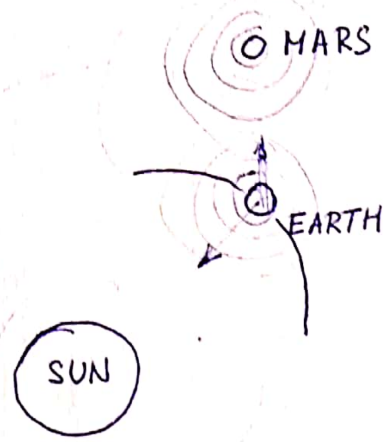
6) LAW OF CONSERVATION OF MECHANICAL ENERGY

Force field



$\vec{F} = \vec{F}(\vec{r})$

$\vec{r}$ ... displacement vector

$d\vec{r} = (dx, dy, dz)$

$\vec{F} = (F_x, F_y, F_z)$

Work of this force

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} = \int_{\vec{r}_1}^{\vec{r}_2} (F_x, F_y, F_z)(dx, dy, dz) =$$

$$= \int_{\vec{r}_1}^{\vec{r}_2} (F_x \cdot dx + F_y\, dy + F_z\, dz) =$$

$$= \int_{\vec{r}_1}^{\vec{r}_2} \left(\frac{\partial U}{\partial x} dx + \frac{\partial U}{\partial y} dy + \frac{\partial U}{\partial z} dz\right) =$$

$$= \int_{\vec{r}_1}^{\vec{r}_2} dU(\vec{r}) = U(\vec{r}_2) - U(\vec{r}_1) =$$

$$= U_2 - U_1$$

If a given force field $\vec{F}(\vec{r})$ has a potential U then mechanical work W of this force between two places 1 and 2 is equal to ~~to~~ a difference of the potential at position 2 and 1. $\Rightarrow$

It does not depend on integrational path

POTENTIAL

$U = U(\vec{r})$

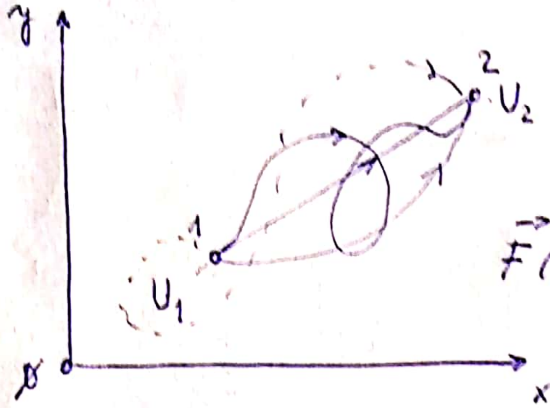
$\vec{F} = \text{grad}\, U$

$\vec{F} = \left(\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z}\right)$

Total differential of U:

$$dU = \frac{\partial U}{\partial x} \cdot dx + \frac{\partial U}{\partial y} \cdot dy + \frac{\partial U}{\partial z} dz$$

U.... potential of force field



$\vec{F}(\vec{r})$ ... has potential U

$$W = U_2 - U_1$$

Conditions of potentiality

$$\boxed{\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}, \quad \frac{\partial F_z}{\partial x} = \frac{\partial F_x}{\partial z}, \quad \frac{\partial F_z}{\partial y} = \frac{\partial F_y}{\partial z}}$$

$\Rightarrow$ then force field has potential

POTENTIAL ENERGY V

$V(\vec{r})$ is given by potential $U(\vec{r})$

$$V(\vec{r}) = -U(\vec{r}) \quad \text{definition}$$

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} d\vec{r} = U_2 - U_1 = -V_2 - (-V_1) = \underbrace{V_1 - V_2}_{\text{difference of potential energy}}$$

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} d\vec{r} = K_2 - K_1 \quad \text{(LAW OF CHANGE OF KINETIC ENERGY)}$$

$$K_2 - K_1 = V_1 - V_2$$

$$\boxed{K_1 + V_1 = K_2 + V_2}$$ Law of conservation of mechanical energy

$$V_2 - V_1 = -\int_{\vec{r}_1}^{\vec{r}_2} \vec{F} d\vec{r}$$