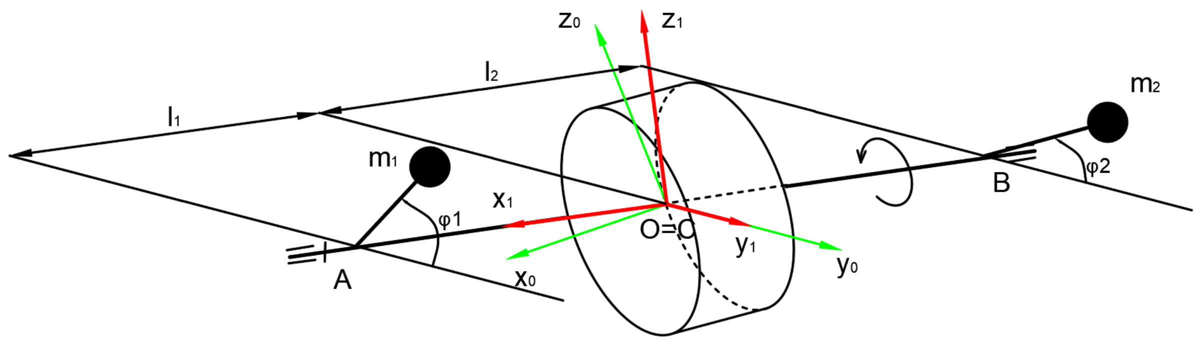
## Balancing of a rotating body

The skew cylinder is attached to the vertical rotary axis as in the figure. The balancing system is created by using two additional masses. They are joined to the rotary axis at joint A and B by weightless bars.

**Given:**

- The dimension of the cylinder *h, r*;
- Mass of the cylinder m;
- Skew angle $\varphi$
- The dimension of the axis $l_1$, $l_2$;




**Task:**

Determine the additional masses $m_1$, $m_2$, the length of bars $r_1$, $r_2$, and the angles $\varphi_1$, $\varphi_2$ to keep the system in static balance and dynamic balance.

**Solution:**

The total mass of the balanced system is:

$$m' = m + m_1 + m_2 \tag{1}$$

Condition of static balance :

$$S'_{xy} = mz_C + m_1 r_1 \sin\varphi_1 + m_2 r_2 \sin\varphi_2 = 0 \tag{1}$$
$$S'_{xz} = my_C + m_1 r_1 \cos\varphi_1 + m_2 r_2 \cos\varphi_2 = 0 \tag{2}$$

Condition of dynamic balance :

$$D'_{xy} = D_{xy} + m_1 l_1 r_1 \cos\varphi_1 - m_2 l_2 r_2 \cos\varphi_2 = 0 \tag{3}$$
$$D'_{xz} = D_{xz} + m_1 l_1 r_1 \sin\varphi_1 - m_2 l_2 r_2 \sin\varphi_2 = 0 \tag{4}$$

where $D_{xy}$, $D_{xz}$ are products of inertia of the cylinder in the coordinate system $O, x_1, y_1, z_1$ .

We already know that

$$y_C = z_C = 0 \tag{5}$$

$$D_{xy} = 0;\ D_{xz} = \frac{m}{4}\cos\varphi\sin\varphi\left(-r^2 + \frac{h^2}{3}\right) \tag{6}$$

Substitute (5),(6) into (1), (2), (3), (4), we get:

$$m_1 r_1 \sin\varphi_1 + m_2 r_2 \sin\varphi_2 = 0 \tag{7}$$

$$m_1 r_1 \cos\varphi_1 + m_2 r_2 \cos\varphi_2 = 0 \tag{8}$$

$$m_1 l_1 r_1 \cos\varphi_1 - m_2 l_2 r_2 \cos\varphi_2 = 0 \tag{9}$$

$$\frac{m}{4}\cos\varphi\sin\varphi\left(-r^2 + \frac{h^2}{3}\right) + m_1 l_1 r_1 \sin\varphi_1 - m_2 l_2 r_2 \sin\varphi_2 = 0 \tag{10}$$

Combine (8) and (9), we have:

$$m_1 r_1 \cos\varphi_1 = m_2 r_2 \cos\varphi_2 = 0 \tag{11}$$

Then we can choose

$$\varphi_1 = \frac{\pi}{2};\varphi_2 = -\frac{\pi}{2} \tag{12}$$

Substitute (12) into (7), we have:

$$m_1 r_1 = m_2 r_2 \tag{13}$$

Substitute (12), (13) into (10), we have:

$$\frac{m}{4}\left(r^2 - \frac{h^2}{3}\right)\sin\varphi\cos\varphi + m_1 r_1 \left(l_1 + l_2\right) = 0 \tag{14}$$

So we get:

$$r_1 = \frac{\frac{m}{4}\left(r^2 - \frac{h^2}{3}\right)\sin\varphi\cos\varphi}{m_1\left(l_1 + l_2\right)} \tag{15}$$

We can choose $m_1 = m_2$ then we get:

$$r_2 = r_1 \tag{16}$$