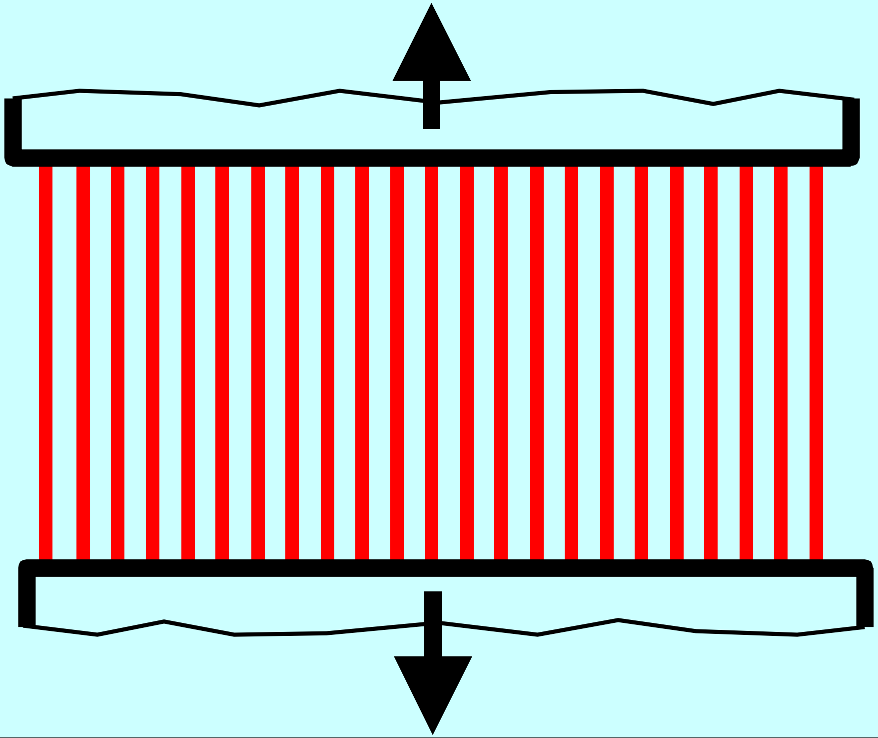
MECHANICS OF PARALLEL FIBER BUNDLES



MECHANICS OF PARALLEL FIBER BUNDLES

Bundle of parallel and more or less independent fibers creates usually an idealized basis of linear textiles - means all sorts of staple and filament yarns, fiber and yarn bundles, e.g. ropes, but also warp yarns for weaving etc. Therefore the regulations, valid for mechanical behavior of such bundle, determine principal properties of this different textiles and knowledge of it is necessary for solving of a lot of special mechanical textile problems. Some models of parallel fiber bundles will be derived in this lecture.

Ideal bundle

Generally:

Assumptions

- Great number of fibers,
- straight (linear) fibers,
- each fiber is griped by both jaws,
- fibers are mutually parallel,
- fibers are mechanically independent to each other

Terminology

Strength of fiber – maximum tensile force in a fiber

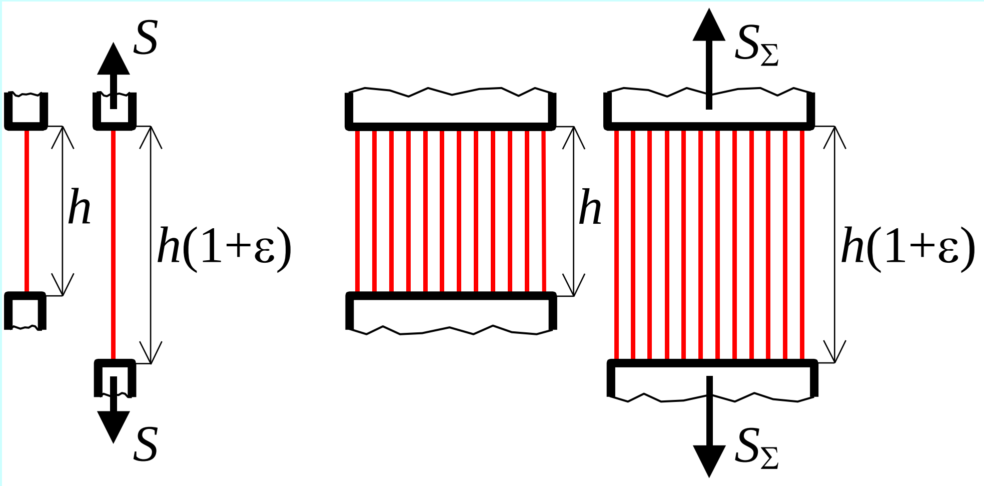
Breaking strain of fiber – strain by fiber strength point

Common variables for one fiber and fiber bundle:

h…gauge length

ε…strain (relative elongation)

One fiber: *Fiber bundle:*



Other variables and functions:

	One fiber	Fiber bundle
Number of fibers:	1	n
Tensile force:	S	S_Σ
Force-strain relation:	$S = S(\varepsilon)$	$S_\Sigma = S_\Sigma(\varepsilon)$
Strength:	P (max. of S)	P_Σ (max. of S_Σ)
Breaking strain:	$a,\ (P = S(a))$	$a_\Sigma,\ (P_\Sigma = S_\Sigma(a_\Sigma))$

CASE 1 (trivial)

Assumptions: All fibers have

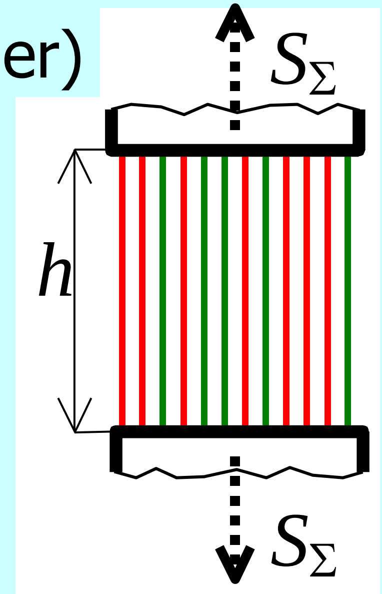
a) same force-strain curve $S = S(\varepsilon)$ and

b) same strength P and same breaking strain a.

Then the following equations are valid evidently:

$$S_\Sigma(\varepsilon) = nS(\varepsilon), \quad P_\Sigma = nP, \quad a_\Sigma = a$$

CASE 2 (blending theory like W. J. Hamburger)



Assumptions:

1. Fiber bundle is a blend (| and |) of 2 types of fibers.
2. All fibers of one type have

a) same force-strain curve $S = S(\varepsilon)$ and

b) same strength P and same breaking strain a.

Convention:

Fiber of one type having <u>smaller value of breaking strain</u> is denoted as <u>No. 1</u> (|), other type of fibers is denoted as No. 2. (|). (This numbers are used as subscripts.)

Variables:	Fiber material No. 1	Fiber material No. 2
Fiber fineness	t_1	t_2
Force-strain relation	$S_1(\varepsilon)$	$S_2(\varepsilon)$
Breaking strain of fiber	$a_1 \le a_2$	
Fiber strength	$P_1 = S_1(a_1)$	$P_2 = S_2(a_2)$
Number o fibers	n_1	n_2
Total number of fibers	$n = n_1 + n_2$	
Mass of fibers	m_1	m_2
Total mass of fibers	$m = m_1 + m_2$	
Bundle fineness (count)	$T = m/h$	
Mass portion	$g_1 = m_1/m$	$g_2 = m_2/m$
Sum of mass portions	$g_1 + g_2 = 1$	

It is valid for the fiber No. 1:

$$m_1 = g_1 m,\ t_1 = m_1/(n_1 h),\ n_1 = m_1/(t_1 h) = (g_1/t_1)(m/h),\quad n_1 = g_1 (T/t_1)$$

($m/h = T$)

For the fiber No. 2, it is valid analogically: $n_2 = g_2 (T/t_2)$

Maximum forces, in a bundle *Force-strain curves:*

a) Interval $\varepsilon \le a_1 \Rightarrow$ max. at $\varepsilon = a_1$

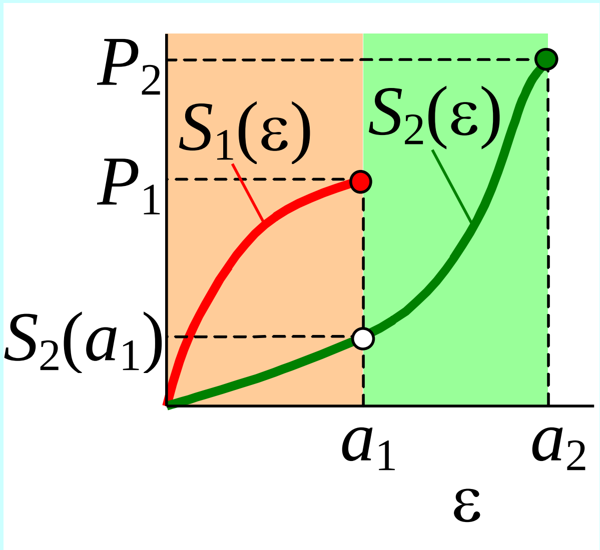
$$S_\Sigma(a_1) = n_1 P_1 + n_2 S_2(a_1)$$

$$S_\Sigma(a_1) = T\left[g_1 P_1/t_1 + g_2 S_2(a_1)/t_2\right]$$

b) Interval $\varepsilon \in \left(a_1, a_2\right\rangle \Rightarrow$ max. at $\varepsilon = a_2$

$$S_\Sigma(a_2) = n_1 \cdot 0 + n_2 P_2$$

$$S_\Sigma(a_2) = T\, g_2 P_2/t_2$$



c) Interval $\varepsilon > a_2 \Rightarrow$ all fibers are broken, $S_\Sigma(\varepsilon > a_2) = 0$

Strength of bundle

$$P_\Sigma = \max\left\{S_\Sigma(a_1), S_\Sigma(a_2)\right\} = T\max\left\{\left[g_1\frac{P_1}{t_1} + g_2\frac{S_2(a_1)}{t_2}\right], \left[g_2\frac{P_2}{t_2}\right]\right\}$$

P_1/t_1…tenacity of fiber No. 1 (e.g. N/tex)

P_2/t_2…tenacity of fiber No. 2 (e.g. N/tex)

$S_2(a_1)/t_2$…specific stress of fiber No. 2 (e.g. N/tex) at $\varepsilon = a_1$

Bundle tenacity P_Σ/T

$$\frac{P_\Sigma}{T} = \max\left\{\left[g_1\frac{P_1}{t_1} + g_2\frac{S_2(a_1)}{t_2}\right], \left[g_2\frac{P_2}{t_2}\right]\right\}$$ (e.g. N/tex)

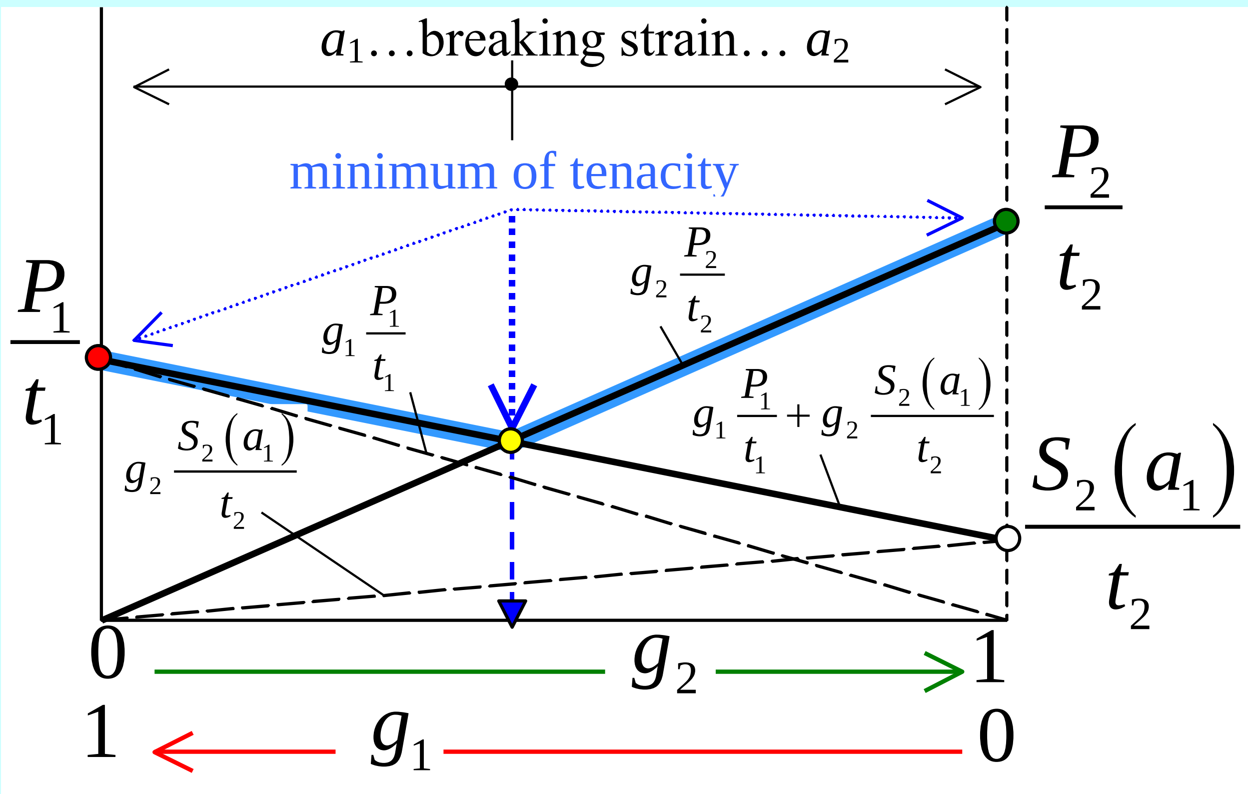
Breaking strain of bundle

a) $a_\Sigma = a_1$ if $P_\Sigma/T = g_1 P_1/t_1 + g_2 S_2(a_1)/t_2$

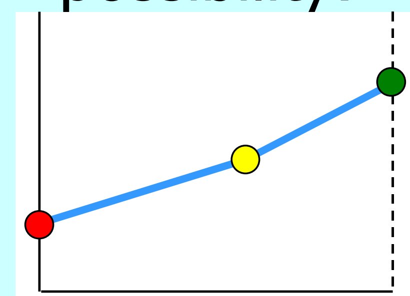
b) $a_\Sigma = a_2$ if $P_\Sigma/T = g_2 P_2/t_2$

Graphical representation of resulting equation

$$P_\Sigma/T = \max\left\{\left[g_1 P_1/t_1 + g_2 S_2(a_1)/t_2\right],\ \left[g_2 P_2/t_2\right]\right\}$$



Other possibility:



Minimum bundle tenacity – two possibilities:

a) $g_2 = 0$ (●) and then $P_\Sigma / T = P_1 / t_1$

b) By point of intersection (○) of two lines, it is

$$\underbrace{g_1}_{=1-g_2} P_1/t_1 + g_2 S_2(a_1)/t_2 = g_2 P_2/t_2,$$

$$P_1/t_1 = g_2 P_1/t_1 + g_2 P_2/t_2 - g_2 S_2(a_1)/t_2,$$

$$▼ \quad g_2 = \frac{P_1/t_1}{P_1/t_1 + P_2/t_2 - S_2(a_1)/t_2}$$

and using this value we get $P_\Sigma / T = g_2 P_2 / t_2$

Now, the minimum bundle tenacity is the **minimum of three calculated values** P_Σ / T.

Note: After addition of fibers having higher tenacity, the tenacity of resulting bundle can ***decrease!***

Note: This theory can be applied for **rough estimation of blended staple yarn** too, but, it is necessary to use analogical values of corresponding individual component yarn in place of fiber parameters. In this case P_1/t_1 means tenacity of single yarn (100% material No. 1), P_2/t_2 means tenacity of single yarn (100% material No.2) and $S_2(a_1)/t_2$ means specific stress of the single yarn (100% material No.2) when the strain is equal to the breaking strain of the single yarn (100% material No. 1), i.e. $(\varepsilon = a_1)$; all e.g. in N/tex.

Original article see

Hamburger,W.J.: The industrial application of the stress-strain relationship. J. Text. Inst. ***40****, 1949, pp. 700-718.*