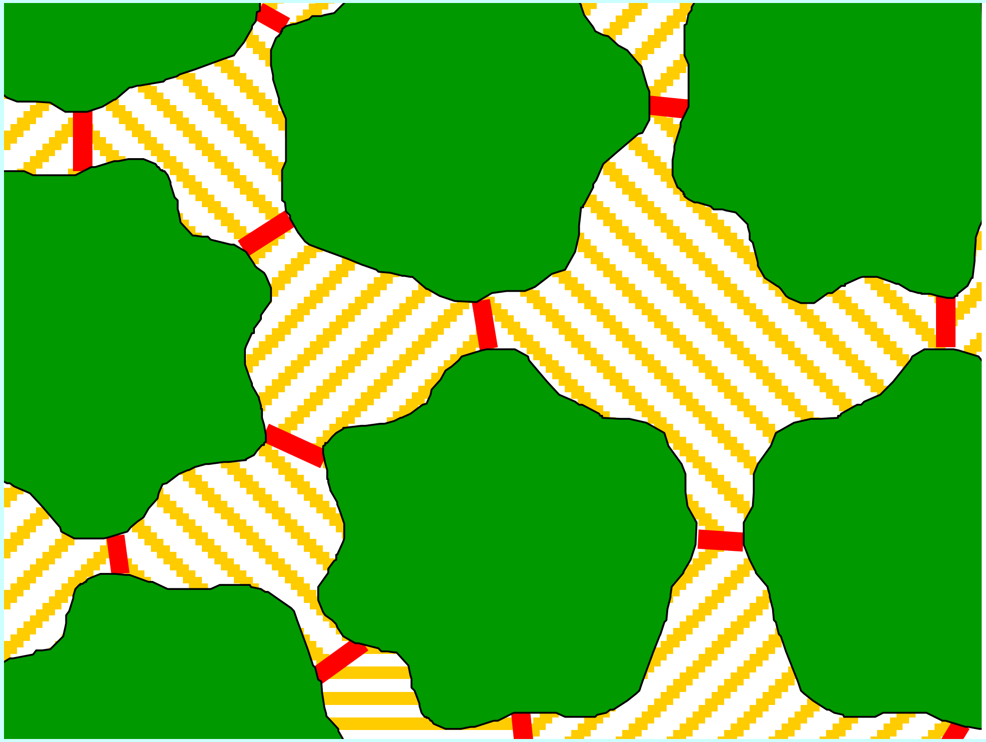
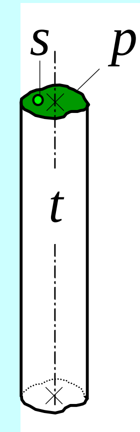
PORES AMONG FIBERS



GENERAL DESCRIPTION



t...fiber <u>fineness</u>, s...fiber <u>cross-sectional area</u>
ρ...material fiber <u>density</u>, d...equivalent fiber <u>diameter</u>,
p...fiber <u>circumference</u>, q...fiber <u>shape factor</u>,
a...<u>specific fiber surface</u> area, L...<u>total length</u> of fibers,
A...<u>total surface</u> area of fibers, V...<u>total volume of fib.</u>,
V_c...<u>total volume of fiber assembly</u>, μ...<u>packing density</u>

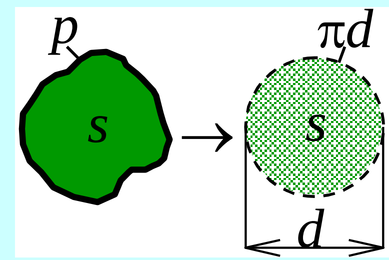
It was derived (lecture 1): :

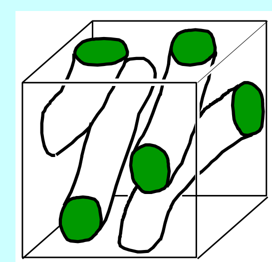
1. $t = s\rho, \quad s = \pi d^2/4, \quad d = \sqrt{4s/\pi} = \sqrt{4t/(\pi\rho)}$

2. $q = p/(\pi d) - 1 \geq 0, \quad p = \pi d(1+q)$

3. $A = pL = \pi d(1+q)L, \quad a = 4(1+q)/(\rho d)$

4. $\mu = V/V_c$, where $V = Ls = L\pi d^2/4$





Furthermore let us define the surface area per unit volume of fiber: $\gamma = \frac{A}{V} = \frac{\pi d\left(1+q\right)L}{L\pi d^2/4}$, $\gamma = A/V = 4\left(1+q\right)/d = a\rho$

Pores and their characteristics

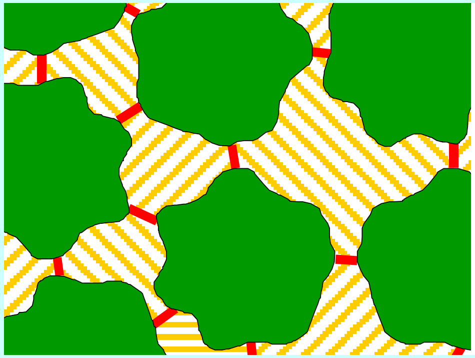
Volume of free space among fibers: $V_p = V_c - V$

Porosity (relative characteristic of this space):

$$\psi = V_p/V_c = \left(V_c - V\right)/V_c = 1 - V/V_c \; (=\mu), \qquad \psi = V_p/V_c = 1-\mu,$$

Porosity characterizes volume of free space among fibers, but not the size of gaps among fibers.

Therefore, we divide these spaces by **fictive (imaginary) borders** (****) to produce some suitable bodies in the form of small tubes or capillaries, called **pores**.



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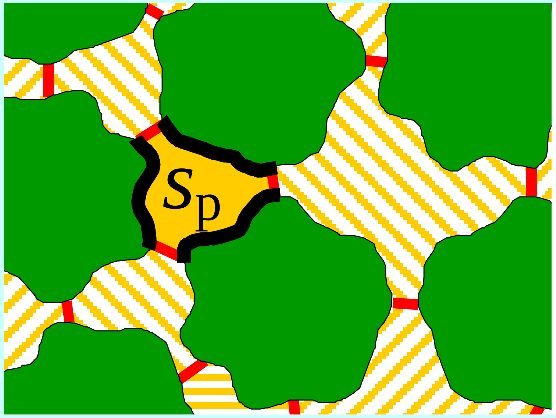
Each created pore (e.g. yellow):



- is in contact with fibers (black - real borders) and also with other pores (red - fictive borders).
- looks like an ***"air fiber"***. (Therefore, all equations derived for real fibers are valid for these "air fibers", too. Subscript "p" like 'pore' will be used for "air fibers".)

Pore sectional area: $s_p = \pi d_p^2/4, \quad d_p = \sqrt{4 s_p/\pi}$

where d_p…**equivalent pore diameter**

Perimeter of pore p_p – is defined as the length of **REAL** (black) **BORDERS ONLY!** (In fact, fictive borders do not exist.) Therefore p_p may be shorter than the perimeter of a circle having the same area; $0 \le p_p \lessgtr \pi d_p^2/4$

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Pore shape factor: $q_{\mathrm{p}} = p_{\mathrm{p}} / (\pi d_{\mathrm{p}}) - 1, \quad q_{\mathrm{p}} \geq -1$

(Considering the definition of p_{p} the pore shape factor could be negative.) It is also valid $p_{\mathrm{p}} = \pi d_{\mathrm{p}} (1 + q_{\mathrm{p}})$.

Assumption (simplification): All pore shapes in a fiber assembly are the same. Then all equations are valid for each pore.

Total pore volume: $V_{\mathrm{p}} = \overset{\pi d_{\mathrm{p}}^2/4}{s_{\mathrm{p}}} L_{\mathrm{p}},$ $\quad V_{\mathrm{p}} = (\pi d_{\mathrm{p}}^2/4) L_{\mathrm{p}}$

where L_{p}...**total length of pores** in a fiber assembly

Total pore surface area: $A_{\mathrm{p}} = \overset{\pi d_{\mathrm{p}}(1+q_{\mathrm{p}})}{p_{\mathrm{p}}} L_{\mathrm{p}},$ $\quad A_{\mathrm{p}} = \pi d_{\mathrm{p}} (1 + q_{\mathrm{p}}) L_{\mathrm{p}}$

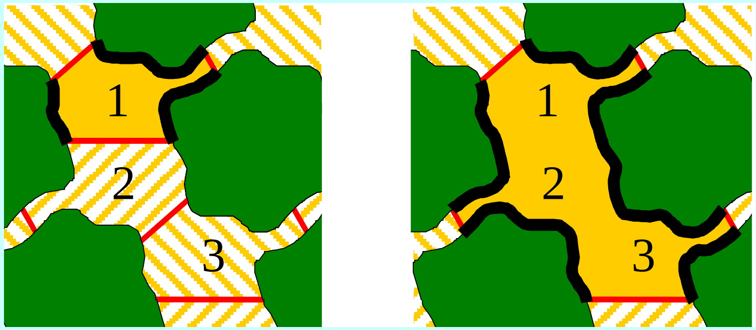
Surface area per unit volume of pore:

$$\gamma_{\mathrm{p}} = \overset{\pi d_{\mathrm{p}}(1+q_{\mathrm{p}})L_{\mathrm{p}}}{A_{\mathrm{p}}} \Big/ \overset{(\pi d_{\mathrm{p}}^2/4)L_{\mathrm{p}}}{V_{\mathrm{p}}} = \left[\pi d_{\mathrm{p}} (1 + q_{\mathrm{p}}) L_{\mathrm{p}}\right] / \left[(\pi d_{\mathrm{p}}^2/4) L_{\mathrm{p}}\right],$$

$$\gamma_{\mathrm{p}} = \frac{4(1 + q_{\mathrm{p}})}{d_{\mathrm{p}}}$$

Effect of pore border choice

All air gaps are divided into n same pores (example: $n = 3$). Each pore has the same parameters s_p, d_p, p_p, q_p, γ_p as pore 1.



If we <u>remove</u> the fictive borders among n pores (among pores 1,2,3) we get a **new big pore** having parameters denoted by ´. It is valid:

<u>Pore sectional area and perimeter</u>: $s'_p = n s_p, \quad p'_p = n p_p$

<u>Equivalent pore diameter</u>: $d'_p = \sqrt{4 \overbrace{s'_p}^{= n s_p} / \pi} = \sqrt{n}\, \overbrace{\sqrt{4 s_p/\pi}}^{= d_p}$, $\quad d'_p = \sqrt{n}\, d_p$

<u>Pore shape factor</u>:

$$1 + q'_p = p'_p / (\pi d'_p) = n p_p / (\pi \sqrt{n}\, d_p) = \sqrt{n}\, \overbrace{p_p / (\pi d_p)}^{= (1 + q_p)}, \quad 1 + q'_p = \sqrt{n}\,(1 + q_p)$$

<u>Total length of (big) pores</u>: $L'_p = L_p / n$

Total pore volume: $V'_p = \underbrace{s'_p}_{=ns_p} \underbrace{L'_p}_{=L_p/n} = \underbrace{s_p L}_{=V_p},$ $\quad V'_p = V_p$

Total pore surface area: $A'_p = \underbrace{p'_p}_{=np_p} \underbrace{L'_p}_{=L_p/n} = \underbrace{p_p L_p}_{=A_p},$ $\quad A'_p = A_p$

Surface area per unit volume of pore: $\gamma'_p = \underbrace{A'_p}_{=A_p} \Big/ \underbrace{V'_p}_{=V_p} = \underbrace{A_p / V_p}_{=\gamma_p},$ $\quad \gamma'_p = \gamma_p$

Values V_p, A_p and γ_p are independent of the choice of fictive borders!

Conventional pore

The inverse value of surface area per unit volume of pore $1/\gamma_p = d_p / 4(1+q_p)$ has a length dimension. So, we introduce a variable $4/\gamma_p$, according to which the pore size will be evaluated. This variable will be called **conventional pore diameter** $\quad d^*_p = 4/\gamma_p = d_p / (1+q_p)$

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Note: In contrary to d_p, the conventional pore diameter d_p^* is <u>independent of the choise of fictive borders</u>, i.e. <u>independent of the shape factor</u> q_p <u>of (real) pore</u>! (We denoted parameters of conventional pore by $*$)

Because $V_p = \left(\pi d_p^2/4\right)L_p$, similarly we use $V_p^* = \left(\pi d_p^{*2}/4\right)L_p^*$ for conventional pore. But $V_p^* = V_p$ (independent of the choice of fictive borders). Then it is valid for **total length of conventional pores**:

$$\underbrace{V_p^*}_{=\left(\pi d_p^{*2}/4\right)L_p^*} = \underbrace{V_p}_{\left(\pi d_p^2/4\right)L_p}, \quad \left(\pi d_p^{*2}/4\right)L_p^* = \left(\pi d_p^2/4\right)L_p, \quad \Big(\underbrace{d_p^*}_{=d_p\left(1+q_p\right)}\Big)^2 L_p^* = d_p^2 L_p, \quad \boxed{L_p^* = L_p\left(1+q_p\right)^2}$$

Because generally $A_p = \pi d_p\left(1+q_p\right)L_p$, analogically we use $A_p^* = \pi d_p^*\left(1+q_p^*\right)L_p^*$ for conventional pore. But $A_p^* = A_p$.

$$\underbrace{A_p^*}_{=\pi d_p^*\left(1+q_p^*\right)L_p^*} = \underbrace{A_p}_{=\pi d_p\left(1+q_p\right)L_p}, \quad \pi \underbrace{d_p^* \left(1+q_p^*\right)}_{=d_p/\left(1+q_p\right)} \underbrace{L_p^*}_{=L_p\left(1+q_p\right)^2} = \pi d_p\left(1+q_p\right)L_p,$$

$$d_\mathrm{p}\left(1+q_\mathrm{p}\right)\left(1+q_\mathrm{p}^*\right)L_\mathrm{p}=d_\mathrm{p}\left(1+q_\mathrm{p}\right)L_\mathrm{p},\quad 1+q_\mathrm{p}^*=1$$

Shape factor of conventional pore: $q_\mathrm{p}^*=0$

(Conventional pore can be considered as air cylinder!)

Sectional area of conventional pore:

$$s_\mathrm{p}^*=\pi\Big(\overbrace{d_\mathrm{p}^*}^{=d_\mathrm{p}/(1+q_\mathrm{p})}\Big)^2\Big/4=\Big(\overbrace{\pi d_\mathrm{p}^2/4}^{=s_\mathrm{p}}\Big)\Big/\left(1+q_\mathrm{p}\right)^2,\qquad s_\mathrm{p}^*=s_\mathrm{p}\Big/\left(1+q_\mathrm{p}\right)^2$$

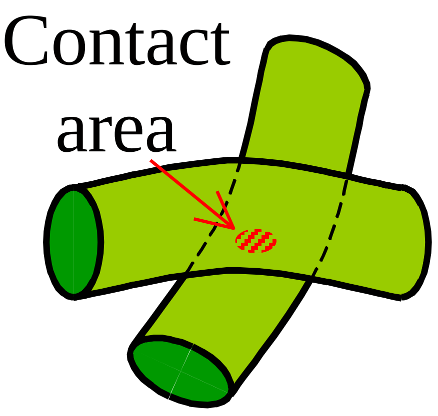
Perimeter of conventional pore:

$$p_\mathrm{p}^*=\pi\overbrace{d_\mathrm{p}^*}^{=d_\mathrm{p}/(1+q_\mathrm{p})}\Big(\overbrace{1+q_\mathrm{p}^*}^{=0}\Big)=\pi d_\mathrm{p}\Big/\left(1+q_\mathrm{p}\right)=\Big(\overbrace{\pi d_\mathrm{p}\left(1+q_\mathrm{p}\right)}^{=p_\mathrm{p}}\Big)\Big/\left(1+q_\mathrm{p}\right)^2,\qquad p_\mathrm{p}^*=p_\mathrm{p}\Big/\left(1+q_\mathrm{p}\right)^2$$

Note: All ***parameters of conventional pore are <u>independent</u> of the choice of fictive borders***, which is of great importance in practice. (Other defined pore parameters depend on the choice of the fictive borders.)

Relationship between fibers and pores

The total fiber surface area A (green) is generally higher than total pore surface area A_p. Namely the contact areas (hatched) are parts of fiber surfaces, but not parts of pore surfaces. But if the contact areas are very small, then it is possible roughly to use the following *assumption* (simplification):



$$A_p = A$$

Using equations derived before we find **surface area per unit volume of pore**:

$$\gamma_p = \overset{=A}{A_p} \Big/ V_p = \overbrace{(A/V)}^{=\gamma}\,\overbrace{(V/V_c)}^{=\mu}\,\overbrace{(V_c/V_p)}^{=1/\psi=1/(1-\mu)},$$

$$\gamma_p = \overset{=4(1+q)/d}{\gamma}\;\mu/(1-\mu),$$

$$\gamma_p = \gamma\,\mu/(1-\mu)$$

$$\gamma_p = \frac{4(1+q)}{d}\,\frac{\mu}{(1-\mu)}$$

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Further $\gamma_p = \gamma\,\mu/(1-\mu)$, $4(1+q_p)/d_p = [4(1+q)/d][\mu/(1-\mu)]$,

(where $\gamma_p = 4(1+q_p)/d_p$, $\gamma = 4(1+q)/d$)

$d_p/(1+q_p) = [d/(1+q)][(1-\mu)/\mu]$,

Equivalent pore diameter:

$$d_p = \frac{1+q_p}{1+q}\,\frac{1-\mu}{\mu}\,d$$

Especially for **conventional pore diameter**

$$d_p^* = d_p\Big/(1+q_p) = \frac{1+q_p}{1+q}\,\frac{1-\mu}{\mu}\,\frac{d}{1+q_p},$$

(where $d_p^* = d_p/(1+q_p)$, $d_p = [(1+q_p)/(1+q)][(1-\mu)/\mu]d$)

$$d_p^* = \frac{1}{1+q}\,\frac{1-\mu}{\mu}\,d$$

It is also possible to use the following rearrangement:

$A_p = \pi\, d_p\,(1+q_p)L_p$, $\pi d(1+q)L = \pi[(1+q_p)/(1+q)][(1-\mu)/\mu]\,d\,(1+q_p)L_p$,

(where $A_p = A = \pi d(1+q)L$, $d_p = \frac{1+q_p}{1+q}\frac{1-\mu}{\mu}d$)

Total length of pores:

$$L_p = \frac{(1+q)^2}{(1+q_p)^2}\,\frac{\mu}{1-\mu}\,L$$

Especially for **total length of conventional pores**

$$L_p^* = L_p\,(1+q_p)^2$$

(where $L_p = \frac{(1+q)^2}{(1+q_p)^2}\frac{\mu}{(1-\mu)}L$)

$$L_p^* = (1+q)^2\,\frac{\mu}{1-\mu}\,L$$

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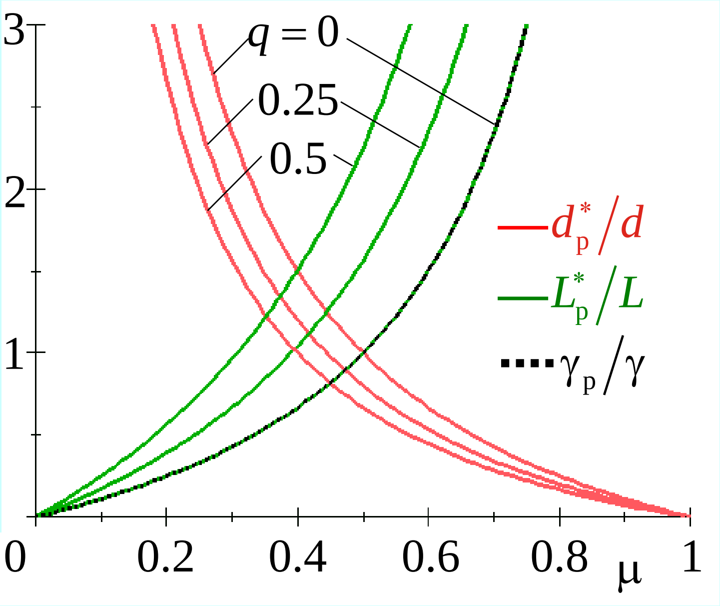
The influences of μ and q on the parameters γ_p, d_p^*, L_p^* of conventional pore are illustrated by the curves on the given graph:



Note: For determination of parameters of (real) pore we need to know the packing density μ, the fiber parameters (q, d, L) and the <u>shape factor of pore</u> q_p, whose value relates to the fictive borders. This value is usually not known. (How the fictive borders will be determined varies from one problem to another problem.) Therefore only some special cases will be presented now.

Pores with a constant shape factor

(Variant I)

Assumption: Pore shape factor q_p is independent of the packing density μ. Then the value of **pore shape factor** is given by the equation $1+q_\mathrm{p}=k\ldots\text{const.}$

Equivalent pore diameter: $d_\mathrm{p}=\frac{\overbrace{1+q_\mathrm{p}}^{=k}}{1+q}\frac{1-\mu}{\mu}d$, $\quad d_\mathrm{p}=\frac{k}{1+q}\frac{1-\mu}{\mu}d$

Total length of pores: $L_\mathrm{p}=\frac{(1+q)^2}{\big(\underbrace{1+q_\mathrm{p}}_{=k}\big)^2}\frac{\mu}{1-\mu}L$, $\quad L_\mathrm{p}=\frac{(1+q)^2}{k^2}\frac{\mu}{1-\mu}L$

Note:

The conventional pore (diameter $d_\mathrm{p}^*=\left[1/(1+q)\right]\left[(1-\mu)/\mu\right]d$) is a special case of pore with constant shape factor, where $q_\mathrm{p}=q_\mathrm{p}^*=0$. (Cylindrical shape of this pore.)

Pores with a constant total length

(Variant II)

Assumption: Total pore length L_p is independent of packing density μ. Because $L_\mathrm{p} = \left[\underbrace{(1+q)^2 \Big/ (1+q_\mathrm{p})^2}_{=k\ldots\text{const.}}\right]\left[\mu/(1-\mu)\right]L$

it is valid $L_\mathrm{p} = \dfrac{(1+q)^2}{(1+q_\mathrm{p})^2}\dfrac{\mu}{1-\mu}L,\quad 1+q_\mathrm{p} = \sqrt{\underbrace{L(1+q)^2}_{\text{fiber parameters}} \Big/ \underbrace{L_\mathrm{p}}_{=\text{const.}}}\sqrt{\dfrac{\mu}{1-\mu}},$

For **pore shape factor** it is valid $1+q_\mathrm{p} = k\sqrt{\dfrac{\mu}{1-\mu}},\quad k\ldots\text{const.}$

(Pore shape factor depends on the packing density now.)

Total length of pores: $L_\mathrm{p} = \dfrac{(1+q)^2}{\Big(\underbrace{1+q_\mathrm{p}}_{=k\sqrt{\mu/(1-\mu)}}\Big)^2}\dfrac{\mu}{1-\mu}L = \dfrac{(1+q)^2}{k^2}\dfrac{1}{\dfrac{\mu}{1-\mu}}\dfrac{\mu}{1-\mu}L,$

$$L_\mathrm{p} = \frac{(1+q)^2}{k^2}L$$

Equivalent pore diameter:

$$d_p = \left[\overbrace{\left(1+q_p\right)}^{k\sqrt{\mu/(1-\mu)}} \Big/ (1+q) \right] \frac{1-\mu}{\mu} d = \frac{k}{1+q} \sqrt{\frac{\mu}{(1-\mu)}} \frac{1-\mu}{\mu} d, \quad d_p = \frac{k}{1+q} \sqrt{\frac{1-\mu}{\mu}} d$$

Generalized pores

(Variant III)

We derived $d_p = \frac{k}{1+q}\left(\frac{1-\mu}{\mu}\right)^{1} d$ for var. (I) and $d_p = \frac{k}{1+q}\left(\frac{1-\mu}{\mu}\right)^{0.5} d$ for var. (II); both are some special "limit" variants. But a right pore (i.e. right in relation to the physical problem studied) need not to follow these variants. Therefore we empirically generalize the equation for **equivalent pore diameter**:

$$d_p = \frac{k}{1+q}\left(\frac{1-\mu}{\mu}\right)^{a} d, \quad k, a \ldots \text{const.}$$

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Generally $d_{\mathrm{p}} = \left[\left(1+q_{\mathrm{p}}\right)/\left(1+q\right)\right]\left[\left(1-\mu\right)/\mu\right] d$ and then now

$$d_{\mathrm{p}} = \left[\left(1+q_{\mathrm{p}}\right)/\left(1+q\right)\right]\left[\left(1-\mu\right)/\mu\right] d = \left[k/\left(1+q\right)\right]\left[\left(1-\mu\right)/\mu\right]^{a} d, \quad 1+q_{\mathrm{p}} = k\left[\left(1-\mu\right)/\mu\right]^{a-1},$$

Pore shape factor:

$$q_{\mathrm{p}} = k\left(\frac{1-\mu}{\mu}\right)^{a-1} - 1,$$

Generally $L_{\mathrm{p}} = \left[\left(1+q\right)^2/\left(1+q_{\mathrm{p}}\right)^2\right]\left[\mu/\left(1-\mu\right)\right] L$ and then now

$$L_{\mathrm{p}} = \left[\left(1+q\right)^2 \Big/ \left(\overbrace{1+q_{\mathrm{p}}}^{=k\left[(1-\mu)/\mu\right]^{a-1}}\right)^2\right]\left[\mu/\left(1-\mu\right)\right] L = \left[\left(1+q\right)^2/k^2\right]\left\{\left[\mu/\left(1-\mu\right)\right]^{a-1}\right\}^2\left[\mu/\left(1-\mu\right)\right] L,$$

Total length of pores:

$$L_{\mathrm{p}} = \frac{\left(1+q\right)^2}{k^2}\left(\frac{\mu}{1-\mu}\right)^{2a-1} L$$

Note: The value of parameter a should lie in the interval $\langle 0.5, 1\rangle$, but generally it need not to lie.

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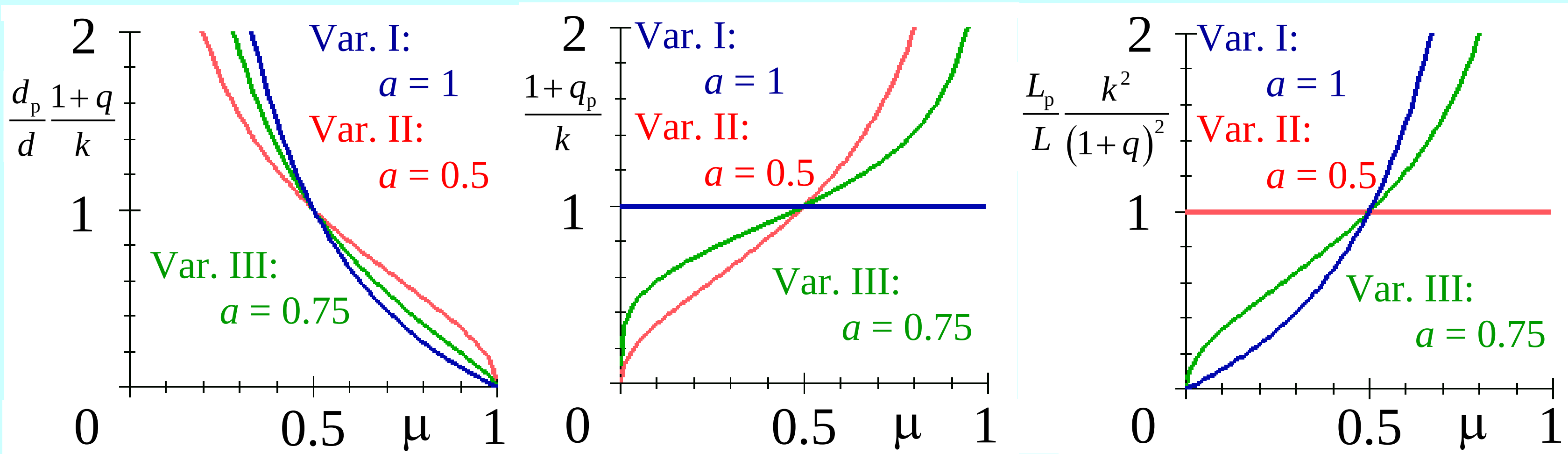
Following graphs show the nature of equivalent pore diameter d_p, pore shape factor q_p and total length of pores L_p in relation to the packing density:



Notes: The given fiber assembly <u>has not only one type of pores</u>. The choice of fictive borders must be always <u>adequate to the solved physical problem</u>. One way how to chose (indirectly) the fictive borders is given by (usually empirical) determination of right parameters a and k.

SOME POSSIBLE APPLICATIONS

Let us know: fiber diameter d, fiber shape factor q, total fiber length L and packing density of the fiber assembly μ. Using equations derived earlier we can estimate: porosity ψ, pore surface area per unit volume γ_p, conventional pore diameter d_p^* and total length of conventional pores L_p^*.

Wicking of textiles (capillarity phenomenon).

...① immersed wall (fiber surface),

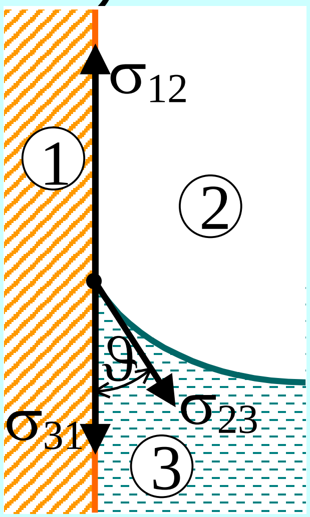
...② air, ...③ fluid (water).

Surface tensions (at the surrounding place of contact – pore circumference p_p):

σ_{12} ...wall-air, σ_{23} ...air-fluid, σ_{31} ...fluid-wall

Tension equilibrium:

$\sigma_{12} - \sigma_{31} = \sigma_{23} \cos \vartheta$, ϑ...constant for given media



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Force, which "lifts" the column of liquid:

$$F_C = \underbrace{p_p}_{=\pi d_p(1+q_p)} \overbrace{(\sigma_{12}-\sigma_{31})}^{=\sigma_{23}\cos\vartheta},$$

$$F_C = \pi d_p (1+q_p)\sigma_{23}\cos\vartheta$$

where p_p...total (real) pore perimeter

Note: The *Young-Laplace equation* for liquid pressure $p_C = F_C / s_p$ is obtained by substituting the pore sectional area $s_p = \pi d_p^2/4$;

$$p_C = \underbrace{F_C}_{=\pi d_p(1+q_p)\sigma_{23}\cos\vartheta} \Big/ \underbrace{s_p}_{=\pi d_p^2/4} = 4\sigma_{23}\cos\vartheta(1+q_p)/d_p$$

Let us denote:

h...height of the fluid column,

ρ_3...fluid mass density,

g...acceleration due to gravity

"Weight" of the "lifted" fluid column:

$$F_g = \overbrace{\Big[\underbrace{\overbrace{s_p h}^{\text{volume}}\,\rho_3}_{\text{mass}}\Big] g}^{\text{"weight"}}$$

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With respect to the force equilibrium $F_C = F_g$ we get

$$\underbrace{F_C}_{=\pi d_p(1+q_p)\sigma_{23}\cos\vartheta} = \underbrace{F_g}_{s_p h \rho_3 g}, \quad \pi d_p(1+q_p)\sigma_{23}\cos\vartheta = \underbrace{s_p}_{=\pi d_p^2/4} h\rho_3 g, \quad \pi d_p(1+q_p)\sigma_{23}\cos\vartheta = \left(\frac{\pi d_p^2}{4}\right) h\rho_3 g,$$

$$h = \frac{4}{d_p}\frac{1}{\rho_3 g}(1+q_p)\sigma_{23}\cos\vartheta = \overbrace{\left(\frac{4\sigma_{23}\cos\vartheta}{\rho_3 g}\right)}^{\text{PARAMETER}} \overbrace{(1+q_p)/d_p}^{=1/d_p^*},$$

Fluid column height:

$$h = \frac{4\sigma_{23}\cos\vartheta}{\rho_3 g}\frac{1}{d_p^*}$$

or since $d_p^* = \frac{1}{(1+q)}\frac{1-\mu}{\mu}d$, hence

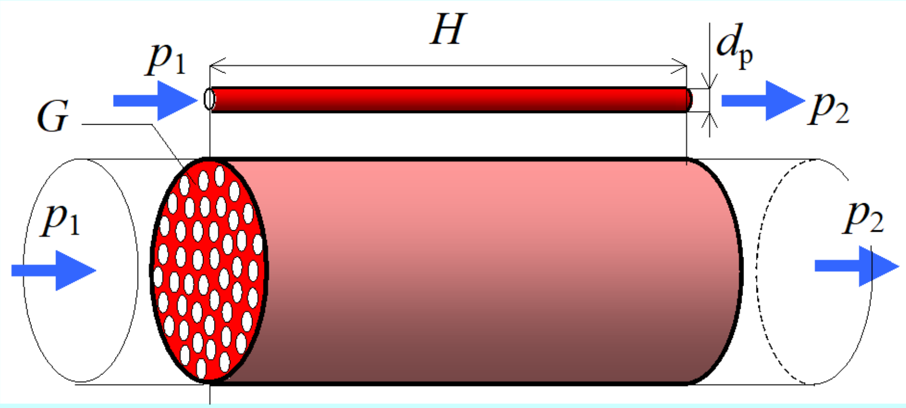
$$h = \frac{4\sigma_{23}\cos\vartheta}{\rho_3 g}\frac{1+q}{d}\frac{\mu}{1-\mu}$$

Because this easy model of capillarity does not respect the pore shape, it is usually better to use <u>more general type</u> of pore diameter (var. I, II or III).
For the most general variant III we get
(k, a...experimental parameters)

$$h = \frac{4\sigma_{23}\cos\vartheta}{\rho_3 g}\overbrace{\frac{1+q}{kd}\left(\frac{\mu}{1-\mu}\right)^a}^{=1/d_p \text{ like var. III}}$$

Flow through a porous fiber assembly

1. **Thin idealized** (cylindrical) **pore**: d_p...pore diameter, H...length, p_1...starting fluid pressure, p_2...final fluid pressure, $\Delta p = p_1 - p_2$...pressure drop



Hagen-Poiseuille law (laminar flow)

Fluid volume per unit time:

where η...dynamic fluid viscosity

$$Q_1 = \frac{\pi d_p^4}{128\eta} \frac{\Delta p}{H}$$

2. **Porous fiber material** (set of cylindrical pores):

G...total cross-sectional area, μ...packing density,

$s_p = \pi d_p^2/4$ sectional area of one pore,

$S_p = G\left(\overset{=\psi\ldots\text{porosity}}{1-\mu}\right)$ ...area of pores in the total cross-sectional area of porous fiber material

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Number of (idealized) pores:

$$n_p = \overbrace{S_p}^{=G(1-\mu)} \Big/ \overbrace{s_p}^{=\pi d_p^2/4}, \qquad n_p = 4G(1-\mu)\big/\left(\pi d_p^2\right)$$

Volume of fluid flow per unit time:

$$Q = \left\{ \overbrace{Q_1}^{=\left[\pi d_p^4/(128\eta)\right]\Delta p/H} \right\} \left\{ \overbrace{n_p}^{=4G(1-\mu)/\left(\pi d_p^2\right)} \right\} = \frac{\pi d_p^4}{\underbrace{128}_{=32\cdot 4}\eta}\frac{\Delta p}{H}\frac{4G(1-\mu)}{\pi d_p^2}, \qquad Q = \frac{G(1-\mu)d_p^2}{32\eta}\frac{\Delta p}{H}$$

Using the equivalent pore diameter $d_p = \left[k/(1+q)\right]\left[(1-\mu)/\mu\right]d$ like var. I and the surface area per unit volume of fiber $\gamma = 4(1+q)/d$ (both derived earlier), we find

$$d_p = k\left[(1-\mu)/\mu\right]\overbrace{\left[d/(1+q)\right]}^{=4/\gamma} = (4k/\gamma)(1-\mu)/\mu,$$ and for volume of fluid per unit time we get

$$Q = \frac{G(1-\mu)\left(\overbrace{d_p}^{=(4k/\gamma)(1-\mu)/\mu}\right)^2}{32\eta}\frac{\Delta p}{H} = \frac{G(1-\mu)}{\underbrace{32}_{=2\cdot 16}\eta}\frac{16k^2}{\gamma^2}\frac{(1-\mu)^2}{\mu^2}\frac{\Delta p}{H},$$

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$$Q=\left(\frac{k^2}{2}\frac{G}{\gamma^2\eta}\right)\frac{\Delta p}{H}\frac{(1-\mu)^3}{\mu^2}$$

This equation is identical to the traditionally well known **Carman -Kozeny equation**.

Of course, more general is to use the equivalent pore diameter $d_\mathrm{p}=\left[k/(1+q)\right]\left[(1-\mu)/\mu\right]^a d$ like var. III. Then

$$d_\mathrm{p}=k\left[(1-\mu)/\mu\right]^a\overbrace{\left[d/(1+q)\right]}^{=4/\gamma}=(4k/\gamma)(1-\mu)^a/\mu^a,$$

$$Q=\frac{G(1-\mu)\Big(\overbrace{d_\mathrm{p}}^{=(4k/\gamma)(1-\mu)^a/\mu^a}\Big)^2}{32\eta}\frac{\Delta p}{H}=\frac{G(1-\mu)}{32\,\eta}\frac{16k^2}{\gamma^2}\frac{(1-\mu)^{2a}}{\mu^{2a}}\frac{\Delta p}{H},$$

$=2\cdot16$

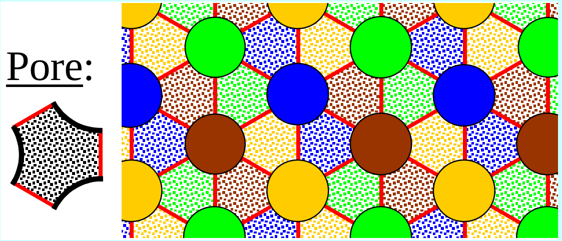
$$Q=\left(\frac{k^2}{2}\frac{G}{\gamma^2\eta}\right)\frac{\Delta p}{H}\frac{(1-\mu)^{2a+1}}{\mu^{a+1}}$$

Generalized Carman-Kozeny equation.

Note: The parameters $k^2G/(2\gamma^2\eta)$ and a are necessary to determine experimentally now.

Aerosol filtration (solid particles)

The pores having constant total length like var. II (but naturally of different size, according to the packing density) are perhaps good for filtration processes.
In spite of random oriented fibers in real fiber bundle, let us study the idealized bundle of cylindrical fibers ($q = 0$), having the hexagonal structure in its cross-section (see lecture 1). The fictive borders (/) can be determined as



shown and then two pores belong to each fiber. It is valid

$$L_p / L = \left(1 + \overset{=0}{q}\right)^2 \Big/ k^2 = 2, \quad \boxed{k = 1/\sqrt{2}} \quad \text{and} \quad d_p = \left[\overset{=1/\sqrt{2}}{k} \Big/ \left(1 + \overset{=0}{q}\right)\right] \sqrt{(1-\mu)/\mu}\; d$$

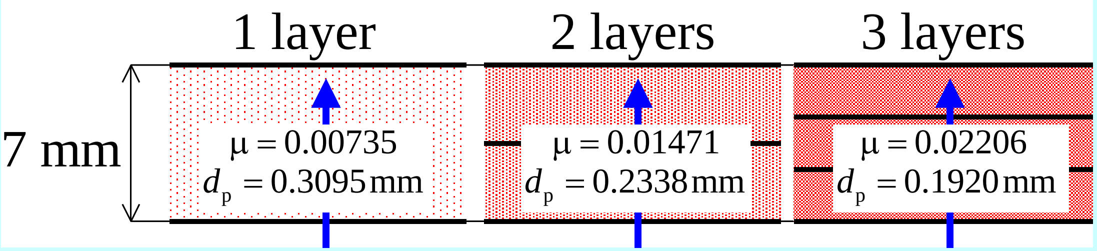
The **equivalent pore diameter** like this model (hexagonal structure) is

$$d_p = \frac{1}{\sqrt{2}} \sqrt{\frac{(1-\mu)}{\mu}}\; d$$

NOTES TO THE PRACTICAL EXPERIENCES

1. The Carman-Kozeny equation is the traditional expression, often used in fluid mechanics for general porous materials. The experiences with this equation as well as with its different generalizations are published in literature (include application for air-flow, i.e. “micronaire” method).

2. The (average) pore diameter is possible to measure using e.g. *POROMETER* (“Porous Material Inc.”). We measured (mean) pore diameter in the 1, 2 and 3 layered webs 70 g/m^2 of PET fibers 6.7 dtex (d = 0.025 mm), compressed to the constant thickness 7 mm. The found values of μ and d_p are shown on the schema (evaluated by *M. Bartáková*).



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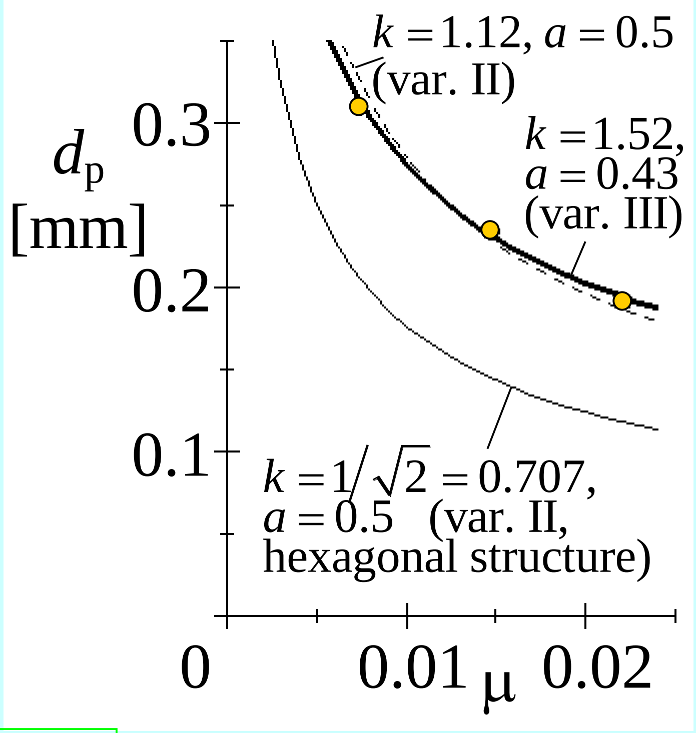

Three yellow points on the picture represent the experimental results graphically.

Thick line shows the equivalent pore diameter like var. III:

$$d_p=\left[\overset{=1.52}{k}\Big/\left(1+\overset{=0}{q}\right)\right]\left[(1-\mu)/\mu\right]^{\overset{=0.43}{a}} d,$$

$$d_p=1.52\left[(1-\mu)/\mu\right]^{0.43} d$$

Thin line indicates the equivalent pore diameter in hexagonal structure like var. II:

$$d_p=\left(1/\sqrt{2}\right)\sqrt{(1-\mu)/\mu}\,d$$

Dotted line tells the equivalent pore diameter like var. II:

$$d_p=\left[\overset{=1.12}{k}\Big/\left(1+\overset{=0}{q}\right)\right]\sqrt{(1-\mu)/\mu}\,d,\quad d_p=1.12\sqrt{(1-\mu)/\mu}\,d$$