

Moment uily v prostora

①

Da'ir: $A[3, 2, 4]$ m

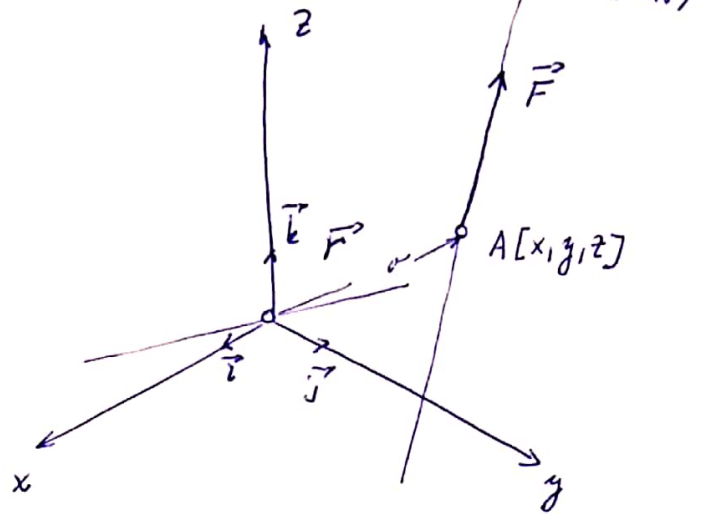
smernice uily v prostora uily

$$\alpha_f = 80^\circ, \beta_f = 60^\circ, \gamma_f < 90^\circ$$

smernice uily osy σ :

$$\alpha_\sigma = 60^\circ, \beta_\sigma = 120^\circ, \gamma_\sigma < 90^\circ$$

velikost uily $F = 40$ N



Ucit:

- 1) Moment uily \vec{F} k smernice osy x, y, z .
- 2) Moment uily \vec{F} k proutku m. s. systému.
- 3) Moment uily \vec{F} k osy σ .

1) Moment uily \vec{F} k smernice osy x, y, z

$$\vec{M} = \vec{r} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 2 & 4 \\ 6,95 & 20 & 33,9 \end{vmatrix} =$$

Urceny uily F :

$$F_x = F \cdot \cos \alpha_f = 40 \cdot \cos 80^\circ = 6,95 \text{ N}$$

$$F_y = F \cdot \cos \beta_f = 40 \cdot \cos 60^\circ = 20 \text{ N}$$

$$F_z = F \cdot \cos \gamma_f = 40 \cdot \cos 31,96^\circ = 33,9 \text{ N}$$

droutku m. s. uily σ :

$$\cos^2 \alpha_f + \cos^2 \beta_f + \cos^2 \gamma_f = 1$$

$$\gamma_f = \arccos \left(\pm \sqrt{1 - \cos^2 \alpha_f - \cos^2 \beta_f} \right) = \arccos \left(\pm \sqrt{1 - \cos^2 80^\circ - \cos^2 60^\circ} \right) =$$

$$= \begin{cases} = 31,96^\circ \\ = 149,04^\circ \end{cases}$$

$$\begin{aligned} &= \vec{i} (2 \cdot 33,9 - 4 \cdot 20) + \\ &- \vec{j} (3 \cdot 33,9 - 4 \cdot 6,95) + \\ &+ \vec{k} (3 \cdot 20 - 2 \cdot 6,95) = \\ &= \underline{\underline{(-12,2 \vec{i} - 73,9 \vec{j} + 46,1 \vec{k}) \text{ Nm}}} \end{aligned}$$

$$\vec{M} = M_x \vec{i} + M_y \vec{j} + M_z \vec{k} = \vec{M}_x + \vec{M}_y + \vec{M}_z$$

$$\vec{M}_x = M_x \vec{i} = -12,2 \vec{i} \text{ Nm} \quad |\vec{M}_x| = M_x = 12,2 \text{ Nm}$$

$$\vec{M}_y = M_y \vec{j} = -73,9 \vec{j} \text{ Nm} \quad |\vec{M}_y| = M_y = 73,9 \text{ Nm}$$

$$\vec{M}_z = M_z \vec{k} = 46,1 \vec{k} \text{ Nm} \quad |\vec{M}_z| = M_z = 46,1 \text{ Nm}$$

2) Moment oleh \vec{F} ke pusatnya tiga dimensi nya

$$\vec{M} = \vec{M}_x + \vec{M}_y + \vec{M}_z$$

selisih sumbu

$$|\vec{M}| = M = \sqrt{M_x^2 + M_y^2 + M_z^2} = \sqrt{(-12,2)^2 + (-73,9)^2 + (46,1)^2} = 87,95 \text{ Nm}$$

menentukan sumbu m:

$$m(\alpha_m, \beta_m, \gamma_m)$$

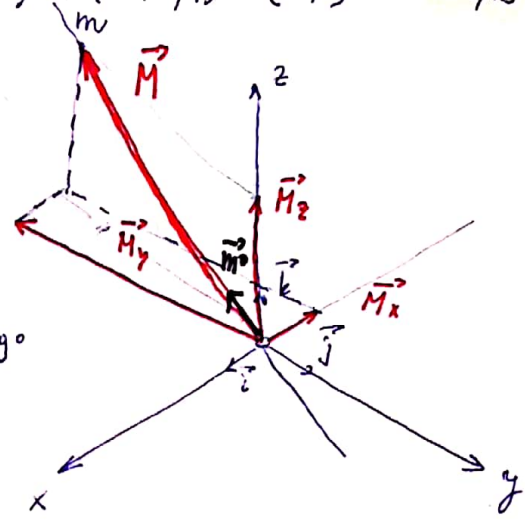
$$M_x = M \cdot \cos \alpha_m \Rightarrow \cos \alpha_m = \frac{M_x}{M}$$

$$\alpha_m = \arccos \frac{M_x}{M} = \arccos \frac{-12,2}{87,95} = 97,9^\circ$$

$$\beta_m = \arccos \frac{M_y}{M} = \arccos \frac{-73,9}{87,95} = 147,2^\circ$$

$$\gamma_m = \arccos \frac{M_z}{M} = \arccos \frac{46,1}{87,95} = 58,4^\circ$$

$$m(97,9^\circ; 147,2^\circ; 58,4^\circ)$$

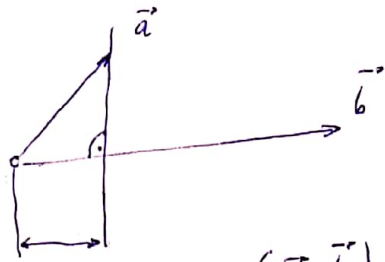


$$\vec{M} = M \cdot \vec{m}^0 = 87,95 \cdot (-0,1387 \vec{i} - 0,8403 \vec{j} + 0,5242 \vec{k}) \text{ Nm}$$

$$\vec{m}^0 (\cos \alpha_m, \cos \beta_m, \cos \gamma_m) = \left(\frac{-12,2}{87,95} ; \frac{-73,9}{87,95} ; \frac{46,1}{87,95} \right) = (-0,1387 ; -0,8403 ; 0,5242)$$

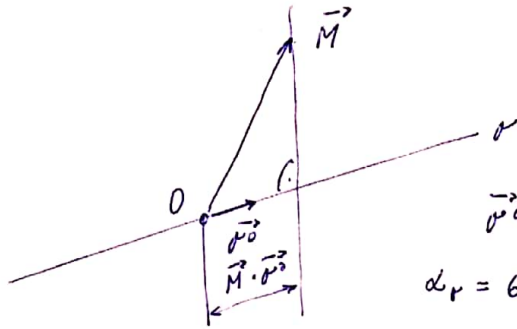
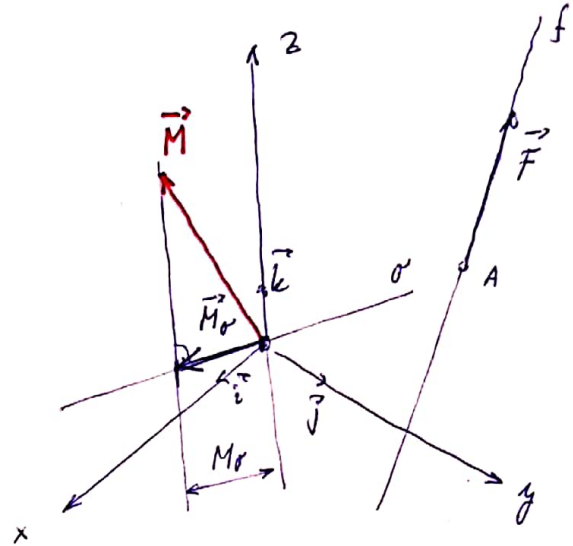
3) Momen oleh \vec{F} ke sumbu σ

3



skalar dari momen ($\vec{a} \cdot \vec{b}$)

$$M_{\sigma} = \vec{M} \cdot \vec{e}_{\sigma}$$



$$\vec{e}_{\sigma} = (\cos \alpha_{\sigma}, \cos \beta_{\sigma}, \cos \gamma_{\sigma})$$

$$\alpha_{\sigma} = 60^{\circ}, \beta_{\sigma} = 120^{\circ}, \gamma_{\sigma} = 45^{\circ}$$

$$\cos^2 \alpha_{\sigma} + \cos^2 \beta_{\sigma} + \cos^2 \gamma_{\sigma} = 1$$

$$\gamma_{\sigma} = \arccos \left(\pm \sqrt{1 - \cos^2 \alpha_{\sigma} - \cos^2 \beta_{\sigma}} \right) = \arccos \left(\pm \sqrt{1 - \cos^2 60^{\circ} - \cos^2 120^{\circ}} \right) =$$

$$= \begin{cases} 45^{\circ} \\ \underline{\underline{135^{\circ}}} \end{cases}$$

$$\vec{e}_{\sigma} = (0,5; -0,5; 0,707)$$

$$M_{\sigma} = \vec{M} \cdot \vec{e}_{\sigma} = (-12,2; -73,9; 46,1) \cdot (0,5; -0,5; 0,707) =$$

$$= 77,66 \text{ Nm}$$

$$\vec{M}_{\sigma} = M_{\sigma} \cdot \vec{e}_{\sigma} = 77,66 \cdot \left(\begin{matrix} 0,5 \\ -0,5 \\ 0,707 \end{matrix} \right) = \left(38,83 \vec{i} - 38,83 \vec{j} + 54,916 \vec{k} \right) \text{ Nm}$$