1.

$$\det(A-\lambda E)=\det\left(\begin{pmatrix}1&-1&4\\3&2&-1\\2&1&-1\end{pmatrix}-\begin{pmatrix}\lambda&0&0\\0&\lambda&0\\0&0&\lambda\end{pmatrix}\right)=$$

$$=\det\begin{pmatrix}1-\lambda&-1&4\\3&2-\lambda&-1\\2&1&-1-\lambda\end{pmatrix}=0$$

$$\begin{matrix}1-\lambda&-1&4\\3&2-\lambda&-1\end{matrix}$$

$$-(1-\lambda^2)(2-\lambda)$$

$$=-(1-\lambda)(2-\lambda)\cdot(1+\lambda)+3\cdot1\cdot4+2\cdot(-1)\cdot(-1)-$$

$$-2\cdot(2-\lambda)\cdot4-(1-\lambda)\cdot1\cdot(-1)-3\cdot(-1)(-1-\lambda)=$$

$$=-\cancel{2}+\cancel{\lambda}+2\lambda^2-\lambda^3+12+\cancel{2}-16+8\lambda+1-\cancel{\lambda}-3-3\lambda=$$

$$=-\lambda^3+2\lambda^2+5\lambda\cancel{+10}=0$$

$$-6$$

2.

$$-\lambda^3 + 2\lambda^2 + 5\lambda \overset{-6}{+10} = 0$$

$$\cancel{\lambda^2(-\lambda + 2)}$$

$$\underline{\underline{\lambda_1 = 1}} \qquad -1 + 2 + 5 - 6 = 0$$

$$\begin{aligned}
&(-\lambda^3 + 2\lambda^2 + 5\lambda - 6) : (\lambda - 1) = \cancel{-\lambda^2 + 2\lambda} \\
& -\lambda^2 + \lambda + 6 \\
&-(-\lambda^3 + \lambda^2) \\
&\overline{\lambda^2 + 5\lambda - 6} \\
&-(\lambda^2 - \lambda) \\
&\overline{6\lambda - 6} \\
&-(6\lambda - 6) \\
&\overline{0}
\end{aligned}$$

3.

$$(\lambda - 1)(-\lambda^2 + \lambda + 6) = 0$$

$$D = 1 + 24 = 25$$

$$\lambda_{2,3} = \frac{-1 \pm 5}{-2} \begin{cases} 3 \\ -2 \end{cases}$$

$\lambda_1 = 1$
$\lambda_2 = 3$
$\lambda_3 = -2$

vlastní čísla matice A

4.

$$A \cdot v - \lambda v = 0$$

$$(A - \lambda E) v = 0$$

5.

$$(A - E) \cdot v = 0$$

$$\left[\begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right] \cdot \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 0 & -1 & 4 & 0 \\ 3 & 1 & -1 & 0 \\ 2 & 1 & -2 & 0 \end{array}\right) \underset{(-1)}{\sim} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & -1 & 4 & 0 \\ 2 & 1 & -2 & 0 \end{array}\right) \begin{matrix} -2 \\ \\ \\ \end{matrix} \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & -1 & 4 & 0 \\ 0 & 1 & -4 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & -1 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad v^1 = \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix}$$

$$x_1 \quad + x_3 = 0 \qquad x_3 = 1 \qquad x_1 = -1$$

$$-x_2 + 4x_3 = 0 \qquad x_2 = 4$$

6.

Vlastnímu číslu $\lambda_1 = 1$ odpovídá vlastní vektor $v_1 = \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix}$

$\lambda_2 = 3$

$(A - \lambda E)v = 0$

$(A - 3E)v = 0$

7.

$$\left(\begin{array}{ccc|c} -2 & -1 & 4 & 0 \\ 3 & -1 & -1 & 0 \\ 2 & 1 & -4 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 3 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \overset{(-3)}{\sim}$$

$$\sim \left(\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 5 & -10 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$x_1 - 2x_2 + 3x_3 = 0$$
$$5x_2 - 10x_3 = 0$$

$$x_3 = 1$$
$$5x_2 - 10 = 0$$
$$x_2 = 2$$
$$x_1 - 4 + 3 = 0$$
$$x_1 = 1$$

$$\mathbb{V}^2 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

vlastní vektor k vl. č. $\lambda_2 = 3$ je $\mathbb{V}^2 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

A. $\mathbb{V}^3 = \begin{pmatrix} 3 \\ 6 \\ 3 \end{pmatrix}$

8.

$\lambda_3 = -2$

$$(A - \lambda E)v = 0$$

$$\left(\begin{array}{ccc|c} 3 & -1 & 4 & 0 \\ 3 & 4 & -1 & 0 \\ 2 & 1 & 1 & 0 \end{array}\right) \overset{(-1)}{\sim} \left(\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 3 & 4 & -1 & 0 \\ 2 & 1 & 1 & 0 \end{array}\right) \begin{array}{c} -3 \quad (-1) \end{array} \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 10 & -10 & 0 \\ 0 & 5 & -5 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x_1 - 2x_2 + 3x_3 = 0$

$x_2 - x_3 = 0$

$x_3 = 1$

$x_2 = 1$

$x_1 - 2 + 3 = 0$

$x_1 = -1$

$$v^3 = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

9.

Vlastnímu číslu $\lambda_3 = -2$ odpovídá vlastní vektor $v^3 = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$.

$$A \cdot v^3 = \begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ -2 \end{pmatrix}$$

$$Av^3 = \begin{pmatrix} 2 \\ -2 \\ -2 \end{pmatrix}$$