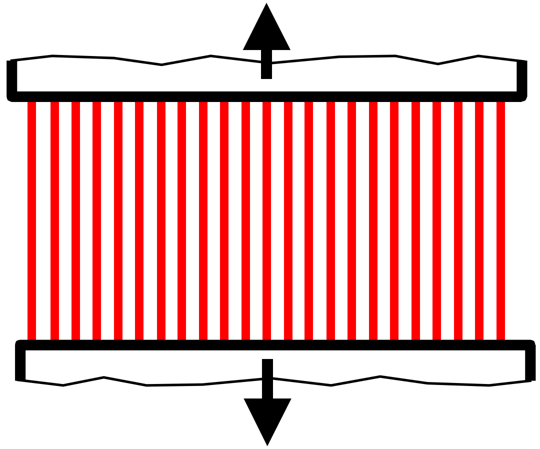
TECHNICAL UNIVERSITY OF LIBEREC
Faculty of Textile Engineering

MECHANICS OF PARALLEL FIBER BUNDLES

„TRIVIAL FIBER BUNDLE, TWO-COMPONENT FIBER BUNDLE“



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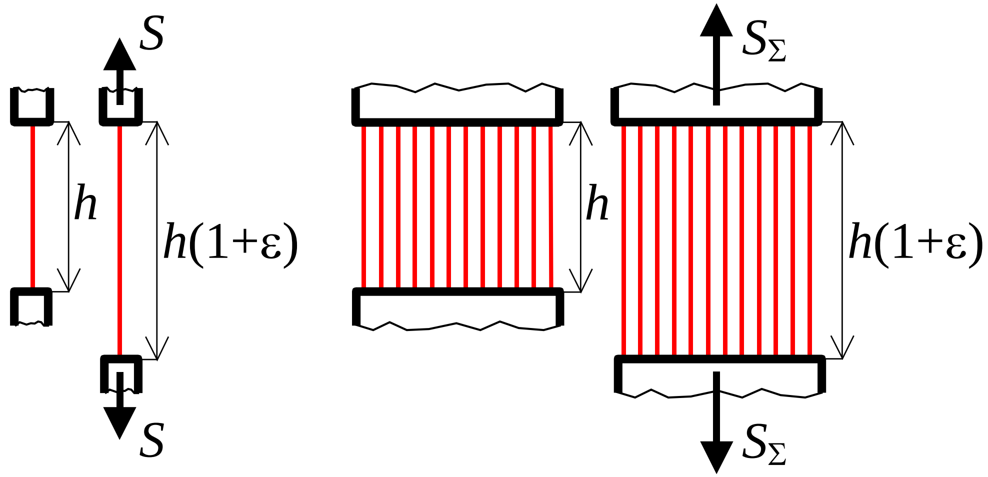
Common variables for one fiber and fiber bundle:

One fiber: *Fiber bundle:*



h…gauge length

ε…strain (relative elongation)

Other variables and functions:

	One fiber	Fiber bundle
Number of fibers:	1	n
Tensile force:	S	S_Σ
Force-strain relation:	$S = S(\varepsilon)$	$S_\Sigma = S_\Sigma(\varepsilon)$
Strength:	P (max. of S)	P_Σ (max. of S_Σ)
Breaking strain:	$a,\ (P = S(a))$	$a_\Sigma,\ (P_\Sigma = S_\Sigma(a_\Sigma))$

CASE 1 (trivial)

Assumptions: All fibers have

a) same force-strain curve $S = S(\varepsilon)$ and

b) same strength P and same breaking strain a.

Then the following equations are valid evidently:

$$S_\Sigma(\varepsilon) = n\,S(\varepsilon), \quad P_\Sigma = n\,P, \quad a_\Sigma = a$$

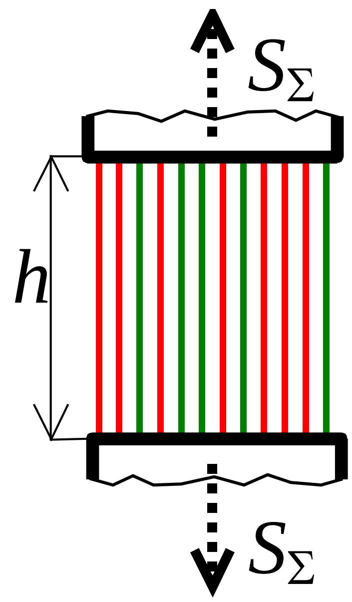
CASE 2 (blending theory like W. J. Hamburger)

Assumptions:

1. Fiber bundle is a blend (| and |) of 2 types of fibers.
2. All fibers of one type have

a) same force-strain curve $S = S(\varepsilon)$ and

b) same strength P and same breaking strain a.



Convention:

Fiber of one type having <u>smaller value of breaking strain</u> is denoted as <u>No. 1</u> (|), other type of fibers is denoted as No. 2. (|). (These numbers are used as subscripts.)

Variables:	Fiber material No. 1	Fiber material No. 2
Fiber fineness	t_1	t_2
Force-strain relation	$S_1(\varepsilon)$	$S_2(\varepsilon)$
Breaking strain of fiber	$a_1 \le a_2$	
Fiber strength	$P_1 = S_1(a_1)$	$P_2 = S_2(a_2)$
Number o fibers	n_1	n_2
Total number of fibers	$n = n_1 + n_2$	
Mass of fibers	m_1	m_2
Total mass of fibers	$m = m_1 + m_2$	
Bundle fineness (count)	$T = m/h$	
Mass portion	$g_1 = m_1/m$	$g_2 = m_2/m$
Sum of mass portions	$g_1 + g_2 = 1$	

It is valid for the fiber No. 1:

$$m_1 = g_1 m,\ t_1 = m_1/(n_1 h),\ n_1 = m_1/(t_1 h) = (g_1/t_1)\underbrace{(m/h)}_{=T},\ \boxed{n_1 = g_1(T/t_1)}$$

For the fiber No. 2, it is valid analogically: $\boxed{n_2 = g_2(T/t_2)}$

Maximum forces, in a bundle *Force-strain curves:*

a) Interval $\varepsilon \le a_1$ max. at $\varepsilon = a_1$

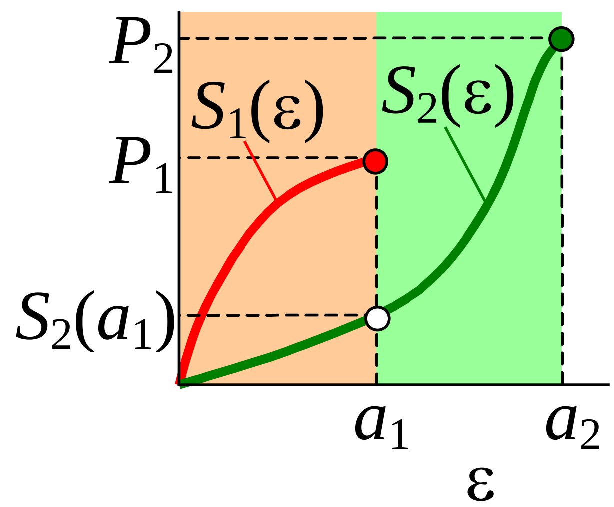
$$S_\Sigma(a_1) = n_1 P_1 + n_2 S_2(a_1)$$

$$\boxed{S_\Sigma(a_1) = T\left[g_1 P_1/t_1 + g_2 S_2(a_1)/t_2\right]}$$

b) Interval $\varepsilon \in \left(a_1, a_2\right\rangle$ max. at $\varepsilon = a_2$

$$S_\Sigma(a_2) = n_1 \cdot 0 + n_2 P_2$$

$$\boxed{S_\Sigma(a_2) = T\, g_2 P_2/t_2}$$



c) Interval $\varepsilon > a_2 \Rightarrow$ all fibers are broken, $\boxed{S_\Sigma(\varepsilon > a_2) = 0}$

Strength of bundle

$$P_\Sigma = \max\left\{S_\Sigma(a_1), S_\Sigma(a_2)\right\} = T\max\left\{\left[g_1\frac{P_1}{t_1} + g_2\frac{S_2(a_1)}{t_2}\right], \left[g_2\frac{P_2}{t_2}\right]\right\}$$

P_1/t_1 ...tenacity of fiber No. 1 (e.g. N/tex)

P_2/t_2 ...tenacity of fiber No. 2 (e.g. N/tex)

$S_2(a_1)/t_2$ ...specific stress of fiber No. 2 (e.g. N/tex) at $\varepsilon = a_1$

Bundle tenacity P_Σ/T

$$\frac{P_\Sigma}{T} = \max\left\{\left[g_1\frac{P_1}{t_1} + g_2\frac{S_2(a_1)}{t_2}\right], \left[g_2\frac{P_2}{t_2}\right]\right\}$$ (e.g. N/tex)

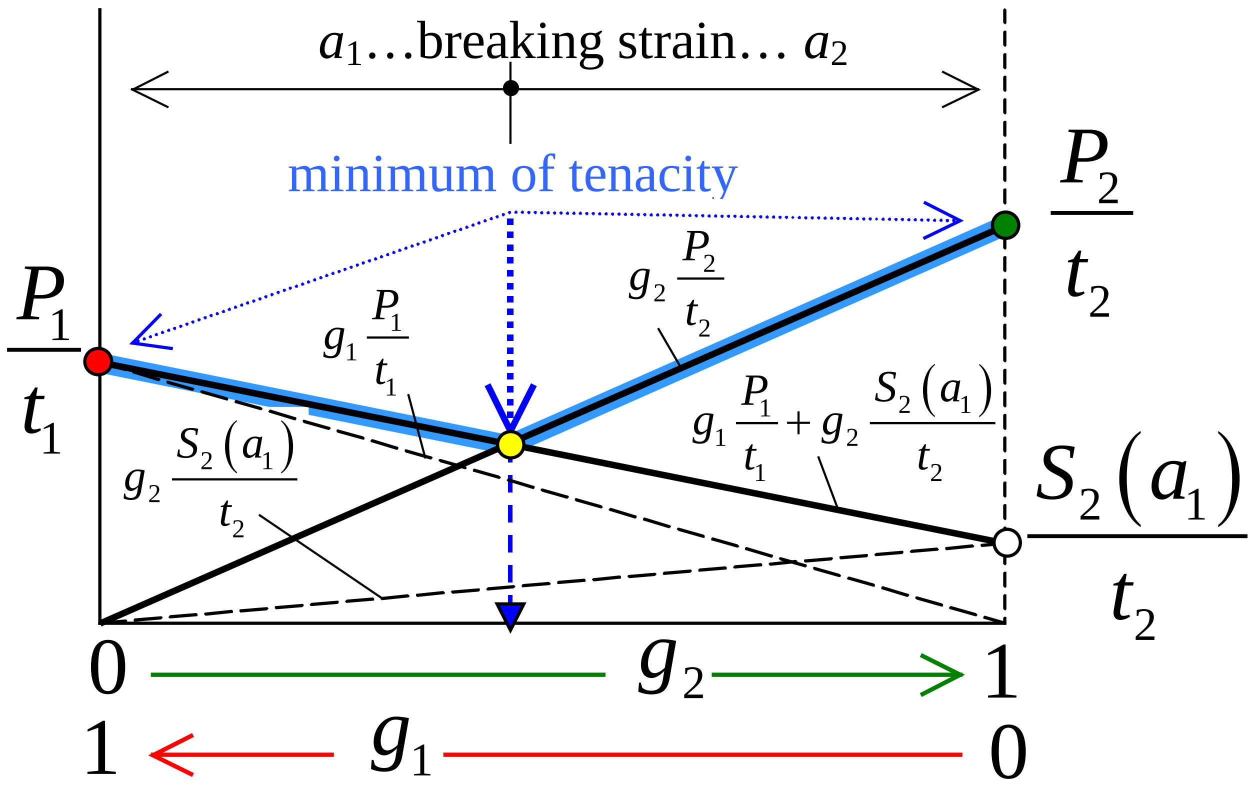
Breaking strain of bundle

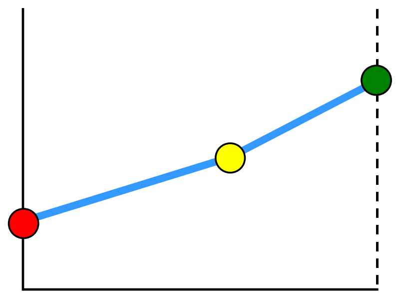
a) $a_\Sigma = a_1$ if $P_\Sigma/T = g_1 P_1/t_1 + g_2 S_2(a_1)/t_2$

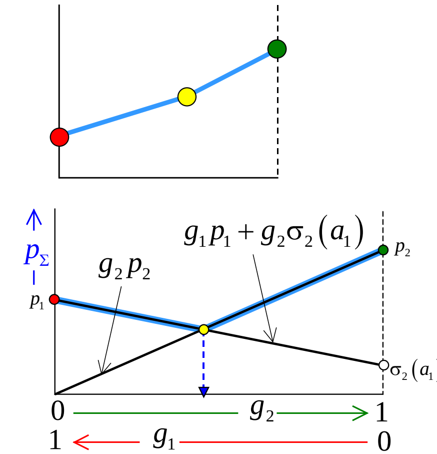
b) $a_\Sigma = a_2$ if $P_\Sigma/T = g_2 P_2/t_2$

Graphical representation of resulting equation

$$P_\Sigma/T = \max\left\{\left[g_1 P_1/t_1 + g_2 S_2(a_1)/t_2\right],\ \left[g_2 P_2/t_2\right]\right\}$$







Minimum bundle tenacity – two possibilities:

a) $g_2 = 0$ (🔴) and then $\boxed{P_\Sigma / T = P_1 / t_1}$

b) By point of intersection (🟡) of two lines, it is

$$\underbrace{g_1}_{=1-g_2} \; P_1/t_1 + g_2\, S_2(a_1)/t_2 = g_2\, P_2/t_2\,,$$

$$\boxed{g_2 = \frac{P_1/t_1}{P_1/t_1 + P_2/t_2 - S_2(a_1)/t_2}}$$

$$P_1/t_1 = g_2 P_1/t_1 + g_2\, P_2/t_2 - g_2\, S_2(a_1)/t_2\,,$$

and using of this value we get $\boxed{P_\Sigma / T = g_2\, P_2 / t_2}$

Now, the <u>minimum bundle tenacity</u> is the **minimum of three calculated values** P_Σ / T.

Note: After addition of fibers having higher tenacity, the tenacity of resulting bundle can ***decrease!***

EMPIRICAL USAGE OF RESULTS FOR YARNS

Instead of fiber parameters, parameters of one component and blended yarns are used.

Quantity	*Instead of FIBERS and BUNDLES*	*we use values of YARNS*
p_1	Relative strength (tenacity)of fiber with lower breaking strain	**Relative strength (tenacity) of one component yarn with lower breaking strain**
p_2	Relative strength (tenacity)of fiber with higher breaking strain	**Relative strength (tenacity)of one component yarn with higher breaking strain**
a_1	Breaking strain of fiber of component with lower breaking strain	Breaking strain of one component yarn with lower breaking strain
a_2	Breaking strain of fiber of component with higher breaking strain	Breaking strain of one component yarn with higher breaking strain
$\sigma_{21}(a\ )$	Relative force in fiber with higher breaking strain by relative elongation $\varepsilon = a_1$	Relative force in one component yarn with higher breaking strain by relative elongation $\varepsilon = a_1$
g_1, g_2	Mass portion of fibers with lower and higher breaking strain in bundle	Mass portion of fibers of one component yarn with lower and higher breaking strain in bundle
p_Σ	Relative strength (tenacity) of bundle from two components	Relative strength (tenacity) of blended yarn from two components
a_Σ	Breaking strain of bundle from two components	Breaking strain of blended yarn from two components

Task 1 Calculate number of fibers in bundle, breaking strain of bundle, breaking strength of bundle and relative breaking strength of bundle.

a) 100% cotton, T=20tex, t=0,17tex, l=26mm, p=0,298Ntex^{-1}, a=9%

b) 100% POP, T=20tex, t=0,188tex, l=40mm, p=0,4Ntex^{-1}, a=63%

a) n=118fibers, a_{bundle}=9%, P_{bundle}=5,98N, p_{bundle}=0,298Ntex^{-1}

b) n=106fibers, a_{bundle}=63%, P_{bundle}=7,97N, p_{bundle}=0,4Ntex^{-1}

Task 2 Calculate relative breaking strength of blended yarn 65CO/35POP, yarn count is 20tex, if you know properties of each component:

100% cotton, t=0,17tex, p_1=0,183Ntex^{-1}, a_1=6,2%

100% POP, t=0,188tex, p_2=0,231Ntex^{-1}, a_2=24,3%

$$\sigma_2(a_1) = a_1 \frac{p_2}{a_2} = 6{,}2 \frac{0{,}231}{24{,}3} = 0{,}0589\, N/\, tex$$

$$G_2 = \frac{p_1}{p_1 + p_2 - \sigma_2(a_1)} = \frac{0{,}183}{0{,}183 + 0{,}231 - 0{,}0589} = 0{,}52$$

$$G_1 = 1 - G_2 = 0{,}48$$

$$g_2 = 0{,}35 \Rightarrow p_\Sigma = g_1 p_1 + g_2 \sigma_2(a_1) = 0{,}65 * 0{,}183 + 0{,}35 * 0{,}0589 = 0{,}1396\, N/\, tex$$

Task 3 Calculate relative breaking strength of blended yarn 50CO/50PES, yarn count is 25tex, if you know properties of each component:

100% cotton, p_1=0,332Ntex^{-1}, a_1=4,9%

100% PES, p_2=0,132Ntex^{-1}, a_2=15%

$$\sigma_2(a_1) = a_1 \frac{p_2}{a_2} = 4{,}9\frac{0{,}132}{15} = 0{,}04312\,N/\,tex$$

$$G_2 = \frac{p_1}{p_1 + p_2 - \sigma_2(a_1)} = \frac{0{,}332}{0{,}332 + 0{,}132 - 0{,}04312} = 0{,}79$$

$$G_1 = 1 - G_2 = 0{,}21$$

$$g_2 = 0{,}5 \Rightarrow p_\Sigma = g_1 p_1 + g_2\sigma_2(a_1) = 0{,}50 * 0{,}332 + 0{,}5 * 0{,}04312 = 0{,}1876\,N/\,tex$$

Task 4 Calculate relative breaking strength of blended yarn 35CO/65POP, yarn count is 20tex, if you know properties of each component:

100% cotton, p_1=0,14Ntex^{-1}, a_1=4,2%

100% POP, p_2=0,42Ntex^{-1}, a_2=9%

$$\sigma_2(a_1) = a_1 \frac{p_2}{a_2} = 4{,}2 \frac{0{,}42}{9} = 0{,}196\, N/tex$$

$$G_2 = \frac{p_1}{p_1 + p_2 - \sigma_2(a_1)} = \frac{0{,}14}{0{,}14 + 0{,}42 - 0{,}196} = 0{,}38$$

$$G_1 = 1 - G_2 = 0{,}62$$

$$g_2 = 0{,}65 \Rightarrow p_\Sigma = g_2 p_2 = 0{,}65 * 0{,}42 = 0{,}273\, N/tex$$