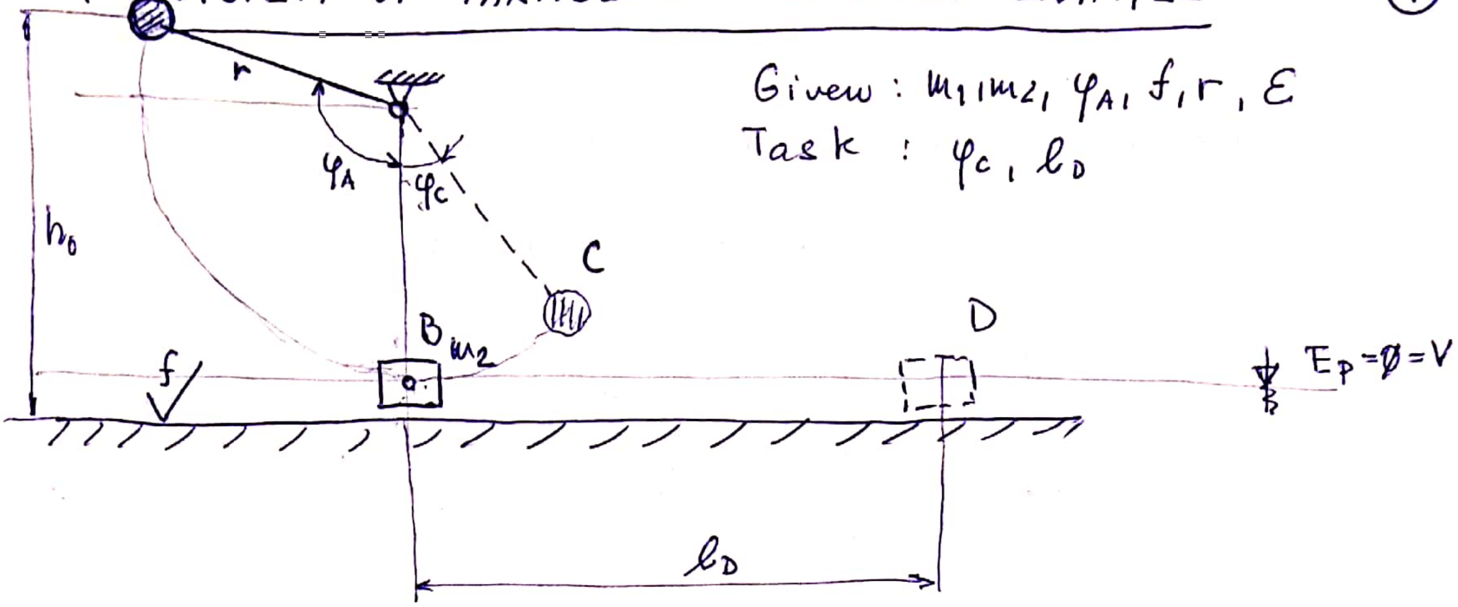
A SYSTEM OF PARTICLES – COMPLEX EXAMPLE

Given : $m_1, m_2, \varphi_A, f, r, \varepsilon$

Task : φ_C, l_D



φ_A.... the angle of pendulum at the beginning

f ... coeff. of friction

$v_{1A} = 0$ ... velocity (initial) of pendulum (particle m_1)

$v_{1B} = 0$ ... initial velocity of particle m_2

at place B we assume the "real" collision (ε)

Solution :

PHASE 1

Particle m_1 is going down getting some energy:

Law of conservation of ME

$$E_{1A} = E_{1B}$$

$$mg \cdot h_0 = \frac{1}{2} m_1 v_{1B}^2$$

$$\cancel{m_1} g \left[r + r\cos(2\pi - \varphi_A) \right] = \frac{1}{2} \cancel{m_1} v_{1B}^2$$

$$gr(1 - \cos\varphi_A) = \frac{1}{2} v_{1B}^2$$

$$v_{1B} = \sqrt{2gr(1 - \cos\varphi_A)}$$

PHASE 2:

Real collision

Newton's theorem $\qquad \mathcal{E} = -\dfrac{v_1^* - v_2^*}{v_1 - v_2}$

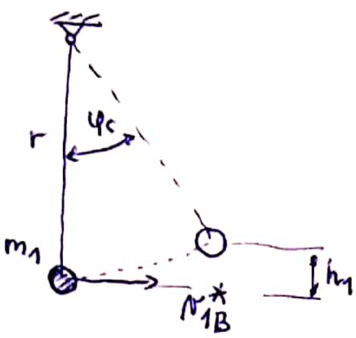
$$v_{1B}^* = \frac{-v_{1B}\,(\mathcal{E}m_2 - m_1)}{m_1 + m_2}$$

$$v_{2B}^* = \frac{(1+\mathcal{E})\,m_1 v_{1B}}{m_1 + m_2}$$

(1) Newton's theorem

(2) Law of conserv. of momentum

PHASE 3:



Energy ballance:

Law of conservation of ME:

$$\frac{1}{2}\cancel{m_1}\, v_{1B}^{*2} = \cancel{m_1} g h_1$$

$$\frac{1}{2} v_{1B}^{*2} = g r (1 - \cos\varphi_c)$$

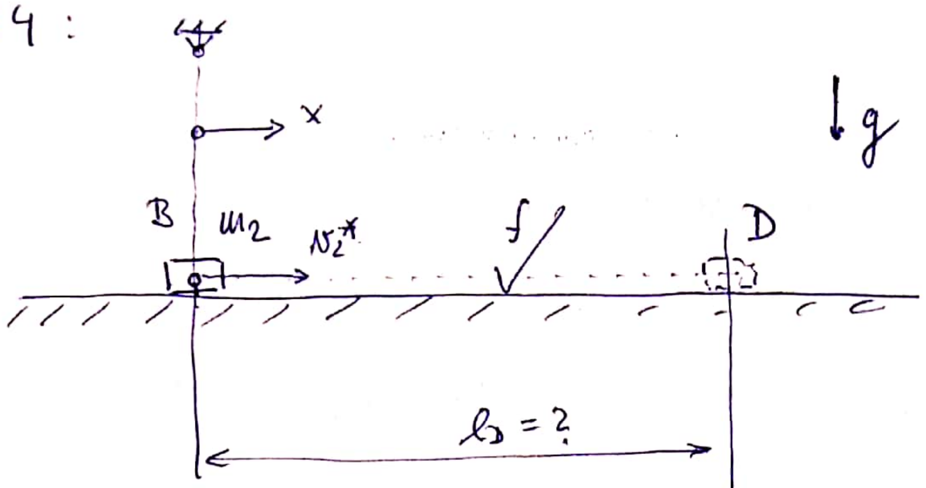
$$v_{1B}^2 \frac{(\mathcal{E}m_2 - m_1)^2}{(m_1+m_2)^2} = 2 g r (1 - \cos\varphi_c)$$

$$\left(v_{1B} = \sqrt{2 g r (1 - \cos\varphi_A)} \right)$$

$$\cancel{2g}r\,(1-\cos\varphi_A)\,\frac{(\mathcal{E}m_2 - m_1)^2}{(m_1+m_2)^2} = \cancel{2g}r\,(1 - \cos\varphi_c)$$

$\varphi_c = \ldots$

PHASE 4 :



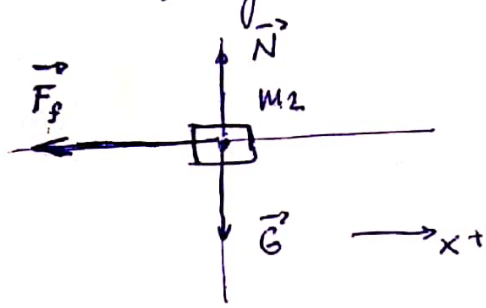
Law of change of kinetic energy (non-potential force field)

$$K_D - K_B = \int_{\vec{r_1}}^{\vec{r_2}} \vec{F} d\vec{r}$$

$$\emptyset - \frac{1}{2} m_2 v_2^{*2} = \int_0^{l_D} F_{RX} dx$$

$\vec{G}$ ... gravity force
$\vec{N}$ ... normal force (reaction)
$\vec{F_f}$ ... friction force

Free body diagram



(1) x: $F_{RX} = - F_f$

(2) y: $F_{RY} = \underline{G - N = \emptyset}$

$N = G = m_2 g$

$\underline{F_f} = f \cdot N = \underline{f m_2 g}$

$$-\frac{1}{2} m_2 v_{2B}^{*2} = \int_0^{l_D} - F_f \, dx$$

$$-\frac{1}{2} m_2 v_{2B}^{*2} = \int_0^{l_D} - f m_2 g \, dx = - f m_2 g \, l_D$$

constants

$$-\frac{1}{2} \cancel{m_2} v_{2B}^{*2} = - f \cancel{m_2} g \, l_D$$

$$l_D = \frac{v_{2B}^{*2}}{2gf}$$