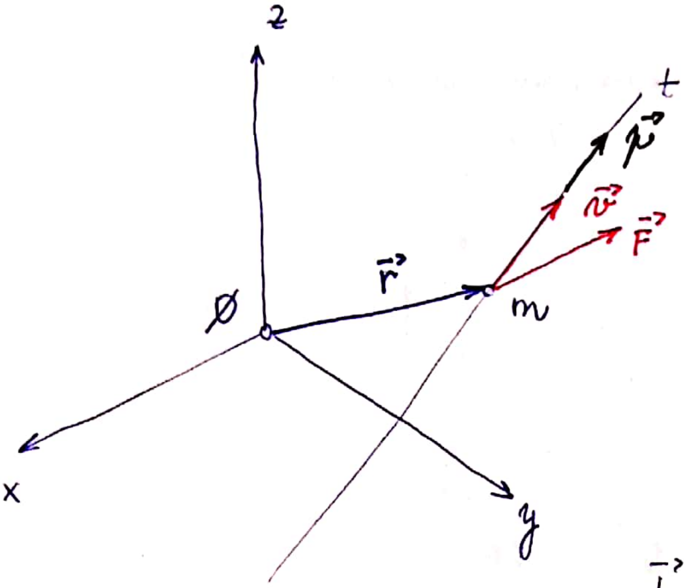
3) LAW OF CHANGE OF ANGULAR MOMENTUM



$\vec{v}$ ... velocity of particle
$\vec{r}$ ... displacement vector
$\vec{p} = m \cdot \vec{v}$
m ... mass of particle

$$\vec{p} = m\vec{v} \quad / \, \vec{r} \times$$

$$\underbrace{\vec{r} \times \vec{p}}_{} = \vec{r} \times (m\vec{v})$$

$\vec{L}$ ... angular momentum

$$\vec{L} = \vec{r} \times (m\vec{v}) \quad / \, \frac{d}{dt}$$

$$\frac{d\vec{L}}{dt} = \underbrace{\frac{d\vec{r}}{dt}}_{\vec{v}} \times (m\vec{v}) + \vec{r} \times \underbrace{m \underbrace{\frac{d\vec{v}}{dt}}_{\vec{a}}}_{\vec{F}}$$

$$\frac{d\vec{L}}{dt} = m \cdot \underbrace{(\vec{v} \times \vec{v})}_{\Longrightarrow \vec{0}} + \underbrace{\vec{r} \times \vec{F}}_{\vec{M}}$$

$$\boxed{\frac{d\vec{L}}{dt} = \vec{M}}$$ differential form

$/ \cdot dt$

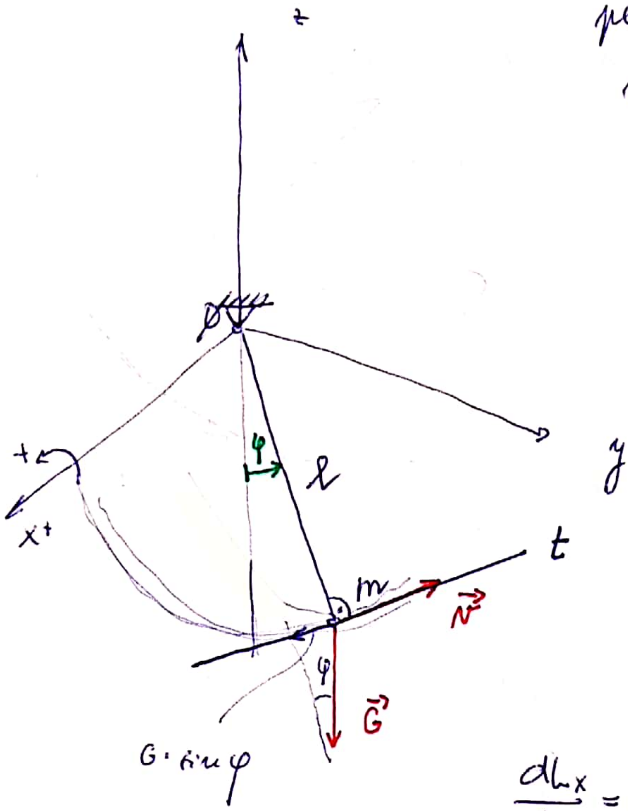
$$\int_{\vec{L}_1}^{\vec{L}_2} d\vec{L} = \int_{t_1}^{t_2} \vec{M} \, dt$$

$$\vec{I}_M - \boxed{\int_{t_1}^{t_2} \vec{M} dt = \vec{L}_2 - \vec{L}_1}$$ integral form

impulse of moment

$$\vec{L}_2 - \vec{L}_1 = \vec{r}_2 \times (\underbrace{m\vec{v}_2}_{\vec{p}_2}) - \vec{r}_1 \times (\underbrace{m\vec{v}_1}_{\vec{p}_1}) = m(\vec{r}_2 \times \vec{v}_2 - \vec{r}_1 \times \vec{v}_1)$$

example: Mathematical pendulum



pendulum makes motion in plane (y, z)

$$\frac{d\vec{L}}{dt} = \vec{M} \quad (*)$$

for 1D case:

$$\frac{dL_x}{dt} = M_x \quad (*')$$

angular momentum

$$L_x = \ell \cdot m \cdot v = \ell \cdot m \cdot \ell \dot{\varphi}$$

$v = \ell \dot{\varphi}$

$\dot{\varphi} = \frac{d\varphi}{dt} = \omega$

$\ddot{\varphi} = \frac{d\dot{\varphi}}{dt} = \alpha$

ω, ℓ, $v = \ell \cdot \omega$

$$\frac{dL_x}{dt} = \underline{\underline{m \cdot \ell^2 \ddot{\varphi}}}$$

$$M_x = -G \sin\varphi \cdot \ell$$

substitution to $(*')$: $\quad (G = mg)$

$$\cancel{m} \cdot \ell^2 \ddot{\varphi} = -\cancel{m} g \cdot \cancel{\ell} \sin\varphi \quad / \frac{1}{\ell}$$

$$\boxed{\ddot{\varphi} + \frac{g}{\ell} \sin\varphi = 0}$$

principal equ. of motion of math. pendulum

$\Downarrow$

$\varphi(t) = \ldots\ldots$