Metoda substituční

$$[\sin(x^2)]' = \cos(x^2)\cdot 2x \qquad [\ln^3 x]' = 3\ln^2 x \cdot \frac{1}{x}$$

$$\int \cos(x^2)\cdot 2x\,dx = \sin(x^2) + c \qquad \int 3\ln^2 x\cdot\frac{1}{x}\,dx = \ln^3 x + c$$

<u>Věta</u>: Nechť $F(t)$ je primitivní funkce k $f(t)$ na (α,β) a funkce $t = g(x)$ má v (a,b) derivaci $g'(x)$ a pro každé $x\in(a,b)$ je $g(x)\in(\alpha,\beta)$. Pak je v (a,b) funkce $F(g(x))$ primitivní k $f(g(x))\cdot g'(x)$ tj.

$$\boxed{\int f(g(x))\cdot g'(x)\,dx = F(g(x)) + c}$$

$$\int f(\overset{t}{g(x)}) \cdot \overset{dt}{\boxed{g'(x)\,dx}} = \int f(t)\,dt = F(t) + c = F(g(x)) + c$$

$$t = g(x)$$
$$dt = g'(x)\,dx$$

$$\int \cos(x^2) \cdot 2x\,dx = \int \cos t\,dt = \sin t + c = \sin(x^2) + c$$

$$t = x^2$$
$$dt = 2x\,dx$$

$$\int \sin(x^2+1) \cdot x\,dx = \frac{1}{2}\int \sin t\,dt = -\frac{1}{2}\cos t + c = -\frac{1}{2}\cos(x^2+1) + c$$

$$t = x^2+1$$
$$dt = 2x\,dx \quad /:2$$
$$\frac{1}{2}\,dt = x\,dx$$

$$\int x^2 \cdot \sqrt{x^3-1}\, dx = \frac{1}{3}\int \sqrt{t}\, dt = \frac{1}{3} \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c =$$

t

$t = x^3 - 1$

$dt = 3x^2 dx \quad / : 3$

$\frac{1}{3} dt = x^2 dx$

$$= \frac{2}{9} (x^3-1)^{\frac{3}{2}} + c$$

$x^3 - 1 > 0 \rightarrow x \in (1, +\infty),\ c \in \mathbb{R}$

$$\int \frac{2x}{x^2+1}\, dx = \int \frac{dt}{t} = \int \frac{1}{t}\, dt = \ln|t| + c = \ln(x^2+1) + c$$

t

$x \in (-\infty, +\infty)$

$t = x^2 + 1$

$dt = 2x\,dx$

$$\int \frac{x}{x^2+1}\, dx = \frac{1}{2}\int \frac{2x}{x^2+1}\, dx$$

$$\int tg(x)\,dx = \int \frac{\sin x}{\cos x}\,dx = -\int \frac{1}{r}\,dr = -\ln|r| + c =$$

$$= -\ln|\cos x| + c$$

$r = \cos x$

$dr = -\sin x\,dx \quad /:(-1)$

$-dr = \sin x\,dx$

$$\int e^{3x+1}\,dx = \frac{1}{3}\int e^{s}\,ds = \frac{1}{3}e^{s} + c = \frac{1}{3}e^{3x+1} + c$$

$s = 3x+1$

$ds = 3dx \quad /:3$

$\frac{1}{3}ds = dx$

$$\int \cos(\underbrace{1-5x}_{t})\,dx = -\frac{1}{5}\int \cos t\,dt = -\frac{1}{5}\sin t + c = -\frac{1}{5}\sin(1-5x) + c$$

$$x \in \mathbb{R},\ c \in \mathbb{R}$$

$$t = 1-5x$$

$$dt = -5\,dx \quad /:(-5)$$

$$-\frac{1}{5}\,dt = dx$$

$$\int (\underbrace{7x+1}_{r})^6\,dx = \frac{1}{7}\int r^6\,dr = \frac{1}{7}\cdot\frac{r^7}{7} + c = \frac{1}{7}\,\frac{(7x+1)^7}{7} + c$$

$$x \in \mathbb{R},\ c \in \mathbb{R}$$

$$r = 7x+1$$

$$dr = 7\,dx \quad /:7$$

$$\frac{1}{7}\,dr = dx$$

Lineární substituce: jestliže $\int f(x)\,dx = F(x) + c$, tak je i

$$\int f(ax+b)\,dx = \frac{1}{a} F(ax+b) + c$$

$$\int e^{5x-1}\,dx = \frac{1}{5} e^{5x-1} + c$$

$$\int \sin(4x+18)\,dx = -\frac{1}{4}\cos(4x+18) + c$$

$$\int (1-3x)^{10}\,dx = -\frac{1}{3}\,\frac{(1-3x)^{11}}{11}$$

$$\int \sqrt{8x-1}\,dx = \int (8x-1)^{\frac{1}{2}}\,dx = \frac{1}{8}\,\frac{(8x-1)^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$\int x^2 e^{3x}\,dx = \frac{1}{3}x^2e^{3x} - \frac{2}{3}\int x e^{3x}\,dx = \frac{1}{3}x^2e^{3x} - \frac{2}{3}\left(\frac{1}{3}xe^{3x} - \frac{1}{3}\int e^{3x}\,dx\right) =$$

$$u = x^2 \qquad u' = 2x$$
$$v' = e^{3x} \qquad v = \frac{1}{3}e^{3x}$$

$$u = x \qquad u' = 1$$
$$v' = e^{3x} \qquad v = \frac{1}{3}e^{3x}$$

$$= \frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{9}\int e^{3x}\,dx = \frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{9}\cdot\frac{1}{3}e^{3x} + c$$

$x \in \mathbb{R},\ c \in \mathbb{R}$