

1.

$$6x_1 + 2x_2 - x_3 + 7x_4 = 0$$

$$4x_1 + 2x_2 - 3x_3 + 5x_4 = -4$$

$$x_1 + x_2 - x_3 - x_4 = 0$$

$$x_1 + x_3 = 3$$

$$A \cdot X = b$$

$$\left( \begin{array}{cccc|c} 6 & 2 & -1 & 7 & 0 \\ 4 & 2 & -3 & 5 & -4 \\ 1 & 1 & -1 & -1 & -1 \\ 1 & 0 & 1 & 0 & 3 \end{array} \right) \xrightarrow{\sim} \left( \begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 4 & 2 & -3 & 5 & -4 \\ 6 & 2 & -1 & 7 & 0 \\ 1 & 0 & 1 & 0 & 3 \end{array} \right) \xrightarrow{\substack{(-4)(-6)(-1) \\ \sim}} \left( \begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 0 & -2 & 1 & 11 & 20 \\ 0 & -4 & 5 & 13 & 6 \\ 0 & -1 & 2 & 1 & 4 \end{array} \right)$$

$$\left( \begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 0 & -2 & 1 & 11 & 20 \\ 0 & -4 & 5 & 13 & 6 \\ 0 & -1 & 2 & 1 & 4 \end{array} \right) \xrightarrow{\sim} \left( \begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 0 & -1 & 2 & 1 & 4 \\ 0 & -4 & 5 & 13 & 6 \\ 0 & -2 & 1 & 9 & 7 \end{array} \right) \xrightarrow{\sim(-1)(-2)} \left( \begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 0 & 1 & -2 & -1 & -4 \\ 0 & -4 & 5 & 13 & 6 \\ 0 & -2 & 1 & 9 & 7 \end{array} \right)$$

$$\begin{pmatrix} 1 & -1 & -1 & -1 & 0 \\ 0 & -1 & 2 & 1 & 3 \\ 0 & 0 & -3 & 9 & -12 \\ 0 & 0 & -3 & 7 & -10 \end{pmatrix} \xrightarrow{F_4 - F_3} \begin{pmatrix} 1 & 1 & -1 & -1 & 0 \\ 0 & -1 & 2 & 1 & 3 \\ 0 & 0 & 1 & -3 & 4 \\ 0 & 0 & 0 & -2 & 2 \end{pmatrix}$$

$$m = 4 \quad h(A) = 4 \quad h(A|b) = 4$$

$$x_1 + x_2 - x_3 - x_4 = 0$$

$$-x_2 + 2x_3 + x_4 = 3$$

$$x_3 - 3x_4 = 4$$

$$-2x_4 = 2$$

$$x_3 + 3 = 4$$

$$-x_2 + 2 - 1 = 3$$

$$x_1 - 2 - 1 + 1 = 0$$

$$\Rightarrow x_4 = -1$$

$$x_3 = 1$$

$$x_2 = -2$$

$$x_1 = 2$$

$$X = \begin{pmatrix} 2 \\ -2 \\ 1 \\ -1 \end{pmatrix}$$

$$x_1 + 2x_2 - (-1+2t) + 2t = 3$$

$$x_3 = 5$$

$$x_1 = -2s + 2$$

$$X = \begin{pmatrix} 2-2s \\ -\frac{1}{2} \\ 5 \\ -1+2t \\ t \end{pmatrix}$$

где  $t, s \in \mathbb{R}$  — свободные,



4.

$$3x_1 - 2x_2 + 6x_3 + 2x_4 - 4x_5 = 5$$

$$x_1 + 2x_3 - x_4 + 2x_5 = 3$$

$$x_1 + 2x_2 + 2x_3 = 1$$

$$2x_1 - 6x_2 + 4x_3 + 2x_4 - 4x_5 = 5$$

$$\left( \begin{array}{ccccc|c} 3 & -2 & 6 & 2 & -4 & 5 \\ 1 & 0 & 2 & -1 & 2 & 3 \\ 1 & 2 & 2 & 0 & 0 & 1 \\ 2 & -6 & 4 & 2 & -4 & 5 \end{array} \right) \sim \left( \begin{array}{ccccc|c} 1 & 0 & 2 & -1 & 2 & 3 \\ 3 & -2 & 6 & 2 & -4 & 5 \\ 1 & 2 & 2 & 0 & 0 & 1 \\ 2 & -6 & 4 & 2 & -4 & 5 \end{array} \right)$$

(+3)(1)R  
-2R  
-3R

$$\sim \left( \begin{array}{ccccc|c} 1 & 0 & 2 & -1 & 2 & 3 \\ 0 & -2 & 0 & 5 & -10 & -4 \\ 0 & 2 & 0 & 1 & -2 & -2 \\ 0 & -6 & 0 & 4 & -8 & -1 \end{array} \right) \sim \left( \begin{array}{ccccc|c} 1 & 0 & 2 & -1 & 2 & 3 \\ 0 & 2 & 0 & 1 & -2 & -2 \\ 0 & 0 & 0 & 6 & -12 & -6 \\ 0 & 0 & 0 & -11 & 22 & 11 \end{array} \right)$$

(+3)R  
-2R  
-2R

$$\begin{pmatrix} 1 & 0 & 2 & -1 & 2 & 3 \\ 0 & 2 & 0 & 1 & -2 & -2 \\ 0 & 0 & 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & -1 & 2 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 2 & -1 & 2 & 3 \\ 0 & 2 & 0 & 1 & -2 & -2 \\ 0 & 0 & 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$m = 5 \quad h(A) = 3 \quad h(A|b) = 3$$

$$x_1 + 2x_3 - x_4 + 2x_5 = 3$$

$$2x_2 + x_4 - 2x_5 = -2$$

$$x_4 - 2x_5 = -1 \quad x_5 = t$$

$$x_4 = -1 + 2t$$

$$2x_2 + (-1 + 2t) - 2t = -2$$

$$2x_2 = -1$$

$$x_2 = -\frac{1}{2}$$

$$x_1 + 2x_2 + 3x_3 + x_4 = 1$$

$$2x_1 + 4x_2 + 7x_3 + 7x_4 = 4$$

$$x_1 + 2x_3 = -2$$

$$3x_1 + 7x_2 + 10x_3 + 6x_4 = 7$$

$$\begin{pmatrix} 1 & 2 & 3 & 1 & 1 & 2 \\ 2 & 4 & 7 & 7 & 3 & 5 \\ 1 & 0 & 2 & 0 & 0 & -2 \\ 3 & 7 & 10 & 6 & 1 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 & 1 & 1 & 2 \\ 0 & 0 & 1 & 5 & 1 & 1 \\ 0 & -2 & -1 & -1 & -3 & -3 \\ 0 & 1 & 1 & 3 & 4 & 4 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 2 & 3 & 1 & 1 & 2 \\ 0 & 1 & 3 & 1 & 3 & 5 \\ 0 & -2 & -1 & -1 & -3 & -3 \\ 0 & 0 & 1 & 5 & 5 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 & 1 & 1 & 2 \\ 0 & 1 & 3 & 1 & 3 & 5 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$



$$\left( \begin{array}{cccc|cccc} 1 & 2 & 3 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 3 & 3 & 4 & 4 & 4 \\ 0 & 0 & 1 & 5 & 5 & 5 & 5 & 5 \\ 0 & 0 & 0 & 0 & 0 & -3 & -3 & -3 \end{array} \right)$$

$$h = 4 \quad h(A) = 3 \quad h(A|b) = 4$$

soy stavc nemc' ve sen'

$$x_1 + 2x_2 + 3x_3 + x_4 = 1$$

$$x_2 + x_3 + 3x_4 = 4$$

$$x_3 + 5x_4 = 5$$

$$0 = -3$$

$$x_1 + x_2 + 2x_3 + 3x_4 + 3x_5 + 3x_6 = 0$$

$$x_1 + x_2 + x_3 + 3x_4 + x_5 + x_6 = 0$$

$$2x_1 + 2x_2 + 2x_3 + 6x_4 + 2x_5 + 8x_6 = 0$$

$$\left( \begin{array}{cccccc|ccc} 1 & 1 & 2 & 3 & 3 & 3 & 0 & 0 & 0 \\ 1 & 1 & 1 & 3 & 1 & 1 & 0 & 0 & 0 \\ 2 & 2 & 2 & 6 & 2 & 8 & 0 & 0 & 0 \end{array} \right) \sim \begin{matrix} (-1) (-2) \\ \uparrow \uparrow \end{matrix}$$

$$\sim \left( \begin{array}{cccccc|ccc} 1 & 1 & 2 & 3 & 3 & 3 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & -2 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & -4 & 2 & 0 & 0 & 0 \end{array} \right) \sim \begin{matrix} (-2) \\ \uparrow \end{matrix}$$

$$\sim \left( \begin{array}{cccccc|ccc} 1 & 1 & 2 & 3 & 3 & 3 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & -2 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 \end{array} \right)$$

m=6

~~r(A)=3~~

h(A)=3

h(A|b)=3



$$x_1 + x_2 + 2x_3 + 3x_4 + 3x_5 + 3x_6 = 0$$

$$-x_3 - 2x_5 - 2x_6 = 0$$

$$6x_6 = 0$$

$$x_6 = 0$$

$$-x_3 - 2x_5 = 0$$

$$x_5 = t$$

$$-x_3 - 2t = 0$$

$$x_3 = -2t$$

$$x_1 + x_2 - 4t + 3x_4 + 3t + 0 = 0$$

$$x_4 = 5 \quad x_2 = t$$

10.

$$x_1 + r - 4t + 3s + 3t = 0$$

$$x_1 = -r + t - 3s$$

$$X = \begin{pmatrix} 0 \\ r \\ 5t \\ -2t \\ -r - 3s + t \end{pmatrix}$$

Kde  $r, s, t$  jsou libovolné,  
reálná čísla.

Elementární úpravy determinantů  
(takové, že se nemění hodnota)

- 1 Záměnou pořadí dvou řádků se změnil znaménko determinantu
- 2 z řádku lze vytknout číslo před determinant.
- 3 k řádku mohou přičíst nenulový násobek jiného řádku.



$$\begin{array}{ccc|ccc} 2 & 3 & 4 & 5 & 2 & 3 & 4 & 5 \\ 0 & 1 & 1 & 0 & = 2 & 0 & 1 & 0 \\ 2 & 0 & 2 & 0 & & 1 & 0 & 1 & 0 \\ 3 & 1 & 2 & 0 & & 3 & 1 & 2 & 0 \end{array}$$

$$= -2$$

$$\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 3 & 2 & 5 & & 0 & 0 & -1 & 5 \\ 0 & 1 & -1 & 0 & & 0 & 0 & -2 & 0 \end{array}$$

$$= -2$$

$$\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & = (-2) \cdot 1 \cdot 1 \cdot (-1) \cdot (-10) & = & & \\ 0 & 0 & -1 & 5 & & & & \\ 0 & 0 & 0 & -10 & & & & \end{array}$$

$$= -20$$

15. c)

$$\begin{array}{c} \left| \begin{array}{cccc|c} 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & -1 & 1 & 1 \end{array} \right| \xrightarrow{R_3 - R_1} \left| \begin{array}{cccc|c} 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 \end{array} \right| \xrightarrow{R_4 - R_2} \left| \begin{array}{cccc|c} 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{array} \right| \end{array}$$

$$\begin{array}{c} \left| \begin{array}{cccc|c} 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right| \xrightarrow{R_1 + R_3} \left| \begin{array}{cccc|c} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right| \xrightarrow{R_1 - R_2} \left| \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right| \end{array}$$

$$= 1 \cdot 1 \cdot 1 \cdot 0 = 0$$