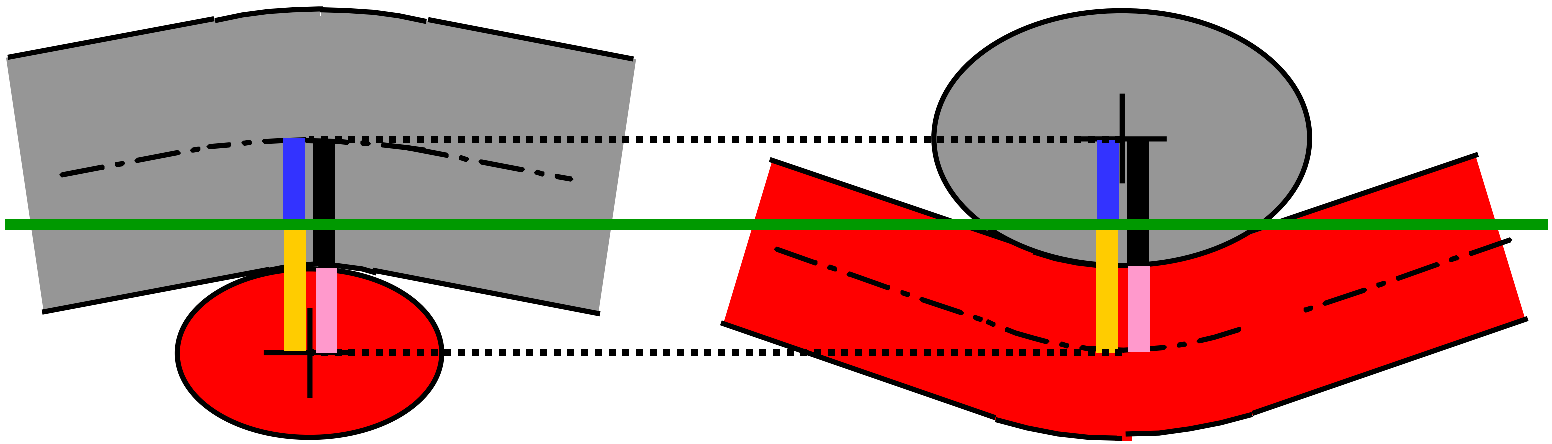
TECHNICAL UNIVERSITY OF LIBEREC
Faculty of Textile Engineering

WOVEN FABRIC

„GEOMETRICAL MODELS“



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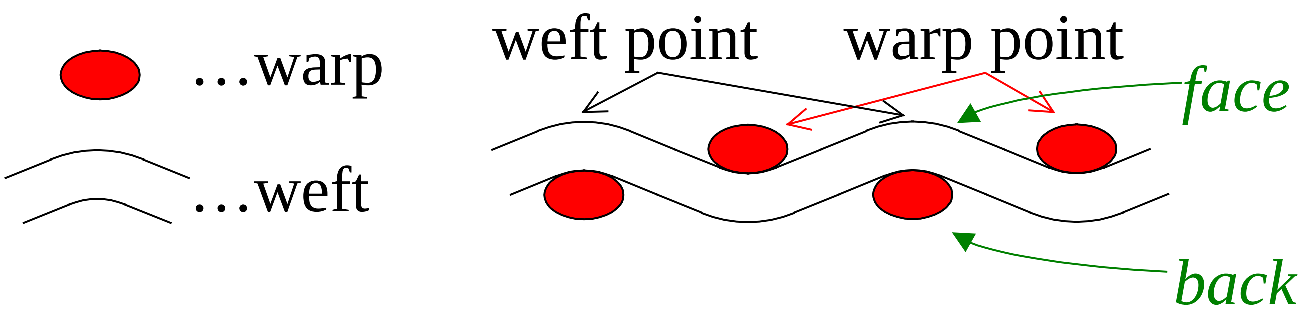
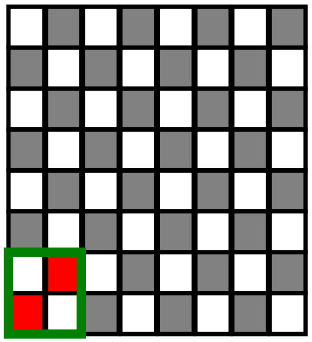
Woven fabric – basic parameters

– input parameters for woven fabric structure description – **Areal geometry of woven fabric**

Warp yarn – count (linear density) T_o [tex], material ρ_o [kgm^{-3}] (fiber density)
Weft yarn – count T_u [tex], material ρ_u [kgm^{-3}] (fiber density)
Weave (pattern)
Warp sett D_o [1/cm], [1/m]
Weft sett D_u [1/cm], [1/m]





Woven fabric – basic parameters

– input parameters for woven fabric structure description – **Spatial geometry of woven fabric**

Warp yarn – count (linear density) T_o [tex], material ρ_o [kgm^{-3}] (fiber density)
Weft yarn – count T_u [tex], material ρ_u [kgm^{-3}] (fiber density)
Weave (pattern)
Warp sett D_o [1/cm], [1/m]
Weft sett D_u [1/cm], [1/m]

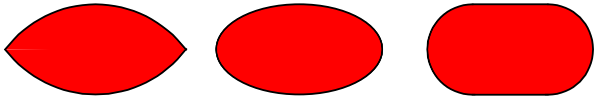
+
Height of warp binding wave h_o [mm]
Height of weft binding wave h_u [mm]
Relative wave height of warp λ_o
Relative wave height of weft λ_u
Yarn enlargement α
Yarn compression β

$$\lambda_o = h_o / (h_o + h_u)$$

$$\lambda_u = h_u / (h_o + h_u)$$

$$\lambda_o + \lambda_u = 1$$

(description of deformation and shape of yarn in binding point, description of waviness)



PEIRCE'S MODEL OF FABRIC STRUCTURE

Type: Prior geometric. Yarn axes: Arches, abscissas.

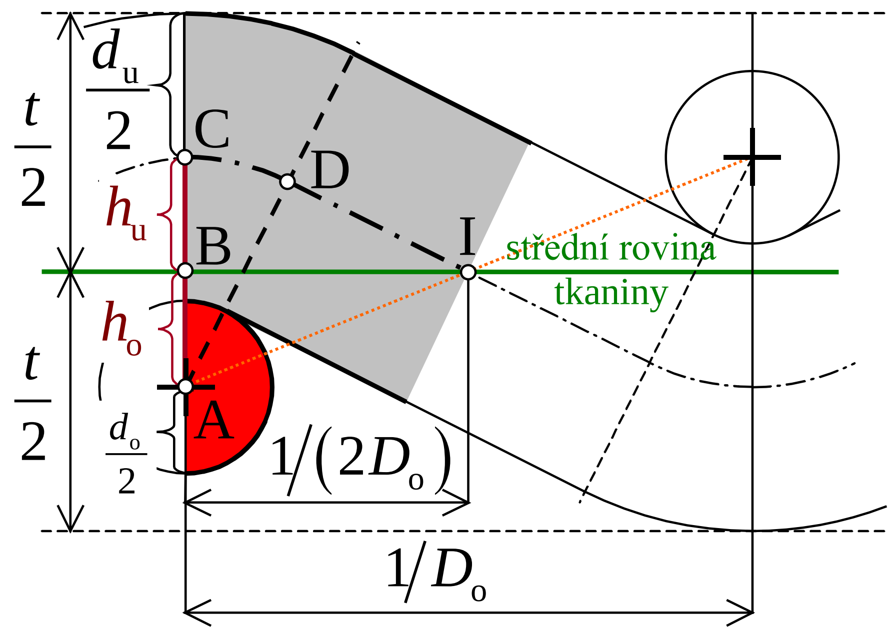
Cross-sections: Ring. Waviness: Non-balanced.

Let us assume that we know (input parameters):

warp sett… D_o, weft sett… D_u ,
warp diameter … d_o , weft diameter… d_u ,
wave height of warp… h_o , wave height of weft… h_u .

It is valid:

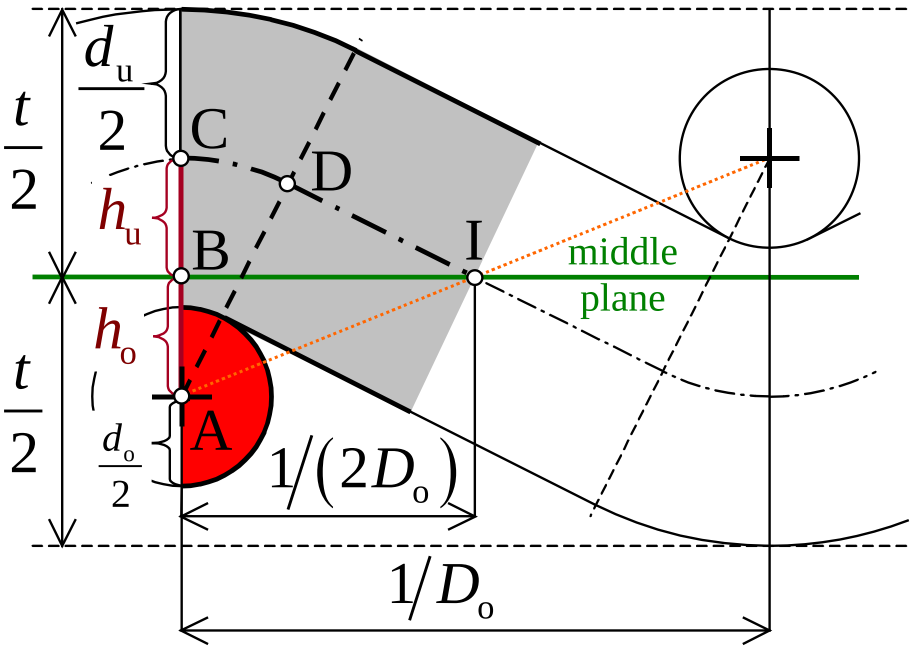
$$h_o + h_u = (d_o + d_u)/2$$



PEIRCE’S MODEL OF FABRIC STRUCTURE

Geometry of crossed segment of weft yarn:

The given relations are also valid for the crossed segment of the warp yarn with the exchange of subscript



$$l_u = C\overset{\frown}{D}_u + a_u$$

$$C\overset{\frown}{D}_u = \alpha_u \frac{d_o + d_u}{2}$$

$$a_u = \sqrt{1/\left(4D_o^2\right) + h_o^2 - \left(h_o + h_u\right)^2}$$

$$\operatorname{tg}\alpha_u = \frac{\left(h_o + h_u\right) - 2D_o h_o \sqrt{1/\left(4D_o^2\right) + h_o^2 - \left(h_o + h_u\right)^2}}{\sqrt{1/\left(4D_o^2\right) + h_o^2 - \left(h_o + h_u\right)^2} + \left(h_o + h_u\right) 2D_o h_o}$$

PEIRCE'S MODEL OF FABRIC STRUCTURE

Weft crimp in crossed segment s_u[-]

$$s_u = \frac{l_u - l_{t,u}}{l_{t,u}} = \frac{\left(C\widehat{D}_u + a_u\right) - \frac{1}{2D_o}}{\frac{1}{2D_o}} = 2D_o\left(C\widehat{D}_u + a_u\right) - 1$$

Fabric thickness t[mm]

$$t = \max\left[2h_o + d_o,\ 2h_u + d_u\right]$$

Task 1

Based on the Peirce's model calculate: length of weft yarn l_u and warp yarn l_o [mm], crimp of weft s_u and warp s_o in crossed segment [%], fabric thickness t [mm], weft covering Z_u, warp covering Z_o, woven fabric covering Z [%].

D_o = 20 cm^{-1}, D_u = 2 mm$^{-1,}$ d_o = 250 mm, d_u = 0,25 mm, h_o = 120 mm

$$l_u = 2\left(C\widehat{D}_u + a_u\right) \qquad = 0{,}5778\,\mathrm{mm}$$

$$a_u = \sqrt{1/\left(4\,D_o^2\right) + h_o^2 - \left(h_o + h_u\right)^2} \qquad = 0{,}12\,\mathrm{mm}$$

$$h_o + h_u = \frac{d_o + d_u}{2} \Rightarrow h_u = 130\,\mu\mathrm{m}$$

$$C\widehat{D}_u = \alpha_u \frac{d_o + d_u}{2} \qquad = 0{,}1689\,\mathrm{mm}$$

$$\mathrm{tg}\,\alpha_u = \frac{\left(h_o + h_u\right) - 2D_o h_o \sqrt{1/\left(4D_o^2\right) + h_o^2 - \left(h_o + h_u\right)^2}}{\sqrt{1/\left(4D_o^2\right) + h_o^2 - \left(h_o + h_u\right)^2} + \left(h_o + h_u\right) 2D_o h_o} \Rightarrow \alpha_u = 0{,}6758\,\mathrm{rad}$$

Crimp of weft yarn

$$s_u = \frac{l_u - l_{t,u}}{l_{t,u}} = 2\,D_o\left(C\widehat{D}_u + a_u\right) - 1 = 0{,}156 = 15{,}6\%$$

Length of warp yarn

$$l_o = 2\left(C\widehat{D}_o + a_o\right) = 0{,}5659\text{mm}$$

$$a_o = \sqrt{1/\left(4\,D_u^2\right) + h_u^2 - \left(h_o + h_u\right)^2} = 0{,}13\text{mm}$$

$$C\widehat{D}_o = \alpha_o \frac{d_o + d_u}{2} = 0{,}1529\text{mm}$$

$$\Rightarrow \alpha_o = 0{,}6118\,\text{rad}$$

Crimp of warp yarn

$$s_o = \frac{l_o - l_{t,o}}{l_{t,o}} = 2\,D_u\left(C\widehat{D}_o + a_o\right) - 1 = 0{,}164 = 13{,}2\%$$

Thickness of woven fabric

$$t = \max\left[2h_{\mathrm{o}} + d_{\mathrm{o}},\ 2h_{\mathrm{u}} + d_{\mathrm{u}}\right]$$

$$2\,h_o + d_o = 490\mu\mathrm{m}$$

$$\Rightarrow t = 510\mu\mathrm{m}$$

$$2\,h_u + d_u = 510\mu\mathrm{m}$$

Covering

$$Z_o = D_o\,d_o = 50\%$$

$$Z_u = D_u\,d_u = 50\%$$

$$Z = Z_o + Z_u - Z_o\,Z_u = 0{,}75 = 75\%$$

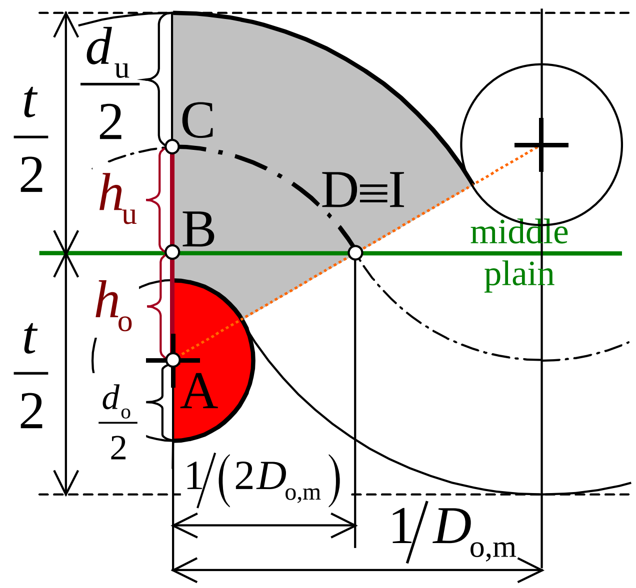
PEIRCE'S MODEL OF FABRIC STRUCTURE

Limit sett of warp (in crossed segments)… $D_{o,m}$ [1/mm]

$$D_{o,m} = \frac{1}{(d_o + d_u)\sqrt{1-\lambda_o^2}}$$

Limit crimp of weft… $s_{u,m}$ [-]

$$s_{u,m} = \frac{\operatorname{arctg}\left(\sqrt{1-\lambda_o^2}/\lambda_o\right)}{\sqrt{1-\lambda_o^2}} - 1$$



Task 2:

Calculate limit sett of weft $D_{u,m}$ and warp $D_{o,m}$ and limit crimp of warp $s_{o,m}$ and weft $s_{u,m}$ for previous woven fabric.

$$D_{o,m} = \frac{1}{(d_o + d_u)\sqrt{1-\lambda_o^2}} = 23\text{cm}^{-1}$$

$$D_{u,m} = \frac{1}{(d_o + d_u)\sqrt{1-\lambda_u^2}} = 23\text{cm}^{-1}$$

$$s_{u,m} = \frac{\text{arctg}\left(\sqrt{1-\lambda_o^2}/\lambda_o\right)}{\sqrt{1-\lambda_o^2}} - 1 = 0{,}2199 = 22\%$$

$$s_{o,m} = \frac{\text{arctg}\left(\sqrt{1-\lambda_u^2}/\lambda_u\right)}{\sqrt{1-\lambda_u^2}} - 1 = 0{,}1988 = 19{,}9\%$$

$$\lambda_o = h_o/(h_o + h_u) = 0{,}48$$

$$\lambda_u = h_u/(h_o + h_u) = 0{,}52$$