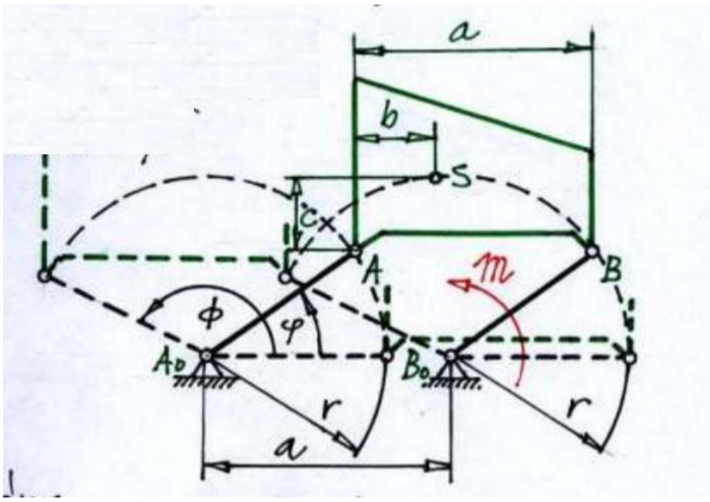
Dynamics of translational motion of a body

A rigid body moves with two bars as shown in the Figure. The body with the center of mass S is connected to the bars by rotary joints at A and B. The first bar rotates by the rotary joint at A_0 and the second one does by the rotary joint at B_0. Two bars rotate parallel from a starting position to another in a given time with the same constant angular velocity by applying a driving moment to the second bar. The gravity and weight of the bars are neglected.

Given:

- The dimensions a, b, c, r;
- The mass of the body m;
- The time of rotation T;
- The angular velocity of bars ω;
- The angle at the ending position is ϕ;
- The applied moment of the second bar: $\mathrm{M(t)} = M_0 \cos(\mathrm{kt})$;
- The boundary condition:
  $\mathrm{v}_s(0) = \mathrm{v}_s(T) = 0;$
  $\varphi(0) = 0, \varphi(T) = \phi;$



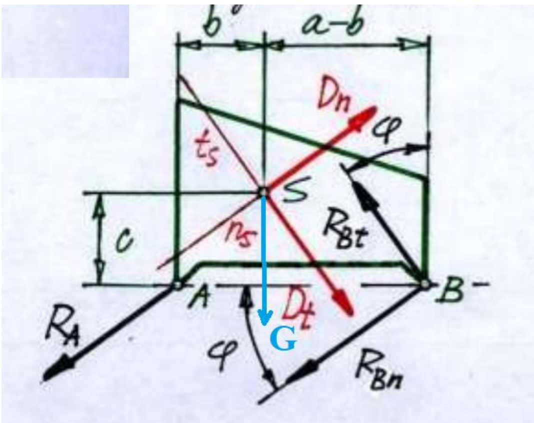
Task:

Find k and M_0;

Find the reaction forces R_A, R_B

Solution:

Firstly, we draw the free body diagram of the rigid body with considering to d'Alembert law in the normal *n*- and tangent *t*-coordinates:



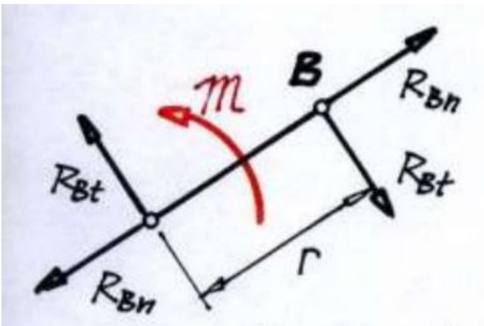
We can write the component equations by the following:

(t) : $-G.\cos\varphi + R_{Bt} - D_t = 0$ (1)

(n) : $G.\sin\varphi + R_A + R_{Bn} - D_n = 0$ (2)

(MS): $\left(R_{Bt}\cos\varphi - R_{Bn}\sin\varphi\right)\left(a-b\right) + R_A b\sin\varphi - \left(R_A\cos\varphi + R_{Bn}\cos\varphi + R_{Bt}\sin\varphi\right)c = 0$ (3)

Next, we consider the second bar:



(MB): $M\left(t\right) - rR_{Bt} = 0$ (4)

Kinematics of the body:

$$v_S = r\frac{d\varphi}{dt} = r\omega \quad (5)$$

$$a_{St} = \frac{dv_s}{dt} = 0 \quad (6)$$

$$a_{Sn} = \frac{v_S^2}{r} = r\omega^2 \quad (7)$$

Specification of D'Alembert forces:

$$D_t = ma_{St} = 0 \quad (8)$$

$$D_n = ma_{Sn} = mr\omega^2 \quad (9)$$

From (1), (4) and (8), we have:

$$R_{Bt} = mg\cos\varphi \quad (10)$$

$$M\left(t\right) = rmg\cos\varphi \quad (11)$$

So

$$M_0 \cos(kt) = mrg \cos(\omega t)$$

We can say that:

$$M_0 = mrg$$
$$k = \omega$$

Then we rewrite the equation (2) and (3):

$$R_A + R_{Bn} = mr\omega^2 - mg\sin\varphi \qquad (12)$$

$$R_A(b\sin\varphi - c\cos\varphi) + R_{Bn}(\mathrm{a}\sin\varphi - b\sin\varphi - c\cos\varphi) \qquad (13)$$
$$+mg\cos\varphi(\mathrm{a}\cos\varphi - b\cos\varphi - c\sin\varphi) = 0$$

Then solve the system of equation (12), (13), we get:

$$R_{Bn} = \frac{mg(\mathrm{b} - \mathrm{a}\cos^2\varphi) - mr\omega^2(\mathrm{b}\sin\varphi - c\cos\varphi)}{(a-2b)\sin\varphi} \qquad (14)$$

$$R_A = mr\omega^2 - mg\sin\varphi - R_{Bn} \qquad (15)$$

In brief, we obtain the reaction forces of joints A and B from (10), (14) and (15).