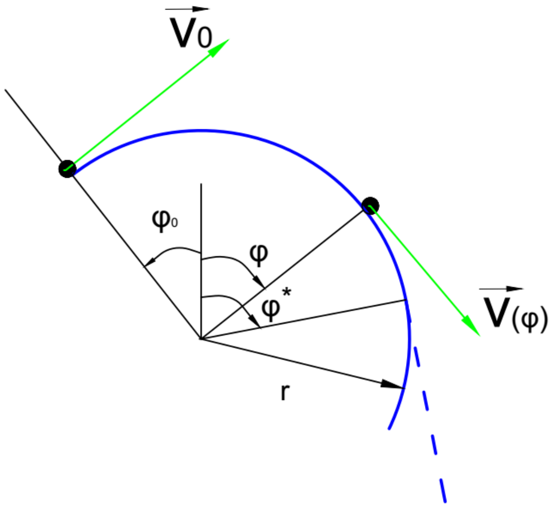
Movement of a particle on a cylindrical surface.



The particle of a given mass moves on the horizontal cylindrical surface from a given starting position with an initial velocity. Assume that no friction between the particle and the cylindrical surface.

Given:

The mass m of the particle.

The initial angle φ_0 between the vertical axis goes through the center of the cylindrical surface and the line contains the particle and the center point.

Initial velocity: v_0

The radius of the cylindrical surface: r

Task:

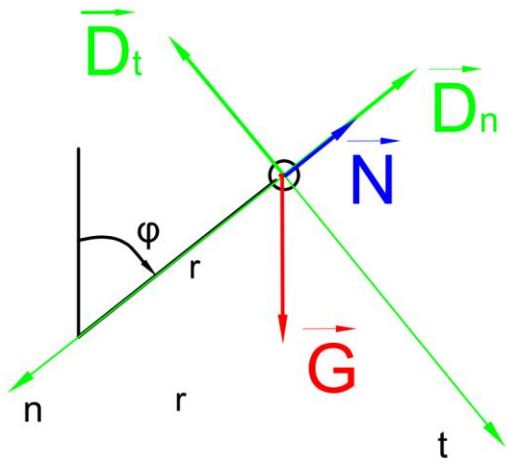
Find the velocity of the particle in dependence of the angle $v(\varphi)$

Find the angle φ^* where the particle leaves out the surface.

Solution:

First, we draw the free body diagram:

In this case, the normal *n*- and tangent *t*-coordinates are considered to move along the path with the particle.



The forces acting on the particle include

Gravity force $\vec{G} = m\vec{g}$ (1)

Normal force $\vec{N}$

D’Alembert forces following the normal *n*- and tangent *t*-coordinates:

$\vec{D}_n = -m\vec{a}_n$ (2)

$\vec{D}_t = -m\vec{a}_t$ (3)

Specification of the D’lembert acceleration components:

$$a_t = \frac{dv}{dt} = \frac{dv}{r.d\varphi}.\frac{r.d\varphi}{dt} = \frac{v.dv}{r.d\varphi} \quad (4)$$

$$a_n = \frac{v^2}{r} \quad (5)$$

Generally, **the equation of motion** can be written:

$$\sum \vec{F} + \vec{D}_t + \vec{D}_n = \vec{0}$$

Or

$$\vec{G} + \vec{N} + \vec{D}_t + \vec{D}_n = \vec{0}$$

Then we can write the component equations by the following:

(t) : $G.\sin\varphi - D_t = 0$ (6)

(n) : $G.\cos\varphi - N - D_n = 0$ (7)

Combine (1), (3), (5) and (6), we have:

$$mg.\sin\varphi - m\frac{vdv}{rd\varphi} = 0$$

Simplify:

$$vdv = rg\sin\varphi.d\varphi \quad (8)$$

Combine (1), (3), and (6), we have:

$$mg.\cos\varphi - N - \frac{mv^2}{r} = 0$$

Then we get the normal force:

$$N = mg.\cos\varphi - \frac{mv^2}{r} \quad (9)$$

Find *v(φ)*

Make the integration of both side of (8), we get:

$$\int_{v_0}^{v(\varphi)} vdv = \int_{\varphi_0}^{\varphi} rg\sin\varphi.d\varphi$$

So:

$$\frac{1}{2}v^2\Big|_{v_0}^{v(\varphi)} = -rg.\cos\varphi\Big|_{\varphi_0}^{\varphi}$$

Then we get:

$$v^2(\varphi) - v_0^2 = 2rg(\cos\varphi_0 - \cos\varphi)$$

$$v^2(\varphi) = 2rg.(\cos\varphi_0 - \cos\varphi) + v_0^2 \quad (10)$$

$$v(\varphi) = \sqrt{2rg.(\cos\varphi_0 - \cos\varphi) + v_0^2} \quad (11)$$

Find φ^*:

When the particle leaves the surface, the normal force is no longer active on the particle, in other word: *N=0.*

From (9) and (10), we can write:

$$N = mg.\cos\varphi^* - \frac{mv^2(\varphi^*)}{r} = 0$$

or

$$g.\cos\varphi^{*} - \frac{2rg.\left(\cos\varphi_0 - \cos\varphi^{*}\right) + v_0^2}{r} = 0$$

Simplify:

$$\cos\varphi^{*} = \frac{2}{3}\cos\varphi_0 + \frac{v_0^2}{3mgr}$$

So:

$$\varphi^{*} = \mathrm{ar}\cos\left(\frac{2}{3}\cos\varphi_0 + \frac{v_0^2}{3mgr}\right) (12)$$