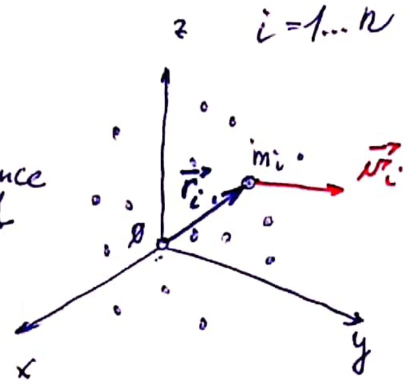
ANGULAR MOMENTUM OF SYSTEM OF PARTICLES

$i = 1 \dots n$



$\vec{r}_i$ ... displacement vector of particle

$\emptyset$ ... origin of coordinate system = reference point

momentum of i-th particle $\vec{p}_i = m_i \vec{v}_i$

angular momentum of i-th particle:

$$\vec{L}_i = \vec{r}_i \times \vec{p}_i \quad / \sum_{(i)}$$

angular momentum of the system of particles:

$$\vec{L} = \sum_{(i)} \vec{r}_i \times \vec{p}_i = \sum_{(i)} \vec{r}_i \times (m_i \vec{v}_i) \quad / \frac{d}{dt}$$

change of angular momentum in time:

$$\frac{d\vec{L}}{dt} = \frac{d}{dt} \sum_{(i)} \vec{r}_i \times (m_i \vec{v}_i) = \overbrace{\sum_{(i)} \underbrace{\frac{d\vec{r}_i}{dt}}_{\vec{v}_i} \times (m_i \vec{v}_i)}^{\vec{\emptyset}} + \sum_{(i)} \vec{r}_i \times (m_i \underbrace{\frac{d\vec{v}_i}{dt}}_{\vec{a}_i})$$

$$\vec{a}_i: \quad \vec{F}_i^E + \sum_{(j)} \vec{F}_{ji}$$

$$\vec{v}_i \parallel m_i \vec{v}_i \Rightarrow \vec{v}_i \times (m_i \vec{v}_i) = \vec{\emptyset}$$

$$\sum_{(i)} \vec{r}_i \times m_i \vec{a}_i = \sum_{(i)} \vec{r}_i \times \left(\vec{F}_i^E + \sum_{(j)} \vec{F}_{ji}\right) = \underbrace{\sum_{(i)} \vec{r}_i \times \vec{F}_i^E}_{\vec{M}^E} + \underbrace{\sum_{(i)} \vec{r}_i \times \sum_{(j)} \vec{F}_{ji}}_{\vec{\emptyset}}$$

$$\boxed{\frac{d\vec{L}}{dt} = \vec{M}^E}$$

(differential form)

LAW OF CHANGE OF ANGULAR MOMENTUM

$$\int_{\vec{L}_1}^{\vec{L}_2} d\vec{L} = \int_{t_1}^{t_2} \vec{M}^E dt \Rightarrow \vec{L}_2 - \vec{L}_1 = \int_{t_1}^{t_2} \vec{M}^E dt$$

(integral form)

LAW OF CONSERVATION OF ANGULAR MOMENTUM

$$\underbrace{\int_{t_1}^{t_2} \vec{M}^E dt}_{\text{Impulse of moments}} = \vec{\emptyset} \Rightarrow \vec{L}_2 - \vec{L}_1 = \vec{\emptyset} \Rightarrow \vec{L}_2 = \vec{L}_1$$