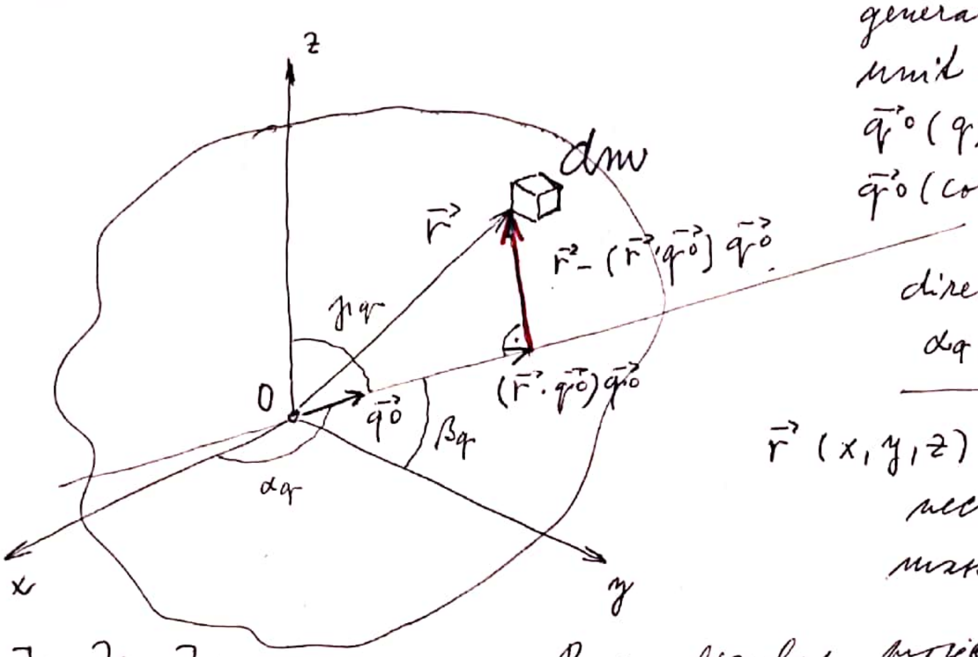
MOMENT OF INERTIA WITH RESPECT TO GENERAL AXIS



general axis q
unit vector $\vec{q}^0$
$\vec{q}^0(q^0_x, q^0_y, q^0_z)$
$\vec{q}^0(\cos\alpha_q, \cos\beta_q, \cos\gamma_q)$

directional angles: $\alpha_q, \beta_q, \gamma_q$

$\vec{r}(x, y, z)$ .... displacement vector of elementary mass dm

J_x, J_y, J_z
J_q

Perpendicular projection of $\vec{r}$ to q:
$(\vec{r}\cdot\vec{q}^0)\cdot\vec{q}^0$

Moment of inertia of body with respect to q axis

$(a-b)^2 = a^2 - 2ab - b^2$
$(\vec{r})^2 = \vec{r}\cdot\vec{r} = r^2$

$$J_q = \int_{(m)} [\vec{r} - (\vec{r}\cdot\vec{q}^0)\vec{q}^0]^2 \, dm =$$

$$= \int_{(m)} \Big[\vec{r}^{\,2} - 2\vec{r}\cdot(\vec{r}\vec{q}^0)\vec{q}^0 + (\vec{r}\vec{q}^0)^2 \cdot \underbrace{\vec{q}^{0\,2}}_{1}\Big] dm =$$

$$= \int_{(m)} [\vec{r}^{\,2} - 2(\vec{r}\vec{q}^0)^2 + (\vec{r}\vec{q}^0)^2] \, dm = \int_{(m)} [\vec{r}^{\,2} - (\vec{r}\vec{q}^0)^2] \, dm =$$

$$= \int_{(m)} [r^2 - (\vec{r}\vec{q}^0)^2] \, dm = *$$

knowing: $\vec{r}\cdot\vec{q}^0 = (x, y, z)\cdot(q^0_x, q^0_y, q^0_z) = x q^0_x + y q^0_y + z q^0_z$
$r^2 = x^2 + y^2 + z^2$

using: $q^{0\,2}_x + q^{0\,2}_y + q^{0\,2}_z = 1$

$$* = J_x q^{0\,2}_x + J_y q^{0\,2}_y + J_z q^{0\,2}_z - 2D_{xy} q^0_x q^0_y - 2D_{yz} q^0_y \cdot q^0_z - 2D_{zx} q^0_z q^0_x$$

$$\boxed{\begin{aligned} J_q = {} & J_x \cdot \cos^2 \alpha_q + J_y \cos^2 \beta_q + J_z \cos^2 \gamma_z + \\ & - 2 D_{xy} \cos\alpha_q \cos\beta_q - 2 D_{xy} \cos\alpha_q \cos\beta_q + \\ & - 2 D_{zx} \cos\gamma_q \cos\alpha_q \end{aligned}}$$

We can rewrite this formula to vector form
(matrix)

$$q^{0} = \begin{bmatrix} \cos\alpha_q \\ \cos\beta_q \\ \cos\gamma_q \end{bmatrix}$$

$$\boxed{J_q = (q^{0})^T \cdot \mathbf{I} \cdot q^{0}}$$