TECHNICAL UNIVERSITY OF LIBEREC
Faculty of Economics

Quality Management

A04. Statistical Process Control

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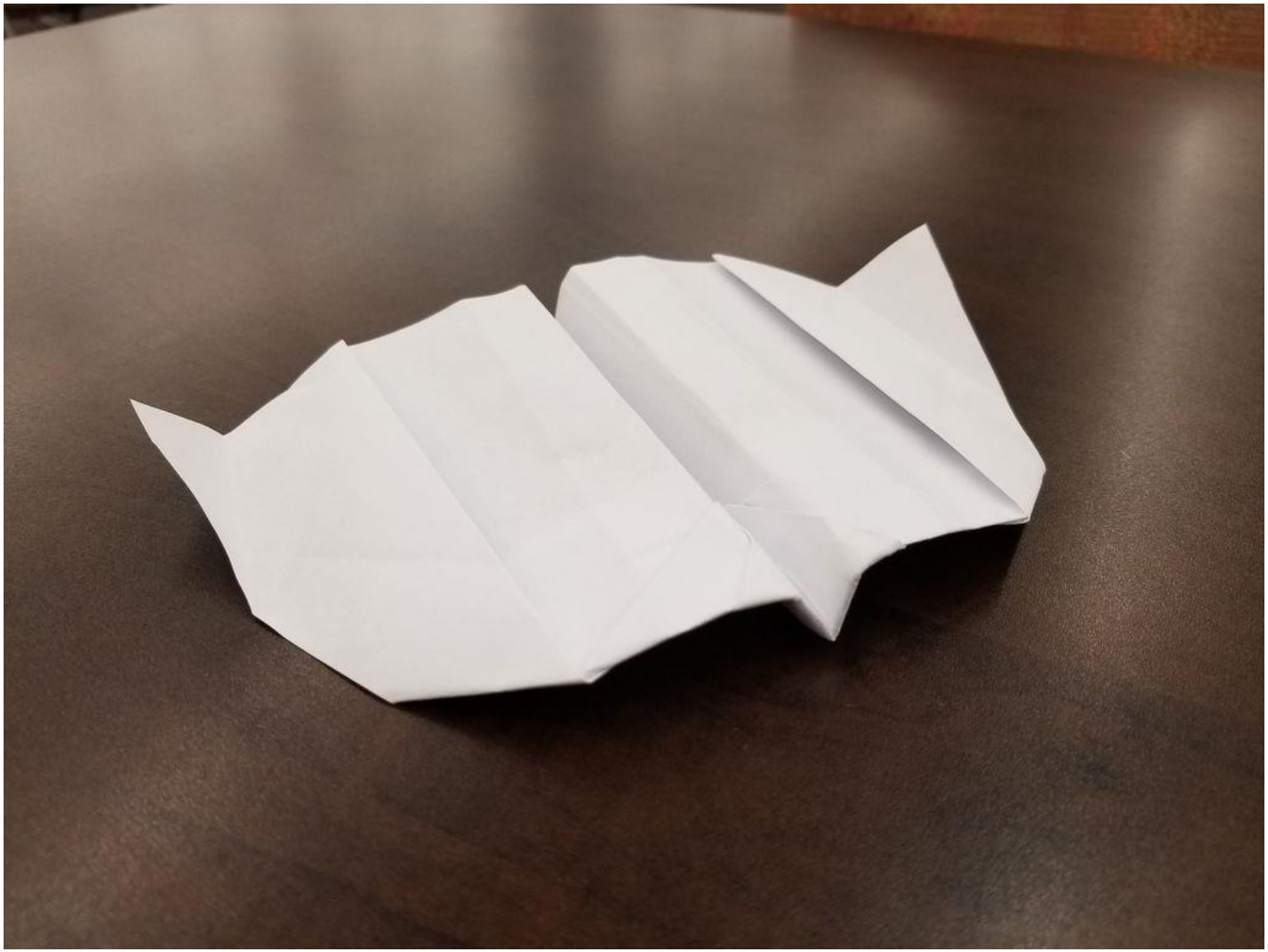
How good is a Quality Level of 99.9999%?

Performance:
Will the product/service do the intended job?
Reliability:
How often does the product/service fail?
Durability:
How long does the product/service last?
Serviceability:
How easy to repair the product / to solve the problems in service?
Aesthetics:
What does the product/service look/smell/sound/feel like?
Features:
What does the product do/ service give?
Perceived Quality:
What is the reputation of the company or its products/services?
Conformance to Standards:
Is the product/service made exactly as the designer/standard intended?

Area	Examples
Airlines	On-time, comfortable, low-cost service
Health Care	Correct diagnosis, minimum wait time, lower cost, security
Food Services	Good product, fast delivery, good environment
Postal Services	fast delivery, correct delivery, cost containment
Academia	Proper preparation for future, on-time knowledge delivery
Consumer Products	Properly made, defect-free, cost effective
Insurance	Payoff on time, reasonable cost
Military	Rapid deployment, decreased wages, no graft
Automotive	Defect-free
Communications	Clearer, faster, cheaper service

Remember?



Aviation Statistics

Source: FAA – US Federal Aviation Administration (2020)

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How good is a Quality Level of **99.9999%?**

0.0001 % = 10^{-4} / 10^{2} = 10^{-6} = 1/1 000 000 = 1 PPM (Part Per Million)

1 in One Million** occurrences **will go WRONG

45 000 daily flights

Example: Take-off and landing processes
1 take-off and 1 landing per flight

90 000 departures and landings per day

1/1 000 000 accidents . 90000 =
90 000/1 000 000 accidents per day =
9/100 accidents per day = 1 / 0.09 ≈ 11.11 =
1 Accident Every 12 Days

How good is a Quality Level of **99.9999%?**

0.0001 % = 10^{-4} / 10^{2} = 10^{-6} = 1/1 000 000 = 1 PPM (Part Per Million)

1 in One Million** occurrences **will go WRONG

2 900 000 passengers/day

Example: Boarding Process in US airports

***2.9 passengers / day** do not board*

***21 passengers / week** do not board*

***104 passengers / year** do not board*

4 000 passengers were denied boarding in the US (2020, Statista.com)

How good is a
Quality Level of **99.9999%?**

Right Answer:
IT DEPENDS

Average annual salary in Modrava village?
8 million CZK!
WHY?

Modrava

Village in the Czech Republic

Modrava is a village and municipality in Klatovy District in the Plzeň Region of the Czech Republic. The municipality covers an area of 81.63 square kilometres, and has a population of around 80. Modrava lies approximately 46 kilometres south of Klatovy, 84 km south of Plzeň, and 140 km south-west of Prague. Wikipedia

Elevation: 985 m

Area: 81.63 km²

Weather: 1°C, Wind W at 11 km/h, 76% Humidity

Population: 58 (2011) czso

Hotels: 3-star averaging CZK 1,625. View hotels

Local time: Tuesday 15:03

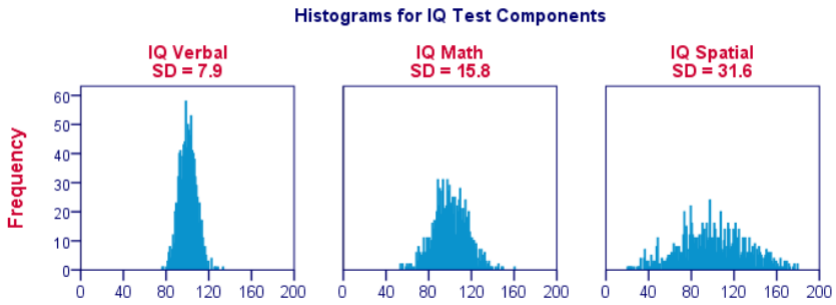
Gross yearly paid wage municipality Modrava: 464 000 000 CZK

Standard deviation shows

how variable the data are

Definition

It is a measure that is used to quantify the amount of variation or dispersion of a set of data values. A low standard deviation indicates that the data points tend to be close to the mean of the set, while a high standard deviation indicates that the data points are spread out over a wider range of values



Sample

$$s = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2},$$

Population

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2}$$

Example: Order Processing Time

We measured 10 random samples of order order processing time of person A in seconds.

Calculate standard deviation of the sample:

Value	Mean	1. Distance	2. Squared
90,2	49,8	40,4	1629,5
28,8	49,8	-20,9	438,4
69,4	49,8	19,6	385,4
31,7	49,8	-18,1	327,0
86,8	49,8	37,0	1372,0
40,1	49,8	-9,7	94,0
26,3	49,8	-23,5	552,2
40,9	49,8	-8,9	79,5
57,5	49,8	7,7	59,3
26,2	49,8	-23,6	557,4

$$s = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2},$$

3. Sum of squares	5494,8
4. Number of samples minus one	9
5. Divide 3/4	610,533
6. Square root of 5	24,709

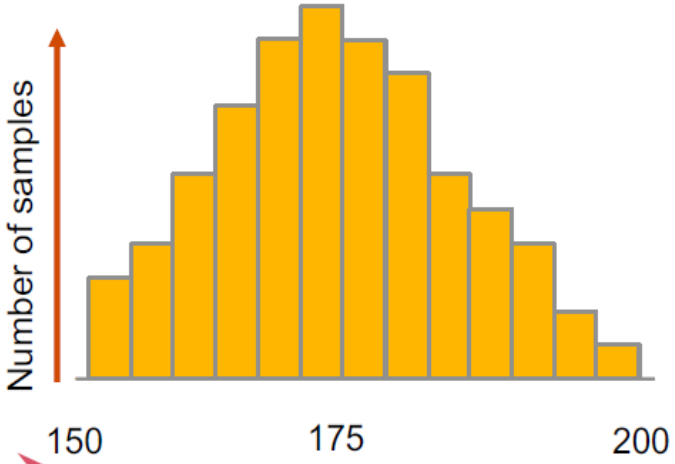
Normal Distribution

Central Limit Theorem

"If the process is influenced by many random variables, the output has a **normal distribution**"

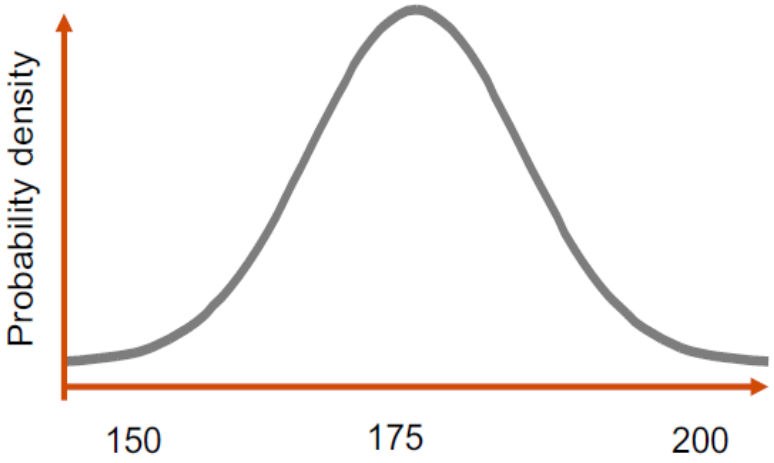
Normal distribution is a **very common probability distribution**

Histogram of height of 100 random men



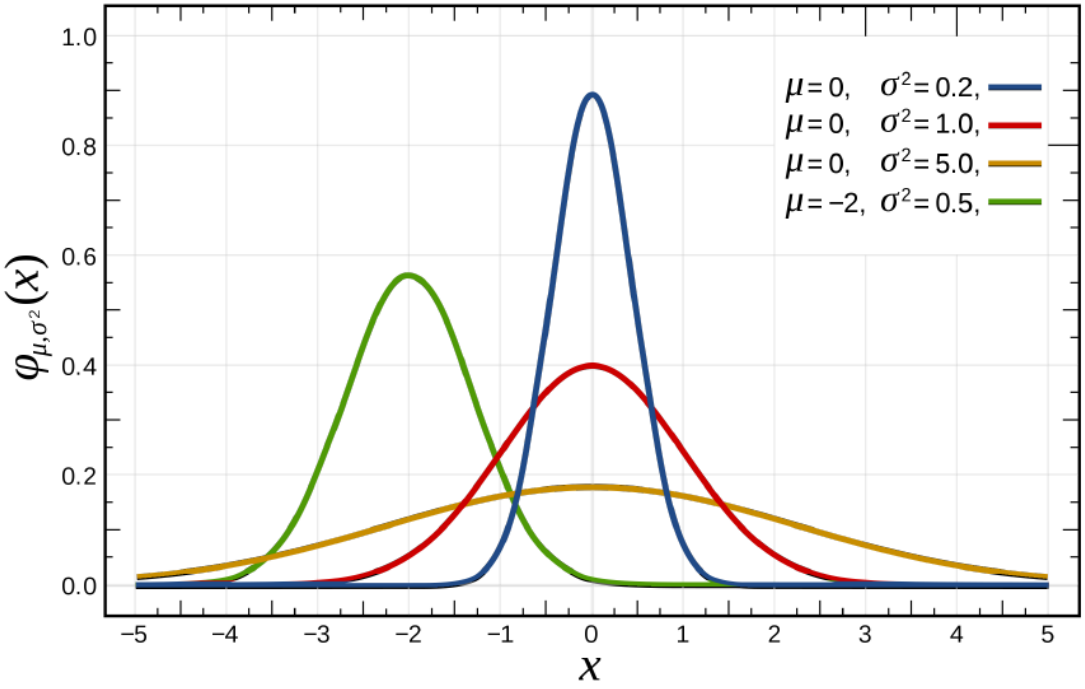
Vertical axis show number of samples

Height of all men is normally distributed



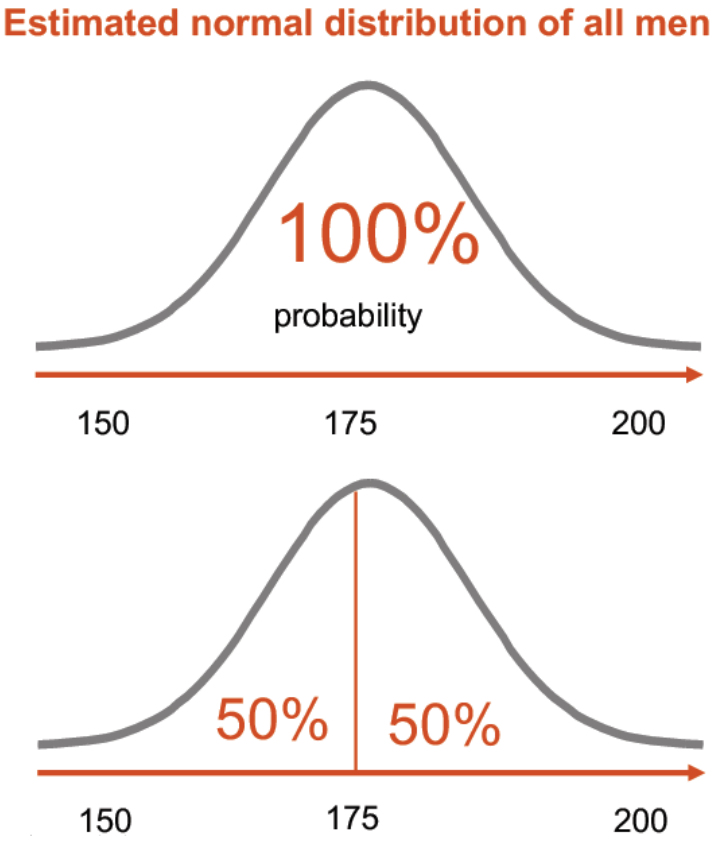
Vertical axis show probability density, abstract and hard to imagine unit

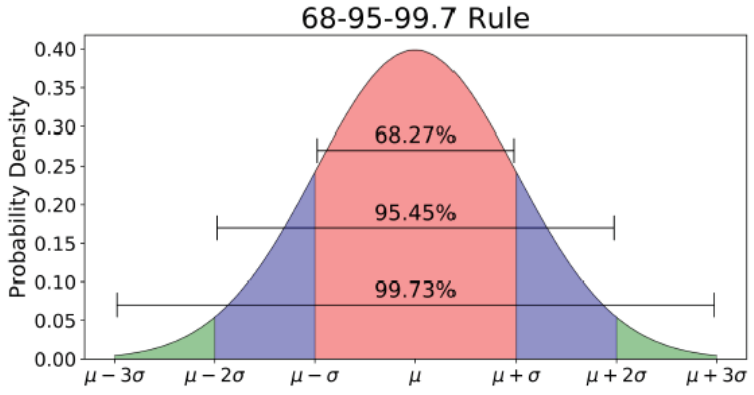
Normal distribution is defined by **mean** (μ = mu [mju]) and **standard deviation** (σ = sigma)

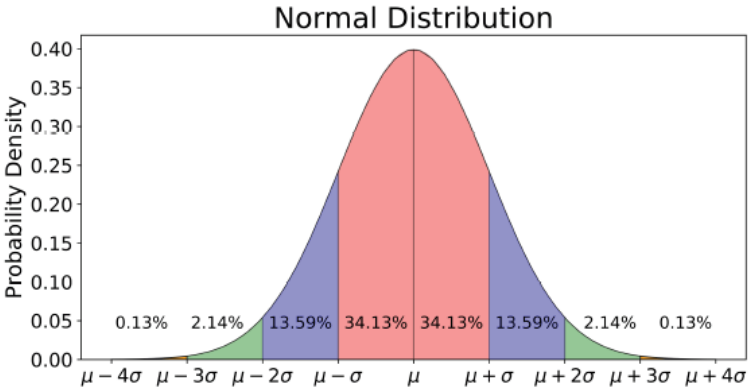


Do not be confused by the word „normal“. Distributions that ar not normal are not abnormal or wrong!

Normal distribution: The area below the curve tells the story







How to estimate normal distribution curve from histogram

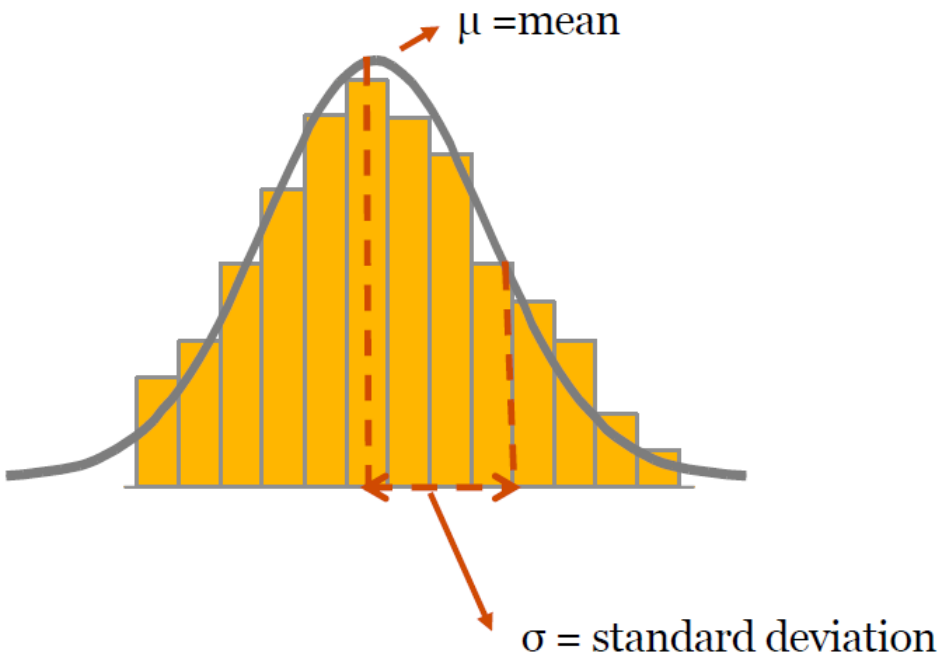
Take a sample of measurements, make a histogram

Does the histogram resemble a bel curve?

Is median very close to average?

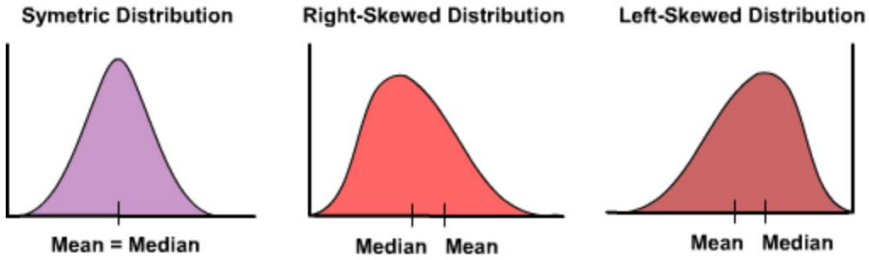
If yes:

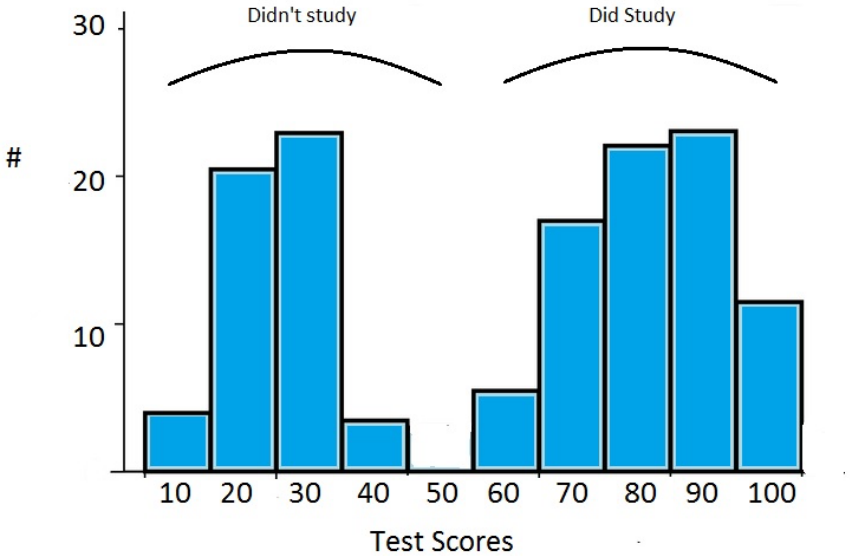
- Estimate mean using sample average
- Estimate standard deviation (sigma) using standard deviation of the sample



Some process outputs are not normal

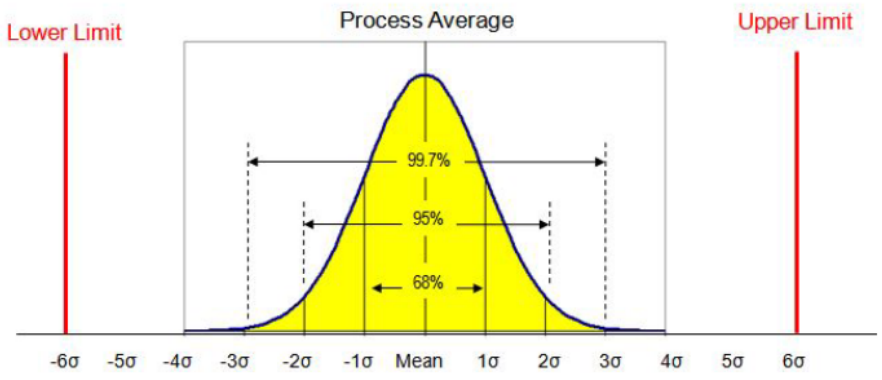
Several segments (male + female, two products)
A strong upper or lower boundary
- Salaries (minimum salary)
- Costs (zero is the limit)
- Delivery time (60min. promised to client)





Sigma Level

Sigma level = how many times you can fit standard deviation into customer limits on each side?

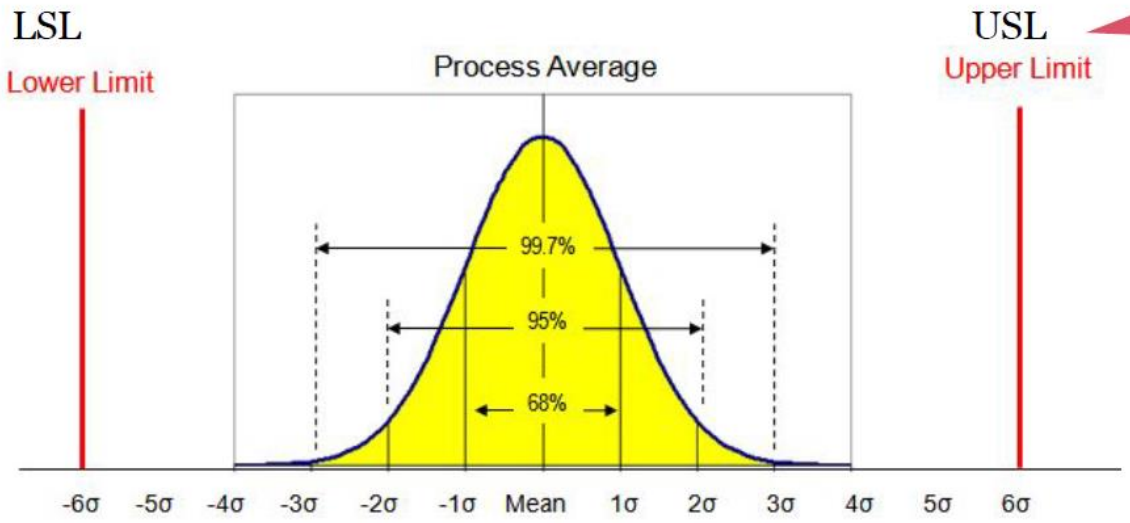


This is a six sigma process

Sigma level (σ)	Process yield	Defect Rate PPM
1	68.27%	317311
2	95.45%	45500
3	99.73%	2700
4	99.99%	63
5	99.9999%	1
6	99.9999998%	0.002

Based on the sigma level, you can calculate defect rate (ppm)

To understand sigma level, we first need to calculate process capability index C_p

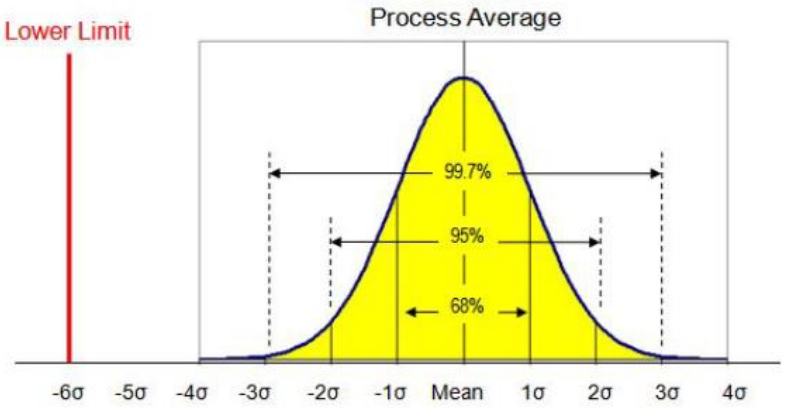


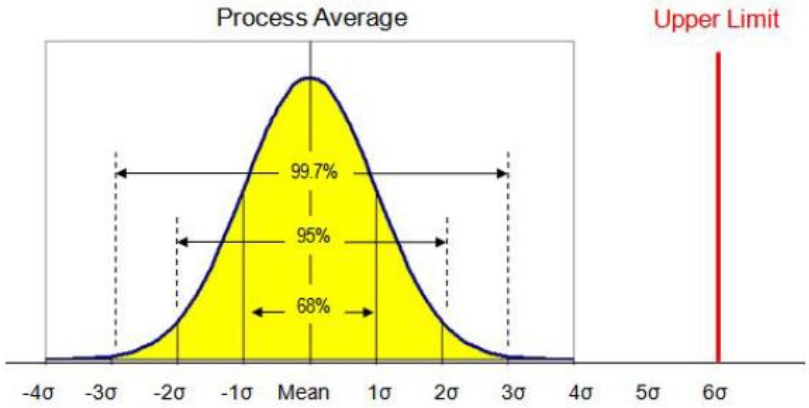
C_p = how many times will six sigmas fit between the specs?

$$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}}$$

C_p	Sigma level (σ)
0.33	1
0.67	2
1.00	3
1.33	4
1.67	5
2.00	6

One-sided process capability index C_p





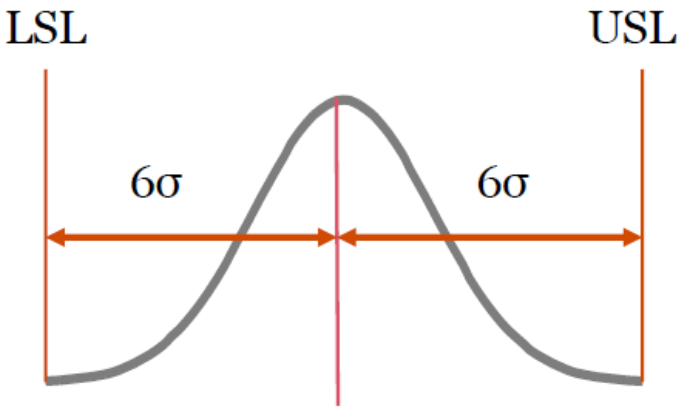
$$\hat{C}_{p,lower} = \frac{\hat{\mu} - LSL}{3\hat{\sigma}}$$

$$\hat{C}_{p,upper} = \frac{USL - \hat{\mu}}{3\hat{\sigma}}$$

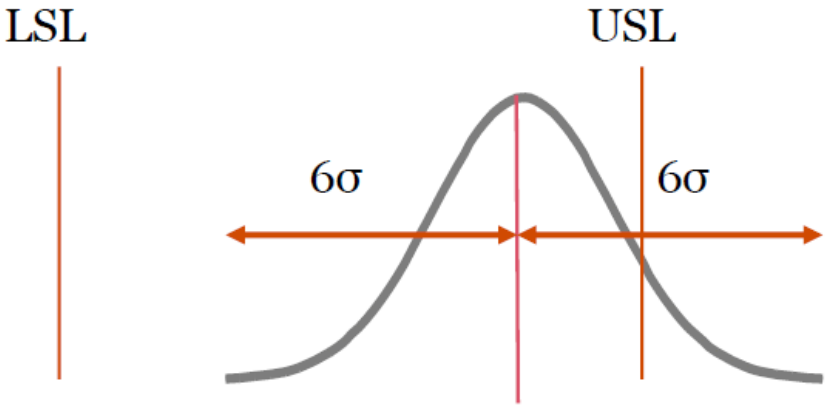
What is the right C_p level?
It is hard to say, these arbitrary numbers are used

Situation	Recommended minimum process capability for two-sided specifications	Recommended minimum process capability for one-sided specification
Existing process	1.33	1.25
New process	1.50	1.45
Safety or critical parameter for existing process	1.50	1.45
Safety or critical parameter for new process	1.67	1.60
Six Sigma quality process	2.00	2.00

If the mean is not centered between the limits, we cannot use the C_p index



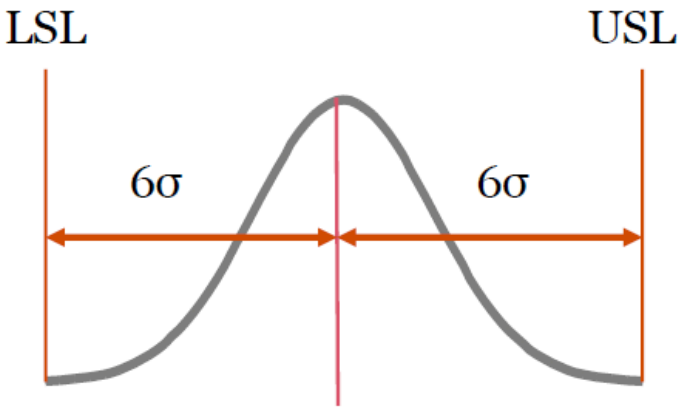
$C_p = 2$
Defect rate= 0.002 PPM



$C_p = 2$
Defect rate= ??????

The defect rate is much higher in this case

If the mean is not centered between the limits, we cannot use the C_p index



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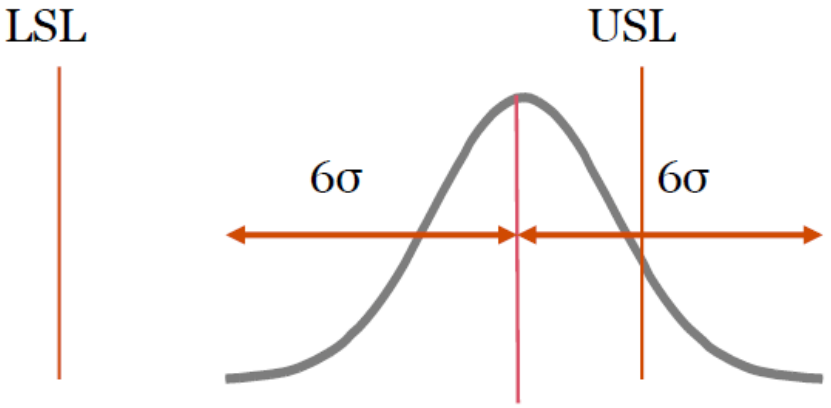
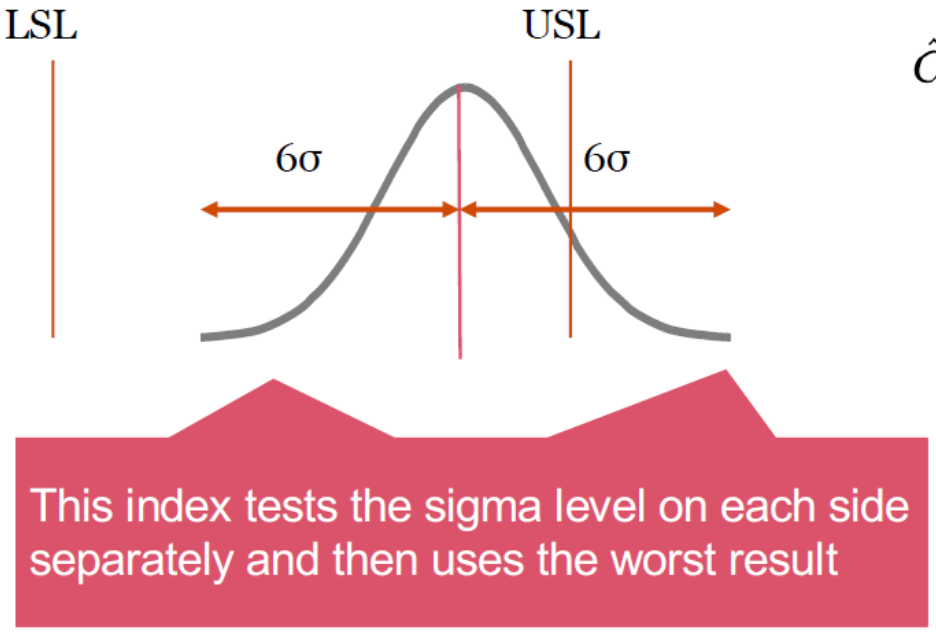


$C_p = 2$
Defect rate= ??????

The defect rate is much higher in this case

If the mean is not centered, we must use the C_{pk}

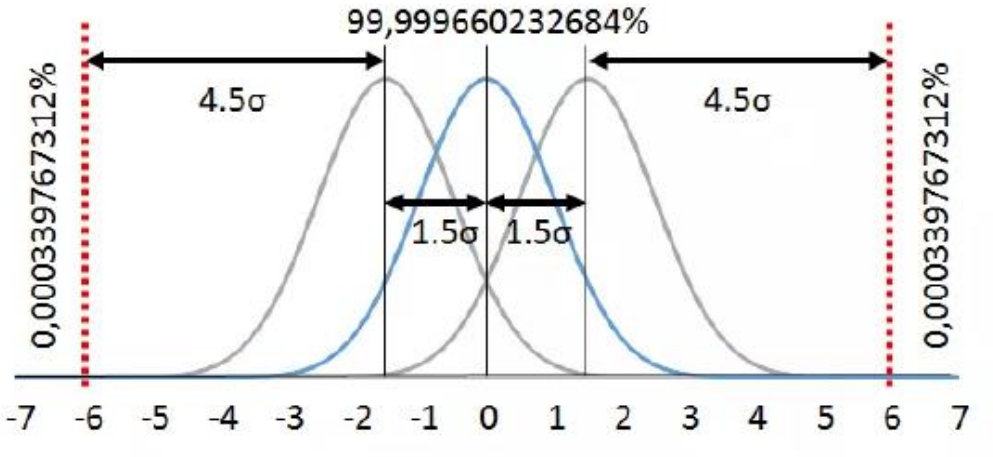


$$\hat{C}_{pk} = \min\left[\frac{USL - \hat{\mu}}{3\hat{\sigma}}, \frac{\hat{\mu} - LSL}{3\hat{\sigma}}\right]$$

C_{pk}	Sigma level (σ)
0.33	1
0.67	2
1.00	3
1.33	4
1.67	5
2.00	6

Long term defect right might be higher due to process drifting

In the long term, the process can “drift“ for various reasons. In Motorola, they observed this drift is 1,5 Sigma towards the upper OR lower limit. Therefore, in long term, defect rate is higher



6 Sigma level (σ)	Defect rate PPM
Short term	0.002
Long term	3,4

Practical Activity

Formulas

$$s = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2},$$

$$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}}$$

C_p	Sigma level (σ)
0.33	1
0.67	2
1.00	3
1.33	4
1.67	5
2.00	6

$$\hat{C}_{p,upper} = \frac{USL - \hat{\mu}}{3\hat{\sigma}}$$

$$\hat{C}_{p,lower} = \frac{\hat{\mu} - LSL}{3\hat{\sigma}}$$

$$\hat{C}_{pk} = \min\left[\frac{USL - \hat{\mu}}{3\hat{\sigma}}, \frac{\hat{\mu} - LSL}{3\hat{\sigma}}\right]$$