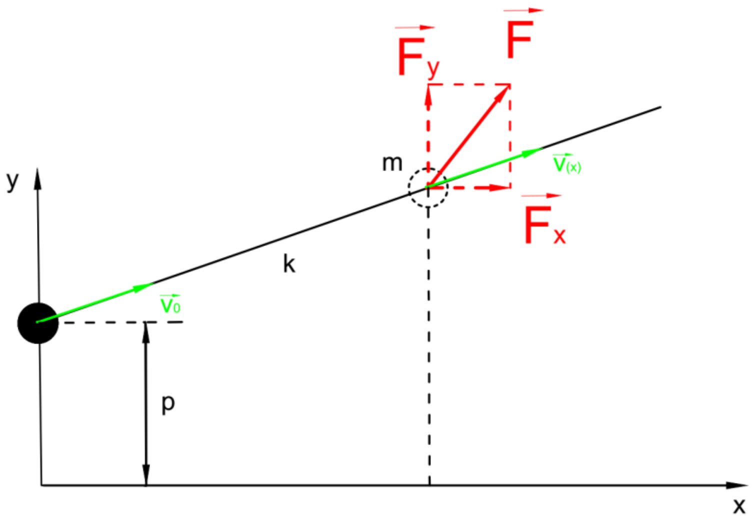
**Dynamics of particle in nonpotential foce field**

**Given:** a particle (mass m) moves on the trajectory which is a line k (with the equation: $y=p+qx$). The particle makes motion in force field given by vector expression:

$\vec{F} = F_x\vec{\imath} + F_y\vec{\jmath}$, in which: $F_x = F_0\frac{x}{y}, F_y = F_0\frac{y}{qx-y}$ and $F_0$, $p$, $q$ are given by constants.

The initial velocity of the particle is $v(0)=v_0$.




**Task:**

1, Is F potential force (potential force field) or not?

2, If F is potential force (force fiel is potential), find potential energy $V(x,y)$=?

3, Find the expression of the velocity of the particle $v(x)$=?

**Solution:**

1, To answer the question, we are going to check the condition of potential force:

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$$

We have:

$$F_x = F_0\frac{x}{y} \rightarrow \frac{\partial F_x}{\partial y} = -F_0\frac{x}{y^2} \text{ (1)}$$

$$F_y = F_0\frac{y}{qx-y} \rightarrow \frac{\partial F_y}{\partial x} = -F_0\frac{qy}{(qx-y)^2} \text{ (2)}$$

It is easy to see $\frac{\partial F_x}{\partial y} \neq \frac{\partial F_y}{\partial x}$. So the force F is not a potential force.

2, Because F is not a potential force so that we can not find the expression of the potential force $V(x,y)$.

3, Accordance with the principle of the work of force, we have:

$$K - K_0 = \int_{\vec{r}_0}^{\vec{r}} \vec{F} d\vec{r} = \int_{\vec{r}_0}^{\vec{r}} \left(F_x dx + F_y dy\right) (3)$$

Where:

K is the kinetic energy: $K = \frac{1}{2}mv^2$ and $K_0 = \frac{1}{2}mv_0^2$, respectively.

In another hand, $y = p + qx \rightarrow dy = qdx$

Substituting to (3), we have:

$$\begin{aligned} \frac{1}{2}m\left(v^2 - v_0^2\right) &= \int_{\vec{r}_0}^{\vec{r}} \left(F_x dx + F_y dy\right) \\ &= F_0 \int_0^x \left( \frac{x}{p+qx} dx + \frac{p+qx}{-p} qdx \right) \end{aligned} \quad (4)$$

Set $u = p + qx$,

$$\rightarrow x = \frac{1}{q}u - \frac{p}{q}$$

$$\rightarrow du = qdx \rightarrow dx = \frac{1}{q}du$$

$$x = 0 \rightarrow u = p$$

we have:

$$\int_0^x \frac{x}{p+qx} dx = \frac{1}{q^2} \int_p^{p+qx} \frac{u-p}{u} du = \frac{1}{q^2}\left(u - p\ln u\right)\Big|_p^{p+qx} = \frac{1}{q}\left( x - \frac{p}{q}\ln\left(1 + \frac{q}{p}x\right)\right) (5)$$

$$\int_0^x \frac{p+qx}{-p} qdx = -q\int_0^x \left(1 + \frac{qx}{p}\right)dx = -q\left(x - \frac{qx^2}{2p}\right) = -qx\left(1 + \frac{qx}{2p}\right) (6)$$

Substituting (5), (6) to (3), we have:

$$\frac{1}{2}m\left(v^2 - v_0^2\right) = \frac{F}{q}\left( x - \frac{p}{q}\ln\left(1 + \frac{q}{p}x\right)\right) - Fqx\left(1 + \frac{qx}{2p}\right)$$

$$\rightarrow v = \sqrt{\frac{2F}{mq}\left( x - \frac{p}{q}\ln\left(1 + \frac{q}{p}x\right)\right) - \frac{2Fq}{m}x\left(1 + \frac{qx}{2p}\right) + v_0^2}$$