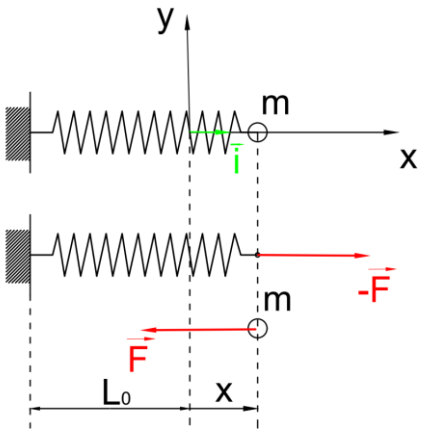
Spring Potential Energy

Given: a model as in the figure. The spring has the stiffness k, x is the deformation. The nature length of the spring is L_0. The expression of the spring force is given: $\vec{F} = -kx\vec{i}$



Task: Is F potential force or not? If it is right, find expression of potential energy *V(x,y)*=?

Solution:

According to the figure, we can rewrite the spring force:

$$\vec{F} = F_x\vec{i} + F_y\vec{j}$$

In which: $F_x = -kx, F_y = 0$

$$\rightarrow \frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x} = 0$$

Hence F is a potential force.

We have:

$$\int_{\vec{r}_0}^{\vec{r}} \vec{F}d\vec{r} = \int_{x_1}^{x_2} F_x dx = -K\int_{x_1}^{x_2} xdx = -\frac{1}{2}K\left(x_2^2 - x_1^2\right) = -\left(\frac{1}{2}Kx_2^2\right) + \left(\frac{1}{2}Kx_1^2\right)$$

The potential energy can be defined as the following:

$$V\left(x_2\right) - V\left(x_1\right) = -\int_{\vec{r}_0}^{\vec{r}} \vec{F}d\vec{r} = \left(\frac{1}{2}Kx_2^2\right) - \left(\frac{1}{2}Kx_1^2\right)$$

Where: $V\left(x\right) = \frac{1}{2}Kx^2$

Hence *V* is the expression of potential energy.