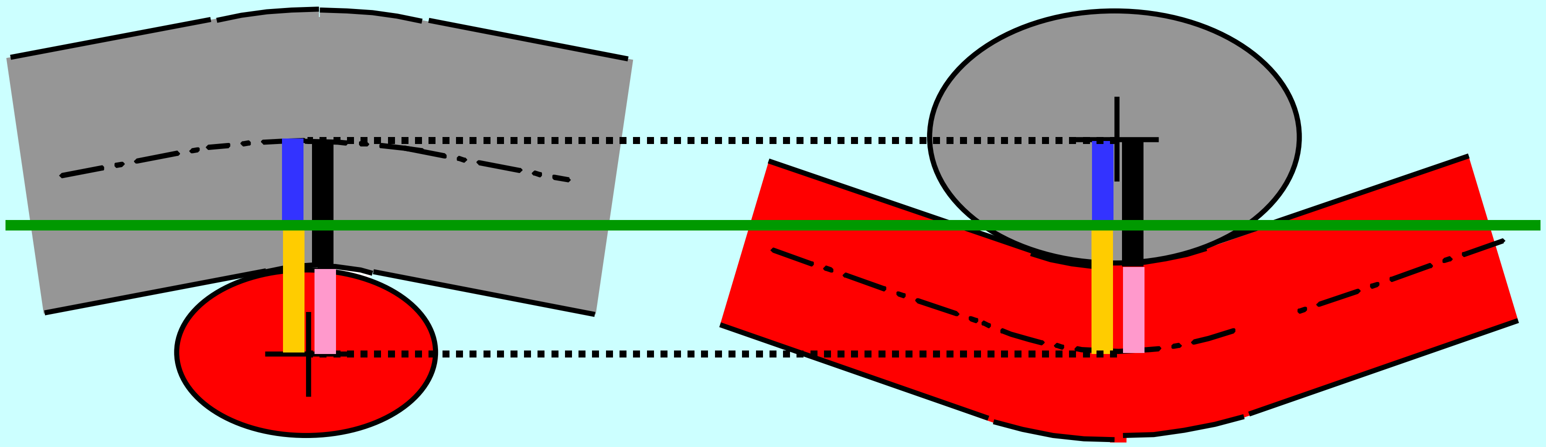
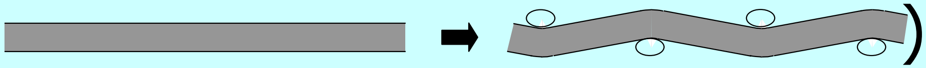
„GEOMETRICAL MODELS“

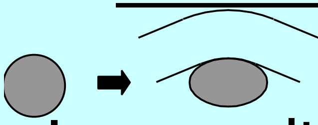


WOVEN FABRICS 2

Geometry of woven fabric – shapes of warp and weft yarns and their mutual spatial form.

Initial **geometry of** ("free") **yarn is changed** by its transformation to a fabric, and so:

Longitudinal shape - initially straight yarn crimps due to interlacing with other yarns (). YARN WAVINES is limited by condition that the yarns must be mutually in contact in binding point.

Transversal shape - initially circular yarn cross-section becomes a flattened shape especially in binding point (). This TRANSVERSAL DEFORMATION of the yarn is a result of mutual compressive forces in binding point

Waviness

There exists the relation between warp and weft waviness, resulting from the contact of both yarns.

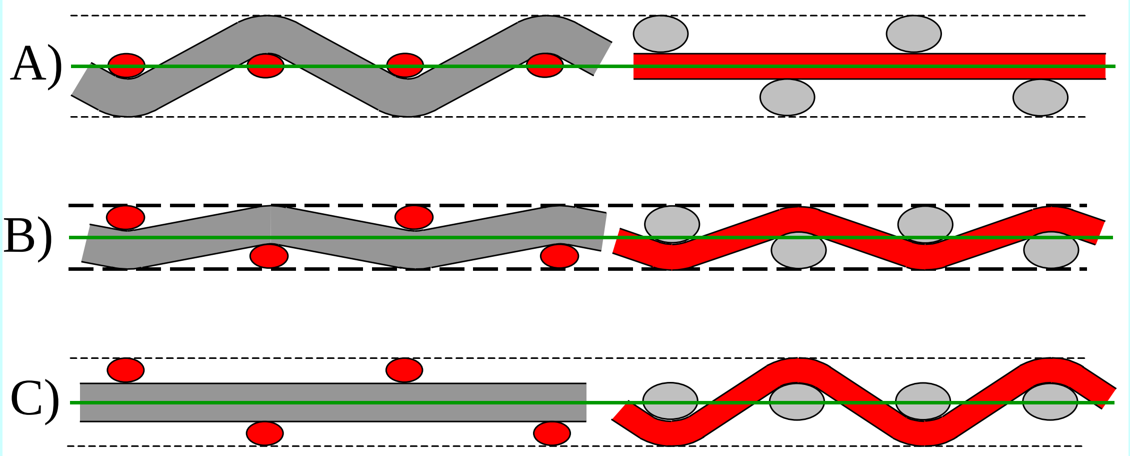
A) *1. Limit case* – straight warp (stick) ⇒ maximum waviness of weft.

C) *2. Limit case* – straight weft (stick) ⇒ maximum waviness of warp.

B) BALANCED FABRIC – warp and weft points are lying in the same height.

(Assumption of easier theoretical models.)

Note: — **central (middle) plane of fabric**



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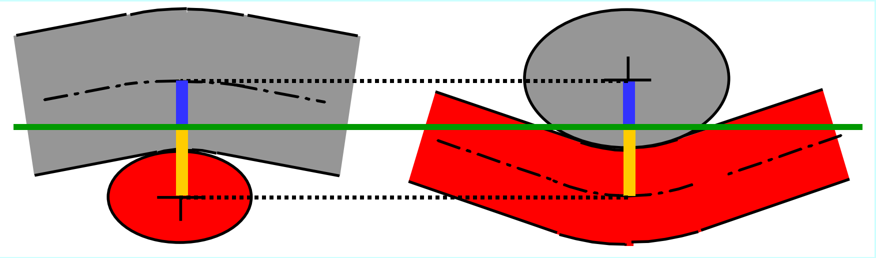
The measure of waviness is **height of crimp wave** – highest distance of yarn axis from the central plane.



Warp height of crimp wave... h_o (|)

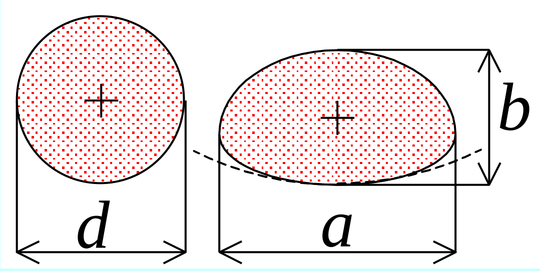
Weft height of crimp wave ... h_u (|)

Transversal deformation

Initial yarn cross-section – circular, diameter d - becomes a flattened shape having **yarn width** a and **yarn height** b. Usually $a > d,\ b < d$ (We suppose that yarn axis is in the middle of a and b.)

We define also

- **yarn enlargement**... $\alpha = a/d$
- **yarn compression**... $\beta = b/d$



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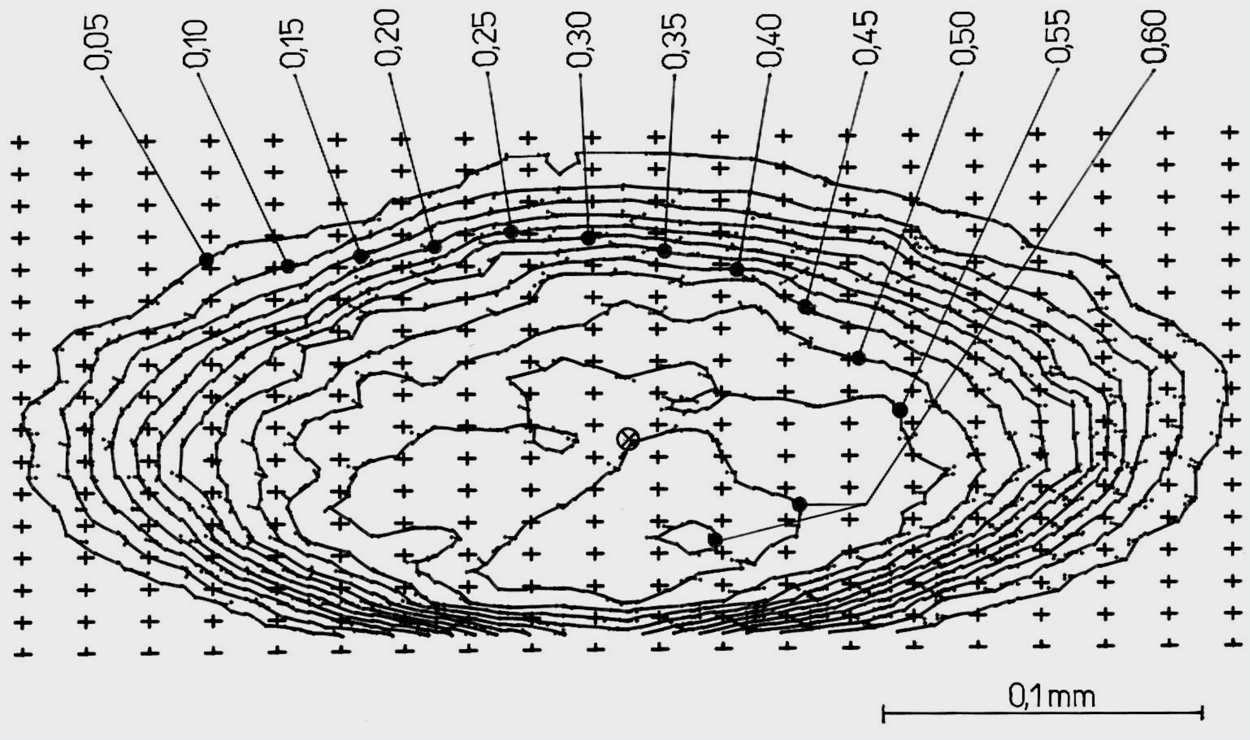
Example: Warp – viscose staple yarn 25 tex (cotton type); fabric – plain weave, setts $D_o = D_u = 2470\ m^{-1}$.

Experimentally determined curves – "isodenses" - connect places of same values of fiber packing density



Waves heights and yarn heights

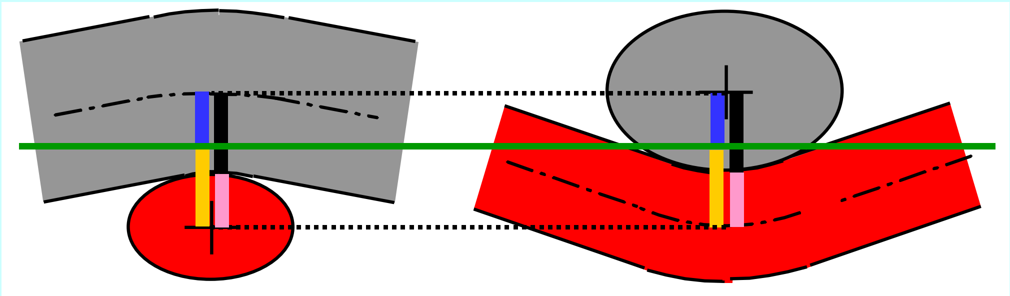
It is in each binding point

Wave height of warp... h_o (|)

Wave height of weft... h_u (|)

Half of height of warp yarn... $b_o/2$ (|)

Half of height of weft yarn... $b_u/2$ (|)



Distance between warp and weft yarn axis is

$$h_o + h_u = \left(b_o + b_u\right)/2$$

Note: This equivalency is valid every time, independently to a theoretical model used.

Models of woven fabric - overview

1. MECHANICAL MODELS – respect, that the yarns are deformed (lengthwise, transversally) by means of mechanical forces ⇒ physical the best, but very difficult.

2. PRIOR GEOMETRIC MODELS – go out from initial geometric assumptions about yarn axes and cross-sections.

Yarn axes – formed only from abscissas
– formed from ring arches and abscissas
– formed from another curves

Yarn cross-sections in binding points of fabric
– circular
– another

Waviness of warp and weft
– balanced fabric
– non-balanced fabric

PEIRCE'S MODEL OF FABRIC STRUCTURE

Type: Prior geometric. *Yarn axes:* Arches, abscissas.
Cross-sections: Ring. *Waviness:* Non-balanced.

Let us assume that we know:

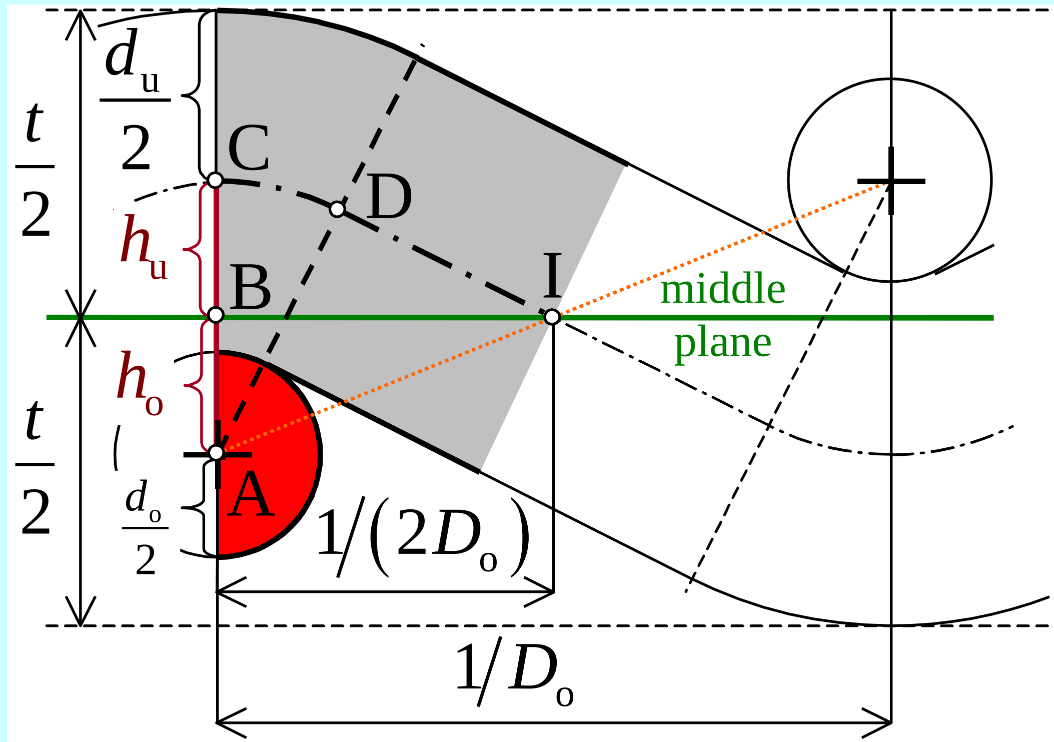
warp sett… D_o, weft sett… D_u,
warp diameter… d_o, weft diameter… d_u,
wave height of warp … h_o,
wave height of weft … h_u

Because of yarns circular cross-section, each yarn height is equal to yarn diameter, $b_o = d_o,\ b_u = d_u$. So it is valid

$$h_o + h_u = \left(\overbrace{b_o + b_u}^{=d_o + d_u}\right)\Big/2, \qquad h_o + h_u = \left(d_o + d_u\right)/2$$

Note: It is sufficient to know only 3 of quantities h_o, h_u, d_o, d_u; the fourth value is given by previous equation.

Geometry on crossed segment of weft yarn



Pitch of warp yarns (distance)_… $1/D_o$

Point I…center of punctual symmetry ("flex point"). It lies on the middle plane and on the join of warp yarn axes;

$$BI = (1/D_o)/2$$

Circular bow CD… center A, radius $h_o + h_u$

Thickness of fabric… t ($t > d_o + d_u$ in non-balanced fabric)

Note: Thenceforth, we shall use only the "half-wave" part

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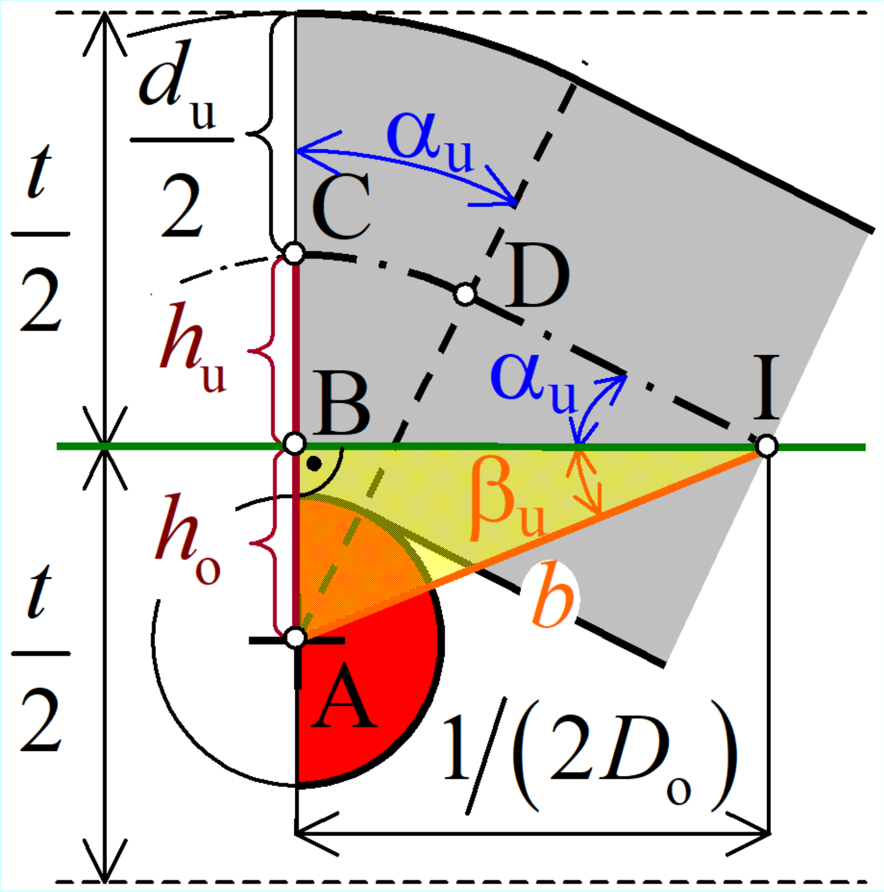
Abscissa AI = b

from the triangle ABI:

$$b^2 = h_o^2 + \left(\frac{1}{2D_o}\right)^2 = $$

$$= h_o^2 + \frac{1}{4D_o^2},$$

$$b = \sqrt{h_o^2 + \frac{1}{4D_o^2}}$$



Angle β_u (AIB):

$$\mathrm{tg}\,\beta_u = \frac{h_o}{1/(2D_o)},$$

$$\mathrm{tg}\,\beta_u = 2D_o h_o$$

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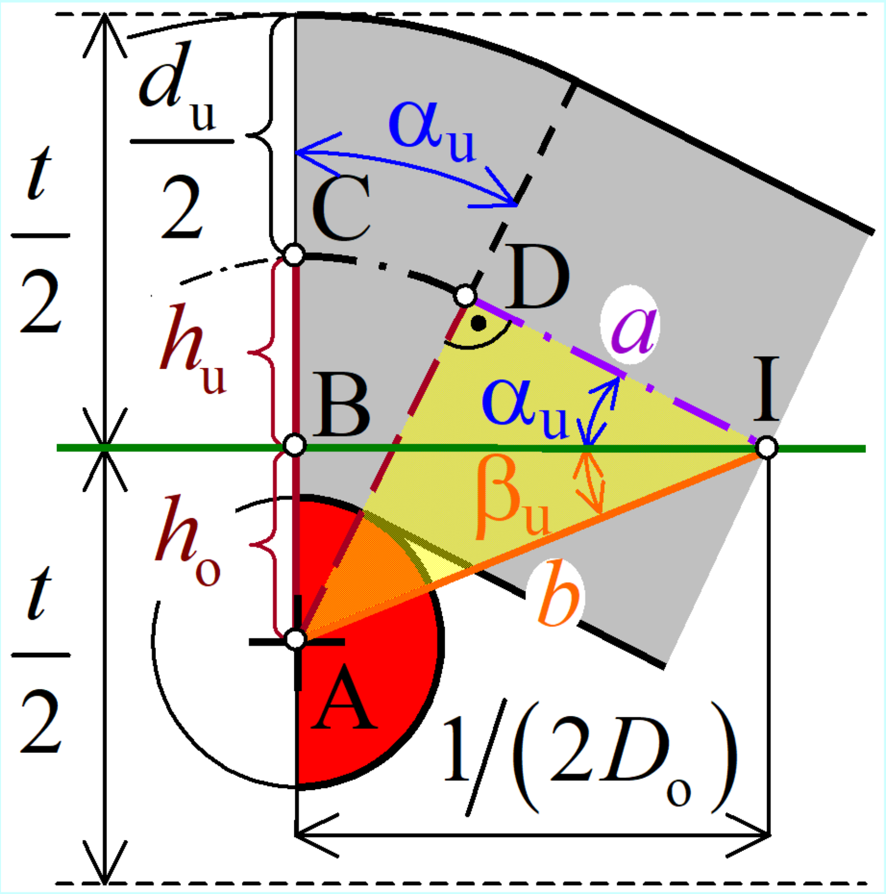
<u>Abscissa</u> DI=a
from the triangle ADI:

$$AD = AC = h_o + h_u$$

$$b^2 = \Big(\underbrace{AD}_{=h_o+h_u}\Big)^2 + a^2 = \\ = \underbrace{(h_o + h_u)^2 + a^2}_{=h_o^2+1/(4D_o^2)},$$

$$a^2 = b^2 - (h_o + h_u)^2 = \\ = h_o^2 + 1/(4D_o^2) - (h_o + h_u)^2,$$

$$a = \sqrt{1/(4D_o^2) + h_o^2 - (h_o + h_u)^2}$$



Angle $\alpha_u+\beta_u$ (AID): $\mathrm{tg}(\alpha_u + \beta_u) = AD / a$, where $AD = h_o + h_u$ and $a = \sqrt{1/(4D_o^2) + h_o^2 - (h_o + h_u)^2}$

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$$\mathrm{tg}\left(\alpha_u+\beta_u\right)=\frac{h_o+h_u}{\sqrt{1/\left(4D_o^2\right)+h_o^2-\left(h_o+h_u\right)^2}}$$

Known formula is valid among angles α_u, β_u and $\alpha_u+\beta_u$.

$\mathrm{tg}\left(\alpha_u+\beta_u\right)=\left(\mathrm{tg}\,\alpha_u+\mathrm{tg}\,\beta_u\right)/\left(1-\mathrm{tg}\,\alpha_u\,\mathrm{tg}\,\beta_u\right)$. Hence

$$\mathrm{tg}\left(\alpha_u+\beta_u\right)-\mathrm{tg}\left(\alpha_u+\beta_u\right)\mathrm{tg}\,\alpha_u\,\mathrm{tg}\,\beta_u=\mathrm{tg}\,\alpha_u+\mathrm{tg}\,\beta_u,$$

$$\mathrm{tg}\left(\alpha_u+\beta_u\right)-\mathrm{tg}\,\beta_u=\mathrm{tg}\,\alpha_u+\mathrm{tg}\left(\alpha_u+\beta_u\right)\mathrm{tg}\,\alpha_u\,\mathrm{tg}\,\beta_u=$$

$$=\mathrm{tg}\,\alpha_u\left[1+\mathrm{tg}\left(\alpha_u+\beta_u\right)\mathrm{tg}\,\beta_u\right],$$

$$\mathrm{tg}\,\alpha_u=\left[\mathrm{tg}\left(\alpha_u+\beta_u\right)-\mathrm{tg}\,\beta_u\right]/\left[1+\mathrm{tg}\left(\alpha_u+\beta_u\right)\mathrm{tg}\,\beta_u\right]$$

And so

$$\mathrm{tg}\,\alpha_u=\left[\overbrace{\mathrm{tg}\left(\alpha_u+\beta_u\right)}^{=\frac{h_o+h_u}{\sqrt{1/\left(4D_o^2\right)+h_o^2-\left(h_o+h_u\right)^2}}}-\overset{=2D_oh_o}{\mathrm{tg}\,\beta_u}\right]\Big/\left[1+\overbrace{\mathrm{tg}\left(\alpha_u+\beta_u\right)}^{=\frac{h_o+h_u}{\sqrt{1/\left(4D_o^2\right)+h_o^2-\left(h_o+h_u\right)^2}}}\overset{=2D_oh_o}{\mathrm{tg}\,\beta_u}\right],$$

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$$\mathrm{tg}\,\alpha_u = \left[\frac{h_o + h_u}{\sqrt{1/(4D_o^2) + h_o^2 - (h_o + h_u)^2}} - 2D_o h_o\right] \Bigg/ \left[1 + \frac{h_o + h_u}{\sqrt{1/(4D_o^2) + h_o^2 - (h_o + h_u)^2}} 2D_o h_o\right]$$

and after multiplication of square root:

$$\mathrm{tg}\,\alpha_u = \frac{(h_o + h_u) - 2D_o h_o \sqrt{1/(4D_o^2) + h_o^2 - (h_o + h_u)^2}}{\sqrt{1/(4D_o^2) + h_o^2 - (h_o + h_u)^2} + (h_o + h_u) 2D_o h_o}$$

Formal simplification of expressions

Let us introduce

- **relative wave height of warp**… $\lambda_o = h_o/(h_o + h_u)$
- **relative wave height of weft**… $\lambda_u = h_u/(h_o + h_u)$

Because $h_o + h_u = (d_o + d_u)/2$ it is valid

$$\lambda_o = \frac{2h_o}{d_o + d_u}, \quad h_o = \lambda_o \frac{d_o + d_u}{2} \qquad \lambda_u = \frac{2h_u}{d_o + d_u}, \quad h_u = \lambda_u \frac{d_o + d_u}{2}$$

It is valid for the angle β_u now

$$\mathrm{tg}\,\beta_u = 2D_o \overbrace{h_o}^{=\lambda_o(d_o+d_u)/2}, \quad \boxed{\mathrm{tg}\,\beta_u = D_o\lambda_o\left(d_o+d_u\right),} \quad \boxed{D_o = \frac{\mathrm{tg}\,\beta_u}{\lambda_o\left(d_o+d_u\right)}}$$

Let us think we know the value of $\mathrm{tg}\,\beta_u$. It is valid

$$\overbrace{\sin^2\beta_u/\sin^2\beta_u}^{=1} + \overbrace{\cos^2\beta_u/\sin^2\beta_u}^{=1/\mathrm{tg}^2\beta_u} = 1/\sin^2\beta_u\,, \quad \boxed{1+\frac{1}{\mathrm{tg}^2\beta_u} = \frac{1}{\sin^2\beta_u}}$$

Using of λ_o, λ_u, $\mathrm{tg}\beta_u$ or $\sin\beta_u$ we find

$$\left[\overbrace{h_o}^{=\lambda_o\frac{d_o+d_u}{2}}\right]^2 + \left[1\Big/\left(2\overbrace{D_o}^{=\frac{\mathrm{tg}\beta_u}{\lambda_o(d_o+d_u)}}\right)\right]^2 = \lambda_o^2\frac{\left(d_o+d_u\right)^2}{4} + \frac{1}{4}\frac{\lambda_o^2\left(d_o+d_u\right)^2}{\mathrm{tg}^2\beta_u} =$$

$$= \frac{\left(d_o+d_u\right)^2}{4}\lambda_o^2\left(\overbrace{1+1/\mathrm{tg}^2\beta_u}^{=1/\sin^2\beta_u}\right)$$

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So $h_o^2 + \frac{1}{(2D_o)^2} = \frac{(d_o + d_u)^2}{4}\frac{\lambda_o^2}{\sin^2\beta_u}$. By using this expression

$$\underbrace{\frac{1}{(2D_o)^2} + h_o^2}_{=\frac{(d_o+d_u)^2}{4}\frac{\lambda_o^2}{\sin^2\beta_u}} - \underbrace{(h_o + h_u)^2}_{=\frac{(d_o+d_u)^2}{4}}, \qquad \frac{1}{(2D_o)^2} + h_o^2 - (h_o + h_u) = \frac{(d_o + d_u)^2}{4}\left(\frac{\lambda_o^2}{\sin^2\beta_u} - 1\right)$$

We can write now

$$b = \sqrt{\underbrace{h_o^2 + 1/(4D_o^2)}_{=\frac{(d_o+d_u)^2}{4}\frac{\lambda_o^2}{\sin^2\beta_u}}}, \qquad b = \frac{d_o + d_u}{2}\frac{\lambda_o}{\sin\beta_u}$$

$\mathrm{tg}\,\beta_u = D_o\lambda_o(d_o + d_u)$ (see earlier); also $\frac{\mathrm{tg}\,\beta_u}{\lambda_o} = D_o(d_o + d_u)$

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$$a=\sqrt{\underbrace{1/\left(4D_o^2\right)+h_o^2-\left(h_o+h_u\right)^2}_{=\frac{(d_o+d_u)^2}{4}\left(\frac{\lambda_o^2}{\sin^2\beta_u}-1\right)}},$$

$$a=\frac{d_o+d_u}{2}\sqrt{\frac{\lambda_o^2}{\sin^2\beta_u}-1}$$

$$\mathrm{tg}\left(\alpha_u+\beta_u\right)=\underbrace{\left(h_o+h_u\right)}_{=(d_o+d_u)/2}\Big/\sqrt{\underbrace{1/\left(4D_o^2\right)+h_o^2-\left(h_o+h_u\right)^2}_{=\frac{(d_o+d_u)^2}{4}\left(\frac{\lambda_o^2}{\sin^2\beta_u}-1\right)}}=\frac{d_o+d_u}{2}\Big/\left(\frac{d_o+d_u}{2}\sqrt{\frac{\lambda_o^2}{\sin^2\beta_u}-1}\right),$$

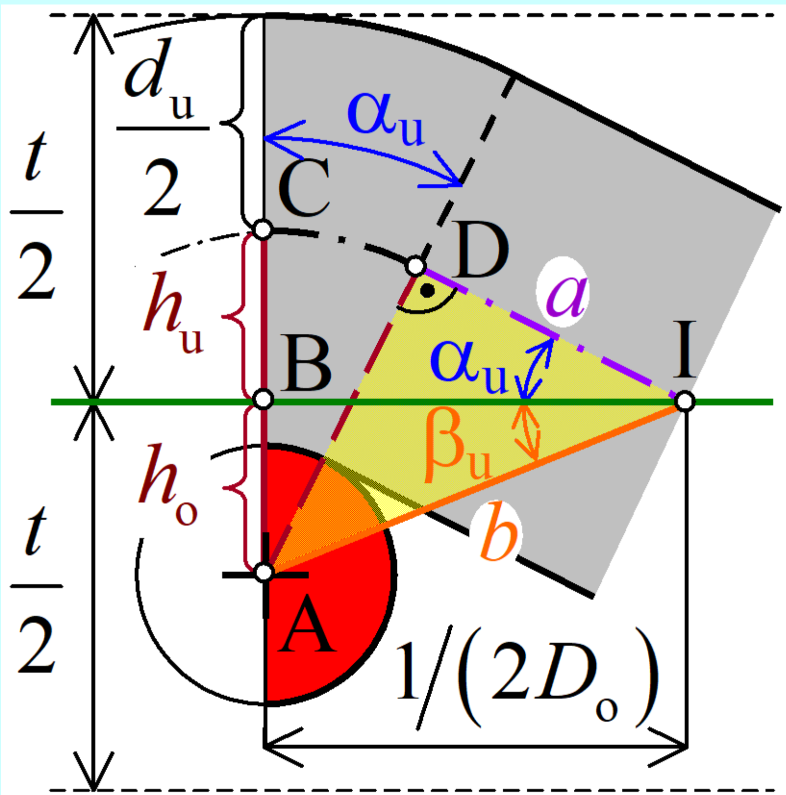
$$\mathrm{tg}\left(\alpha_u+\beta_u\right)=1\Big/\sqrt{\frac{\lambda_o^2}{\sin^2\beta_u}-1}$$

and finally

$$\mathrm{tg}\,\alpha_u=\frac{\underbrace{\left(h_o+h_u\right)}_{=(d_o+d_u)/2}-\underbrace{2\,D_o h_o}_{=\mathrm{tg}\,\beta_u/2}\sqrt{\underbrace{1/\left(4D_o^2\right)+h_o^2-\left(h_o+h_u\right)^2}_{=\frac{(d_o+d_u)^2}{4}\left(\frac{\lambda_o^2}{\sin^2\beta_u}-1\right)}}}{\sqrt{\underbrace{1/\left(4D_o^2\right)+h_o^2-\left(h_o+h_u\right)^2}_{=\frac{(d_o+d_u)^2}{4}\left(\frac{\lambda_o^2}{\sin^2\beta_u}-1\right)}}+\underbrace{\left(h_o+h_u\right)}_{=(d_o+d_u)/2}\underbrace{2\,D_o h_o}_{=\mathrm{tg}\,\beta_u/2}}$$

$$\operatorname{tg}\alpha_u = \frac{\frac{d_o+d_u}{2} - \operatorname{tg}\beta_u \frac{d_o+d_u}{2}\sqrt{\frac{\lambda_o^2}{\sin^2\beta_u}-1}}{\frac{d_o+d_u}{2}\sqrt{\frac{\lambda_o^2}{\sin^2\beta_u}-1} + \frac{d_o+d_u}{2}\operatorname{tg}\beta_u},$$

$$\operatorname{tg}\alpha_u = \frac{1-\operatorname{tg}\beta_u\sqrt{\frac{\lambda_o^2}{\sin^2\beta_u}-1}}{\sqrt{\frac{\lambda_o^2}{\sin^2\beta_u}-1}+\operatorname{tg}\beta_u}$$



Crimping of weft (crossed)

It is: IB⊥CB, ID⊥AD, so that

$$\alpha_u = \angle DIB = \angle DAC$$

Then the <u>length of bow</u> CD is

$$CD = \alpha_u \Big(\underbrace{h_o + h_u}_{=(d_o+d_u)/2}\Big), \quad CD = \alpha_u \frac{d_o+d_u}{2}$$

Note: Angles in <u>radians</u>!

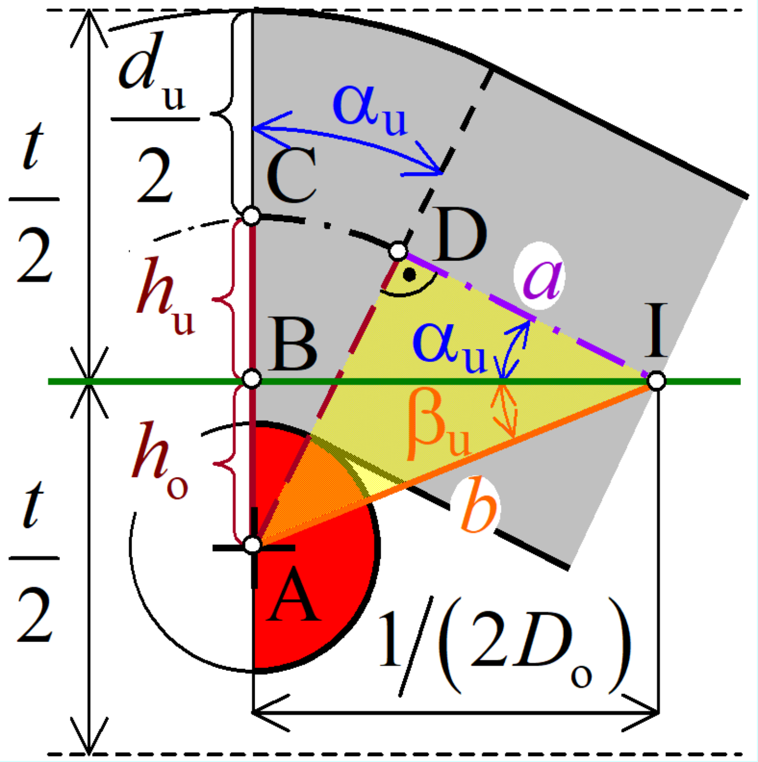
It is in illustrated "half-wave" part:

- length of weft yarn $l_u = CD + a$

$CD = \alpha_u \frac{d_o + d_u}{2}$, $a = \frac{d_o + d_u}{2}\sqrt{\frac{\lambda_o^2}{\sin^2\beta_u} - 1}$

$$l_u = CD + a = \frac{d_o + d_u}{2}\left(\alpha_u + \sqrt{\frac{\lambda_o^2}{\sin^2\beta_u} - 1}\right),$$

- length of fabric $l_{t,u} = 1/(2D_o)$



Crimping of weft

$$s_u = \frac{l_u - l_{t,u}}{l_{t,u}} = l_u / l_{t,u} - 1 = \frac{2D_o(d_o + d_u)}{2}\left(\alpha_u + \sqrt{\frac{\lambda_o^2}{\sin^2\beta_u} - 1}\right) - 1,$$

where $l_u = \frac{d_o + d_u}{2}\left(\alpha_u + \sqrt{\frac{\lambda_o^2}{\sin^2\beta_u} - 1}\right)$, $l_{t,u} = 1/(2D_o)$, $D_o(d_o + d_u) = \mathrm{tg}\,\beta_u/\lambda_o$

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$$s_u = \frac{\operatorname{tg}\beta_u}{\lambda_o}\left(\alpha_u + \sqrt{\frac{\lambda_o^2}{\sin^2\beta_u} - 1}\right) - 1 \,, \qquad \alpha_u = \operatorname{arctg}\left[\frac{1 - \operatorname{tg}\beta_u\sqrt{\frac{\lambda_o^2}{\sin^2\beta_u} - 1}}{\sqrt{\frac{\lambda_o^2}{\sin^2\beta_u} - 1} + \operatorname{tg}\beta_u}\right]$$

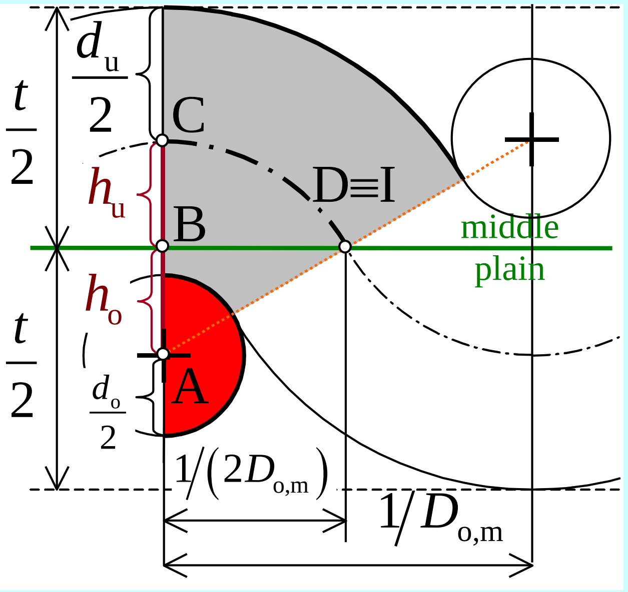
Limit sett of warp

(in crossed segment)

Let us increase the warp sett D_o at still constant values of h_o, h_u, d_o, d_u. We come upon some "barrier limit" in a moment. This highest warp sett is so called

limit sett of warp… $D_{o,m}$

In the case of limit sett the bows of weft yarn are mutually connected, so that the length DI is equal to 0.



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In such case it is

$$DI = a = \frac{d_o + d_u}{2}\sqrt{\frac{\lambda_o^2}{\sin^2\beta_u} - 1} = 0$$

and it must be

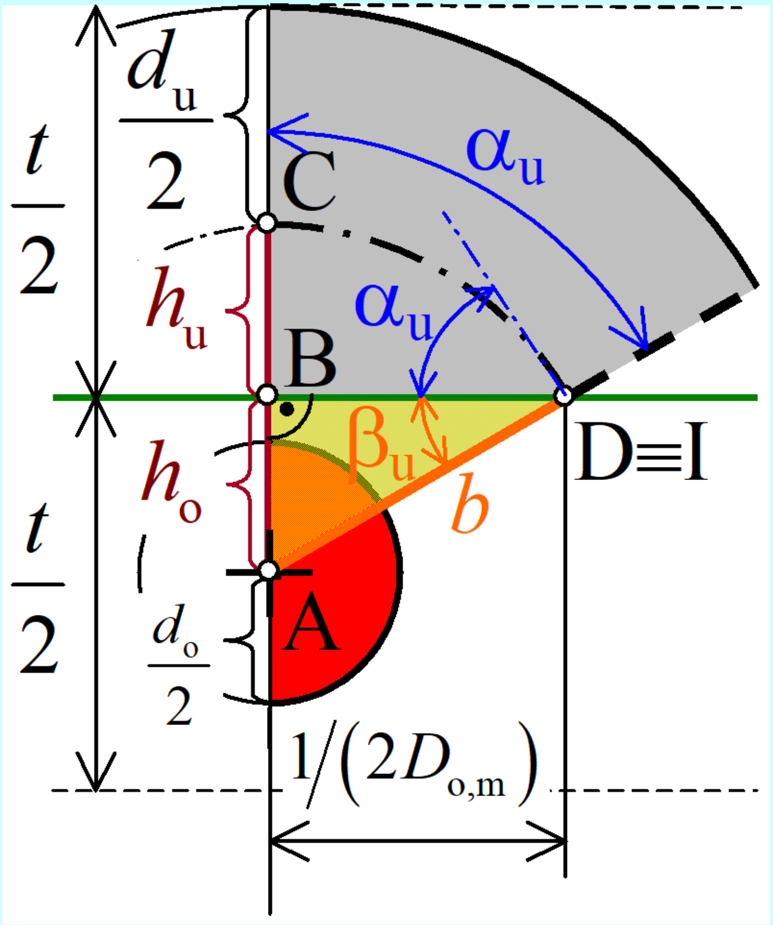
$$\frac{\lambda_o^2}{\sin^2\beta_u} - 1 = 0 \qquad \frac{\lambda_o^2}{\sin^2\beta_u} = 1, \quad \frac{\lambda_o}{\sin\beta_u} = 1,$$

$$\sin\beta_u = \lambda_o$$

$$\mathrm{tg}\,\beta_u = \frac{\sin\beta_u}{\cos\beta_u} = \frac{\overset{=\lambda_o}{\sin\beta_u}}{\sqrt{1-\left(\overset{=\lambda_o}{\sin\beta_u}\right)^2}}, \qquad \cos\beta_u = \sqrt{1-\sin^2\beta_u}$$

$$\mathrm{tg}\,\beta_u = \frac{\lambda_o}{\sqrt{1-\lambda_o^2}}$$

Half-wave by limit sett:

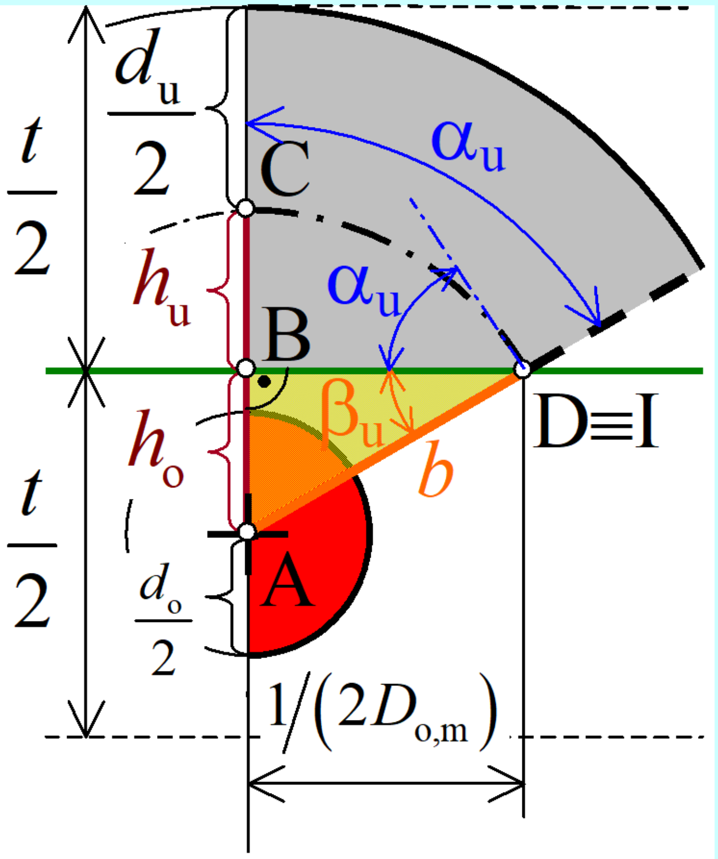


$$\mathrm{tg}(\alpha_u+\beta_u)=1\Bigg/\sqrt{\underbrace{\frac{\lambda_o^2}{\sin^2\beta_u}-1}_{=0}}=\frac{1}{0}=\infty$$

$$\alpha_u+\beta_u=\pi/2$$

$$\mathrm{tg}\,\alpha_u=\frac{1-\mathrm{tg}\,\beta_u\sqrt{\underbrace{\frac{\lambda_o^2}{\sin^2\beta_u}-1}_{=0}}}{\sqrt{\underbrace{\frac{\lambda_o^2}{\sin^2\beta_u}-1}_{=0}}+\mathrm{tg}\,\beta_u}=\frac{1}{\mathrm{tg}\,\beta_u}$$

$$\mathrm{tg}\,\beta_u=\frac{\lambda_o}{\sqrt{1-\lambda_o^2}}$$



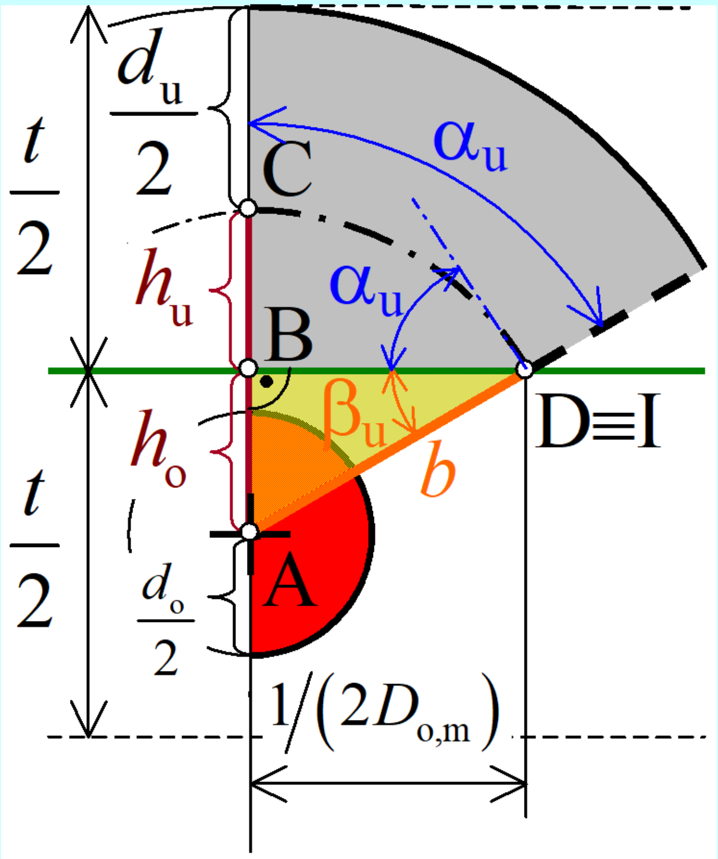
$$\mathrm{tg}\,\alpha_u=\frac{\sqrt{1-\lambda_o^2}}{\lambda_o},\quad \alpha_u=\mathrm{arctg}\left(\frac{\sqrt{1-\lambda_o^2}}{\lambda_o}\right)$$

Limit crimping of weft… $s_{u,m}$

$$s_{u,m} = \underbrace{\frac{\operatorname{tg}\beta_u}{\lambda_o}}_{=\frac{\lambda_o}{\sqrt{1-\lambda_o^2}}}\left(\underbrace{\alpha_u}_{=\operatorname{arctg}\left(\frac{\sqrt{1-\lambda_o^2}}{\lambda_o}\right)} + \sqrt{\underbrace{\frac{\lambda_o^2}{\sin^2\beta_u} - 1}_{=0}}\right) - 1,$$

$$s_{u,m} = \frac{\operatorname{arctg}\left(\sqrt{1-\lambda_o^2}/\lambda_o\right)}{\sqrt{1-\lambda_o^2}} - 1$$

The limit sett we find from the expression $\operatorname{tg}\beta_u = D_o\lambda_o\left(d_o + d_u\right)$.



Limit sett of warp…

$$D_{o,m} = \frac{1}{\left(d_o + d_u\right)\sqrt{1-\lambda_o^2}}$$

Geometry on crossed segment of warp yarn, limit sett of weft

The equations, describing the geometry of warp yarn segment incl. the sett of weft yarns, can be derived similarly; but

these equations we obtain after change of subscripts 'o' and 'u' in all previous equations!

E.g. it is valid

- Limit crimping of warp on a crossed segment

$$s_{o,m} = \frac{\mathrm{arctg}\left(\sqrt{1-\lambda_u^2}/\lambda_u\right)}{\sqrt{1-\lambda_u^2}} - 1$$

- Limit sett of weft

$$D_{u,m} = \frac{1}{(d_o + d_u)\sqrt{1-\lambda_u^2}}$$

etc.

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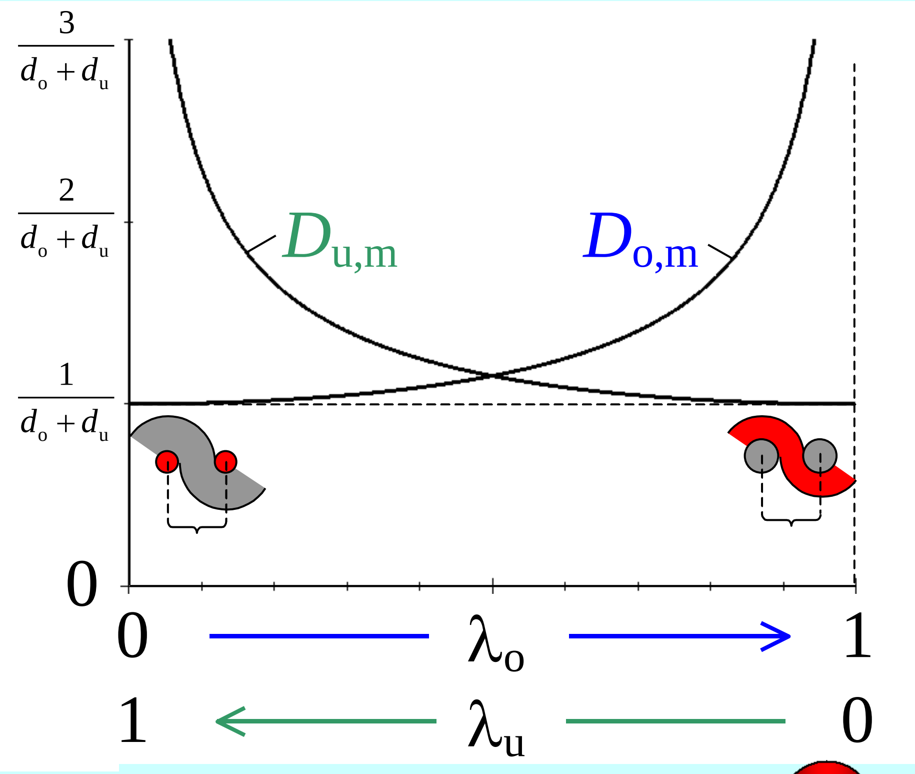
Mutual dependence of limit setts:

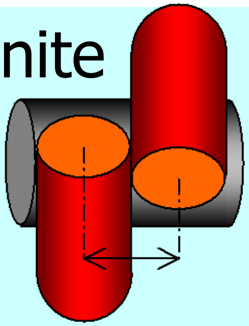
$$D_{o,m} = \frac{1}{(d_o + d_u)\sqrt{1-\lambda_o^2}}$$

$$D_{u,m} = \frac{1}{(d_o + d_u)\sqrt{1-\lambda_u^2}}$$



If the limit sett of such system increases then the limit sett of second one decreases!

Note: Limit setts can reach theoretically to the infinite value. But – with respect to yarn crossing – the maximum of each sett cannot be higher then reciprocal value of yarn diameter.



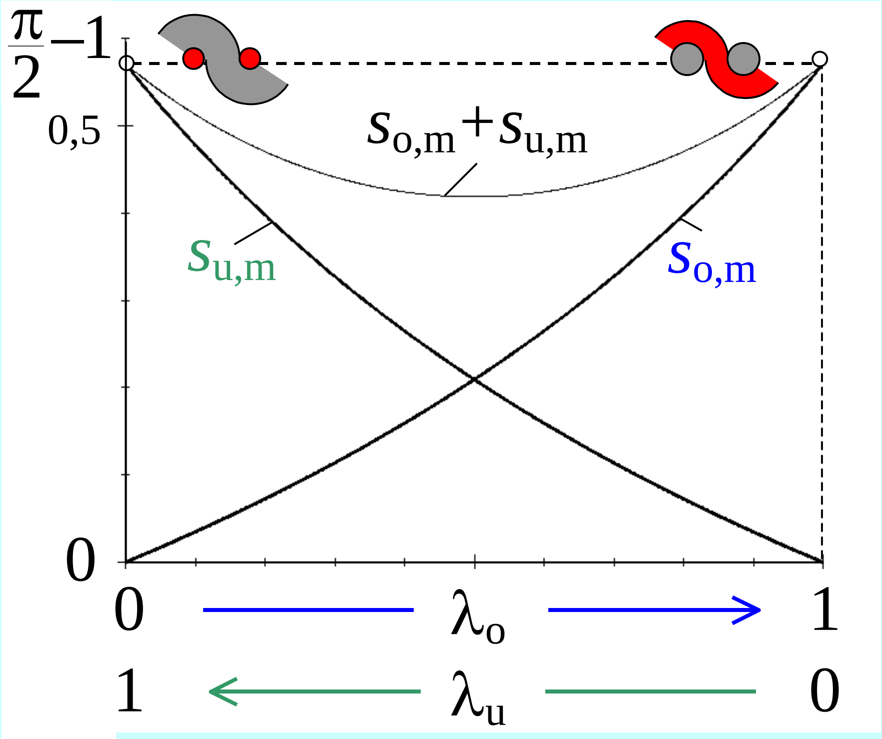
WOVEN FABRICS 2

Limit crimping in relation to relative wave heights:

$$s_{o,m} = \frac{\operatorname{arctg}\left(\sqrt{1-\lambda_u^2}/\lambda_u\right)}{\sqrt{1-\lambda_u^2}} - 1$$

$$s_{u,m} = \frac{\operatorname{arctg}\left(\sqrt{1-\lambda_o^2}/\lambda_o\right)}{\sqrt{1-\lambda_o^2}} - 1$$

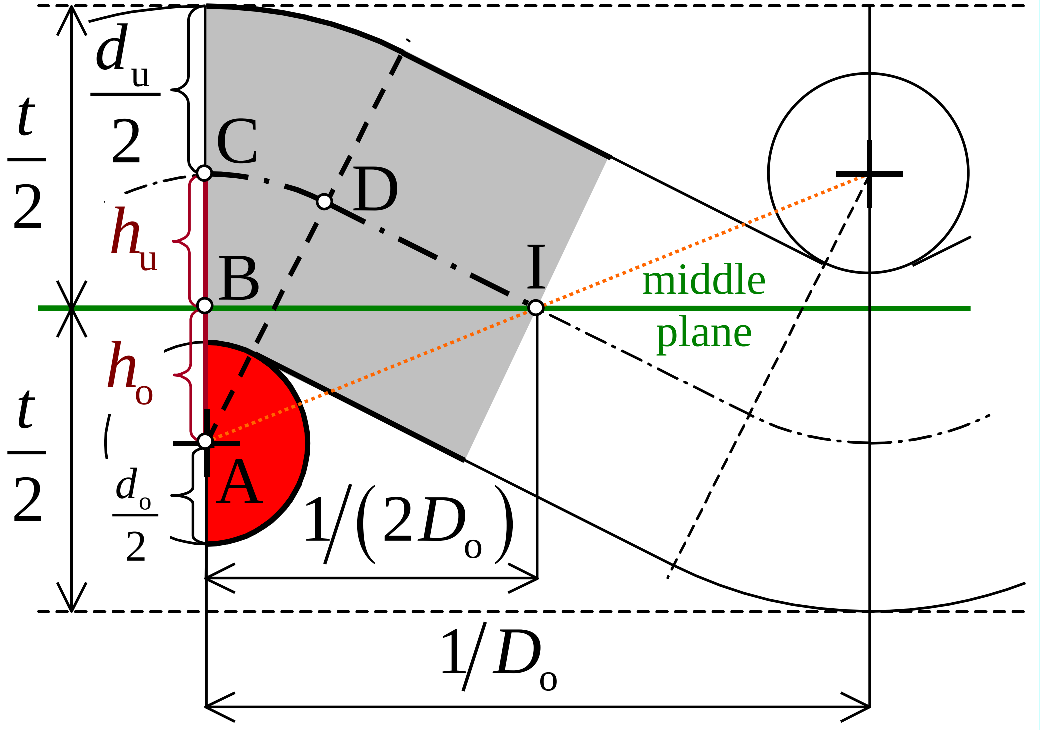
If the limit crimping of such system increases then the limit crimping of second one decreases!



Note: Near the value $\lambda_o = 1 - \lambda_u = 0.5$ the sum of both limit crimping is roughly constant.

Thickness of fabric

It is the double value of maximum from values $h_o+d_o/2$ and $h_u+d_u/2$. (Higher value is $h_u+d_u/2$ on our scheme.)



Generally

$$\frac{t}{2}=\max\left[\left(h_o+\frac{d_o}{2}\right),\left(h_u+\frac{d_u}{2}\right)\right]$$

Thickness of fabric $t=\max\left[2h_o+d_o,\ 2h_u+d_u\right]$

WOVEN FABRICS 2

The relative wave heights $\lambda_o = 2h_o/(d_o + d_u),\ \lambda_u = 2h_u/(d_o + d_u)$ can be used in the last equation.

Further we define

- **relative diameter of warp yarn**… $\delta_o = d_o/(d_o + d_u)$
- **relative diameter of weft yarn**… $\delta_u = d_u/(d_o + d_u)$

So, we can express the thickness of fabric by following equation

$$t = (d_o + d_u)\cdot \max\left[\underbrace{\left(\frac{2h_o}{d_o + d_u}\right)}_{=\lambda_o} + \underbrace{\left(\frac{d_o}{d_o + d_u}\right)}_{=\delta_o},\ \underbrace{\left(\frac{2h_u}{d_o + d_u}\right)}_{=\lambda_u} + \underbrace{\left(\frac{d_u}{d_o + d_u}\right)}_{=\delta_u}\right],$$

$$t = (d_o + d_u)\ \max\left[\lambda_o + \delta_o,\ \lambda_u + \delta_u\right]$$

Balanced fabric

Usually we don't know the values h_o, h_u (and/or λ_o, λ_u), but we know *empirically* that the warp and weft binding points often lies "almost" in the same height ⇒ model of balanced fabric. The warp and weft binding points (▼) lies in the same plane in the case of balanced fabric.

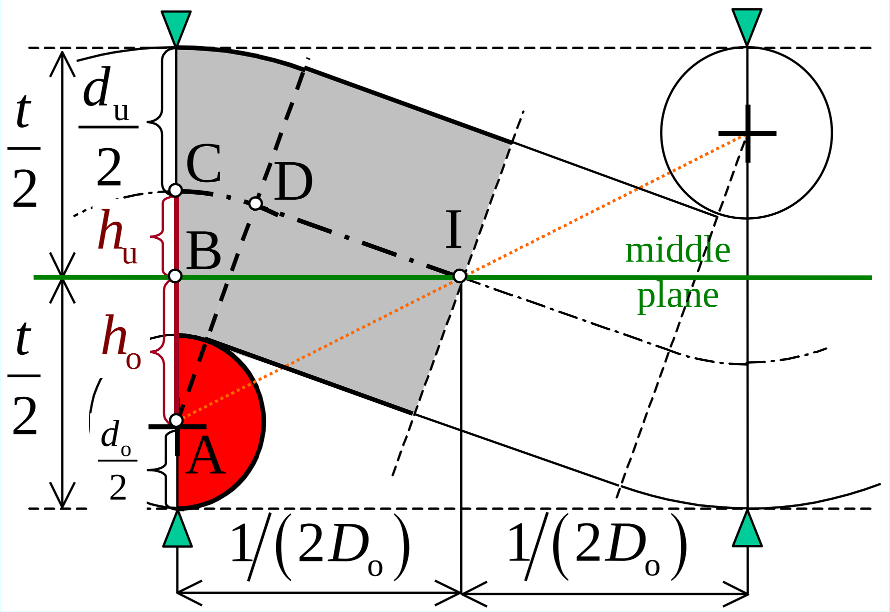
Then it is valid

$$h_o + \frac{d_o}{2} = h_u + \frac{d_u}{2}$$

By using of expressions

$$\lambda_o = 2h_o / (d_o + d_u),$$

$$\lambda_u = 2h_u / (d_o + d_u)$$



and $\delta_o = d_o/(d_o + d_u)$, $\delta_u = d_u/(d_o + d_u)$ we find

$$\underbrace{\left(\frac{2h_o}{d_o + d_u}\right)}_{=\lambda_o} + \frac{2}{2}\underbrace{\left(\frac{d_o}{d_o + d_u}\right)}_{=\delta_o} = \underbrace{\left(\frac{2h_u}{d_o + d_u}\right)}_{=\lambda_u} + \frac{2}{2}\underbrace{\left(\frac{d_u}{d_o + d_u}\right)}_{=\delta_u},$$

($=1-\delta_u$, $=1-\lambda_o$)

$\lambda_o + \delta_o = \lambda_u + \delta_u$. Further $\lambda_o + \delta_o = \lambda_u + \delta_u$,

$\lambda_o + 1 - \delta_u = 1 - \lambda_o + \delta_u$, ($=\delta_u$) $2\lambda_o = 2\delta_u$, $\lambda_o = \delta_u$

and also $\lambda_o + \delta_o = \lambda_u + \delta_u$, $\delta_u + \delta_o = \lambda_u + \delta_u$, $\lambda_u = \delta_o$

All earlier derived relations are valid also for each balanced fabric by using $\lambda_o = \delta_u$ and $\lambda_u = \delta_o$.

E.g.:

Thickness of fabric

$$t=(d_o+d_u)\,\max\Big[\lambda_o+\overbrace{\delta_o}^{=\delta_u\;=1},\ \lambda_u+\overbrace{\delta_u}^{=\delta_o\;=1}\Big],\qquad t=d_o+d_u$$

Limit sett of warp

$$D_{o,m}=1\Big/\left[(d_o+d_u)\sqrt{1-\left(\frac{\delta_u}{\lambda_o}\right)^2}\right],\qquad D_{o,m}=\frac{1}{(d_o+d_u)\sqrt{1-\delta_u^2}}$$

Limit covering by warp… $Z_{o,m}$

$$Z_{o,m}=\overbrace{D_{o,m}}^{=1/\left[(d_o+d_u)\sqrt{1-\delta_u^2}\right]}\,d_o=\overbrace{\left(\frac{d_o}{d_o+d_u}\right)}^{=\delta_o}\frac{1}{\sqrt{1-\delta_u^2}},\qquad Z_{o,m}=\frac{\delta_o}{\sqrt{1-\delta_u^2}}$$

Similarly we obtain for limit sett of weft and **limit covering by weft**

$$D_{u,m}=\frac{1}{(d_o+d_u)\sqrt{1-\delta_o^2}}\qquad Z_{u,m}=\frac{\delta_u}{\sqrt{1-\delta_o^2}}$$

Square balanced fabric of plain weave

(special case)

Square balanced fabric:

The same setts... $D_o = D_u = D_s$

The same yarn diameters... $d_o = d_u = d_s$

The same relative yarn diameters... $\delta_o = \delta_u = \delta_s = 1/2$

The same relative wave heights... $\lambda_o = \lambda_u = \lambda_s = 1/2$

Note: Each quantity with subscript 's' ("system") has the same value for warp as well as for weft system.

Covering by system... $Z_s = D_s d_s$

Angle β... $\operatorname{tg}\beta_o = \operatorname{tg}\beta_u = \operatorname{tg}\beta_s = D_s \underbrace{\lambda_s}_{=1/2} \left(\underbrace{d_s + d_s}_{=2d_s} \right) = D_s d_s = Z_s$

Limit sett...

$$D_{o,m} = D_{u,m} = D_{s,m} = 1 \Big/ \left[\left(\underbrace{d_s + d_s}_{=2d_s} \right) \sqrt{1 - \left(\underbrace{\delta_s}_{=1/2} \right)^2} \right] = \frac{1}{2 d_s \sqrt{3/4}} = \frac{1}{d_s \sqrt{3}}$$

WOVEN FABRICS 2

Limit covering by system...

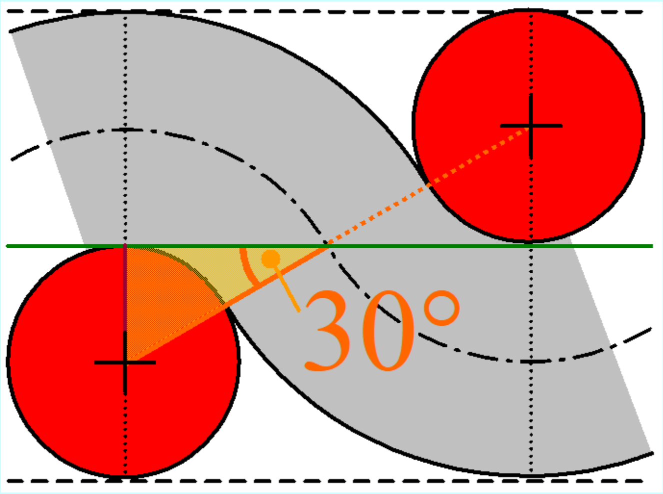
$$Z_{o,m} = Z_{u,m} = Z_{s,m} = \overset{=1/2}{\delta_s} \Big/ \sqrt{1-\left(\overset{=1/2}{\delta_s}\right)^2} = \frac{1}{2\sqrt{\frac{3}{4}}} = \frac{1}{\sqrt{3}} \doteq 0.577$$

Limit covering of the (whole) fabric...

$$Z_{c,m} = Z_{o,m} + Z_{u,m} - Z_{o,m} Z_{u,m} = \overset{=1/\sqrt{3}}{Z_{s,m}} + \overset{=1/\sqrt{3}}{Z_{s,m}} - \left(\overset{=1/\sqrt{3}}{Z_{s,m}}\right)^2 = \frac{2}{\sqrt{3}} - \frac{1}{3} \doteq 0,821$$

Limit value of the angle β...

$$\operatorname{tg}\beta_{s,m} = Z_{s,m},\ \ \beta_{s,m} = \operatorname{arctg} \overset{=1/\sqrt{3}}{Z_{s,m}} = $$
$$= 30^\circ$$



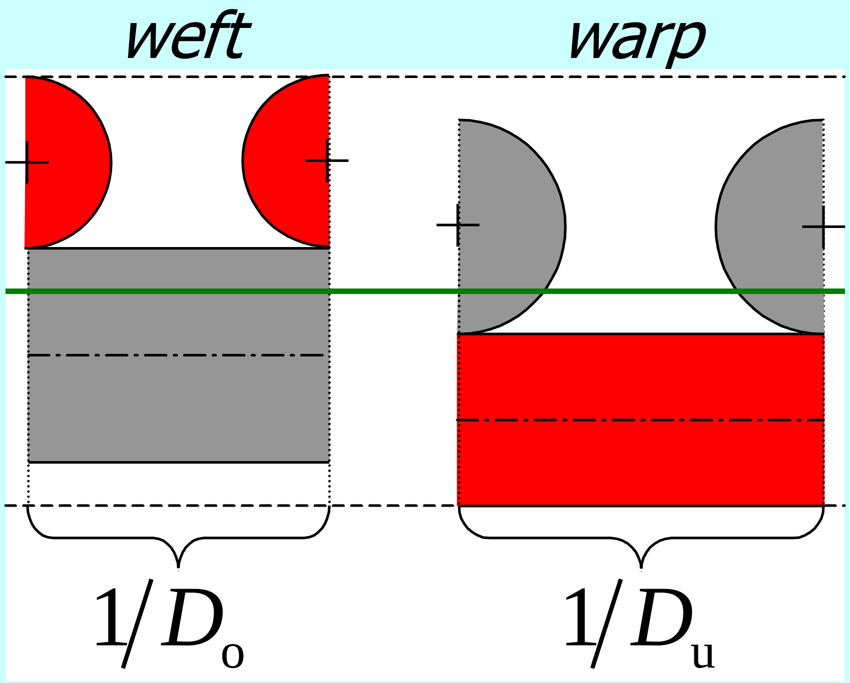
Notes to weaves with non-crossed segments:

Foregoing equations can be used also for weaves with non-crossed segments. The warp and weft crossing factors determine proportions of crossed segments.

Rest segments of given system are "straight", non- crossed. Length of such one is $1/D_o$ and/or $1/D_u$.

For crimping calculation we must sum lengths of crossed and non-crossed segments together

Non-crossed segments



Some problems of application of Peirce's model

1. What are (equivalent) diameters d_o, d_u of warp and weft yarns, how they are related to original yarn diameters, setts and other parameters of fabrics structure? What is the influence of yarn flattening?
2. What are wave heights of warp and weft h_o, h_u (λ_o, λ_u), how they relate to the other parameters of fabric?
3. To which degree the simple idea of balanced fabric is acceptable, which fabric structures can be consider as balanced?

Note: More complicated models often choose an empirical solving of such problems. (Fully exact model of fabric structure don't exist up to date.)