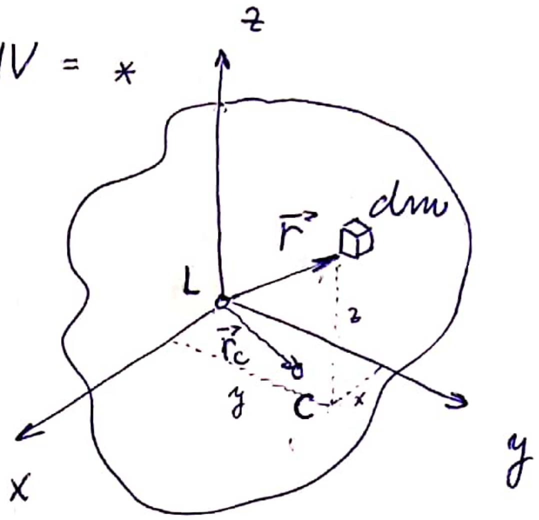
MASS PROPERTIES OF BODIES

0-th order

- mass $\quad m = \int_{(m)} dm = \int_{(V)} \rho \cdot dV = *$



dm ... elementary mass
dV ... — " — volume

$dm = \rho \cdot dV$

if $\rho = const. \; \Big| \; * = \rho \cdot \int_{(V)} dV$

(if) const

1-st order

$\vec{r}$ ... displacement vector

STATIC MOMENT OF MASS

$$\vec{S} = \int_{(m)} \vec{r}\, dm = \int_{(V)} \rho\, \vec{r}\, dV$$

in component expression:

$$S_{yz} = \int_{(m)} x\, dm$$

$$S_{zx} = \int_{(m)} y\, dm$$

$$S_{xy} = \int_{(m)} z\, dm$$

(x, y, z) ... coordinates of displacement vector:

$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

calculation of center of mass:

$$\vec{r}_c \cdot m = \int_{(m)} \vec{r}\, dm = \vec{S} \Rightarrow \vec{r}_c = \frac{\vec{S}}{m}$$

$$x_c = \frac{S_{yz}}{m} \; ; \; y_c = \frac{S_{xz}}{m} \; ; \; z_c = \frac{S_{xy}}{m}$$

$$\vec{r}_c = x_c\vec{i} + y_c\vec{j} + z_c\vec{k}$$

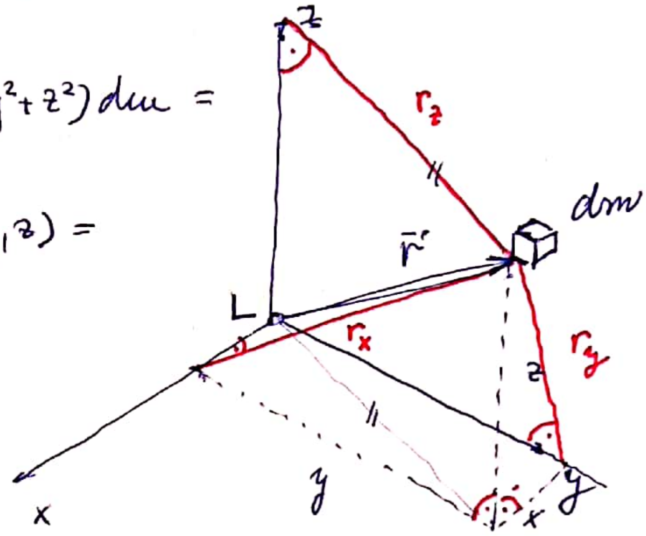
2-nd order

MOMENTS OF INERTIA

a) With respect to the point (origin of coordinates)

$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

$J_L = \int_{(m)} (\vec{r})^2 dm = \int_{(m)} (x^2 + y^2 + z^2)\, dm =$



$(\vec{r})^2 = \vec{r} \cdot \vec{r} = (x, y, z) \cdot (x, y, z) =$

$= x^2 + y^2 + z^2$

$= \int_{(m)} (x^2 + y^2 + z^2)\, \rho\, dV$

if $\rho = \text{const.}$ $\quad J_L = \rho \int_{(m)} (x^2 + y^2 + z^2)\, dm$

b) With respect to the coordinate axis

$J_x = \int_{(m)} r_x^2\, dm \quad ; \quad r_x = \sqrt{y^2 + z^2}$

$J_x = \int_{(m)} (y^2 + z^2)\, dm$

$J_y = \int_{(m)} r_y^2\, dm = \int_{(m)} (x^2 + z^2)\, dm \qquad r_y = \sqrt{x^2 + z^2}$

$J_z = \int_{(m)} r_z^2\, dm = \int_{(m)} (x^2 + y^2)\, dm \qquad r_z = \sqrt{x^2 + y^2}$

c) With respect to plane
(a plane of coordinate system)

$$J_{yz} = \int_{(m)} x^2 \, dm$$

$$J_{xz} = \int_{(m)} y^2 \, dm$$

$$J_{xy} = \int_{(m)} z^2 \, dm$$

Mutual associations :

$$J_L = \int_{(m)} (x^2+y^2+z^2) \, dm = \int_{(m)} x^2 \, dm + \int_{(m)} y^2 \, dm + \int_{(m)} z^2 \, dm =$$

$$= J_{yz} + J_{xz} + J_{xy}$$

$$J_x = \int_{(m)} (y^2+z^2) \, dm = \int_{(m)} y^2 \, dm + \int_{(m)} z^2 \, dm = J_{xz} + J_{yx}$$

$$J_y = J_{xy} + J_{yz}$$

$$J_z = J_{xz} + J_{yz}$$

$$2J_L = J_x + J_y + J_z = \underline{J_{xz}} + \overline{J_{xy}} + \overline{J_{xy}} + \widetilde{J_{yz}} + \underline{J_{xz}} + \widetilde{J_{yz}}$$