CV3 – Př. 1

UKAŽTE, ŽE SILOVÉ POLE JE POTENCIÁLNÍ A NALEZNĚTE JEHO POTENCIÁL (POTENCIÁLNÍ ENERGII)

a) GRAVITAČNÍ HOMOGENNÍ POLE

b) GRAVITAČNÍ POLE Z NEWTONOVA GRAV. ZÁKONA

c) SILOVÉ POLE VINUTÉ PRUŽINY

VÝCHOZÍ VZTAHY: – POTENCIÁLOVOST POLE: $\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$

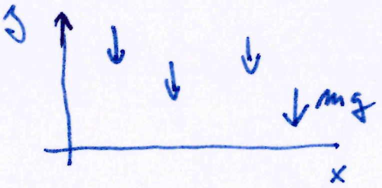
(ROVINNÝ PROBLÉM)

$$\vec{F} = (F_x; F_y)$$

$$-\overrightarrow{\mathrm{grad}}\, V = \vec{F} \iff \vec{\nabla} V_{(x;y)} = \vec{F}$$

↳ POTENCIÁL $V(x;y)$

ad a) $\vec{F} = (0; -mg)$



$$\frac{\partial F_x}{\partial y} = 0 = \frac{\partial F_y}{\partial x}$$

$$V'_x = 0 \Rightarrow V_x = \int V'_x\, dx = C(y)$$

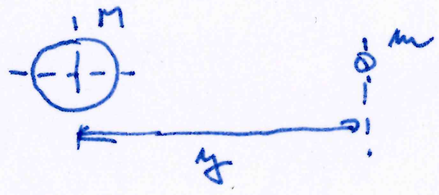
$$-(V'_x; V'_y) = \vec{F} \qquad V'_y = +m\cdot g \Rightarrow V_y = \int V'_y\, dy = +m\cdot g\, y + C(x)$$

$$\Rightarrow V_{(x;y)} = +m\cdot g\cdot y + C(x) + C(y)$$

$$V_{(x;y)} = +m\cdot g\cdot y + C$$

$$\boxed{V_{(y)} = +m\cdot g\cdot y + C}$$

ad b) $\vec{F} = \left(0; -\varkappa \frac{mM}{y^2}\right)$



$$\frac{\partial F_x}{\partial y} = 0 = \frac{\partial F_y}{\partial x}$$

$$-V'_x = 0 \Rightarrow V_x = C(y)$$

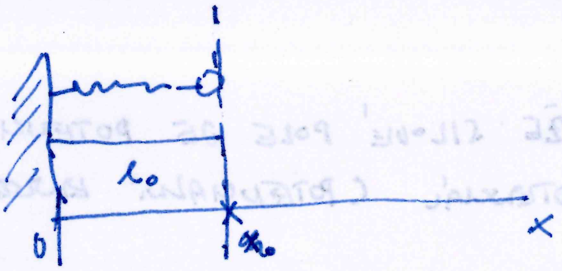
$$-V'_y = -\varkappa \frac{mM}{y^2} \Rightarrow V_y = +\int \varkappa \frac{mM}{y^2}\, dy = -\varkappa \frac{mM}{y} + C(x)$$

$$\Bigg\} \; V_{(x;y)} = -\varkappa \frac{mM}{y} + C$$

c) $\vec{F} = (-kx; 0)$

$\downarrow$

PŮSOBÍ PROTI SMĚRU POHYBU



$\frac{\partial F_x}{\partial y} = 0 \quad \frac{\partial F_y}{\partial x}$ $\quad V'_x = +kx \Rightarrow V(x) = \int +kx\,dx = +\frac{1}{2}kx^2 + C(y)$

$-\vec{\nabla} V(x;y) = \vec{F}$ $\quad V'_y = 0 \Rightarrow V(y) = C(x)$

$\underline{V(x) = +\frac{1}{2}kx^2 + C}$

PP:

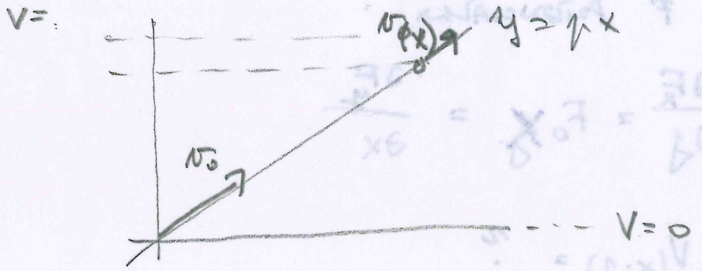
CV 3 – PŘ. 2 – VIZ CV2 PŘ. 2 – SROVNEJ!!

HB SE POHYBUJE PO PŘÍMCE $k: y = p x$ V TÍH. POLI.

URČETE ZÁVISLOST RYCHLOSTI NA SOUŘADNICI X. POČ. RYCHLOST JE NENULOVÁ.

D: v_0; g; p; bez tření

U: $v(x)$



VYUŽIJEME ZZME

$$E_{M_1} = E_{M_2}$$

$$K_1 + U_1 = K_2 + U_2$$

$$\frac{1}{2} m v_0^2 + 0 = \frac{1}{2} m v_{(x)}^2 + m \cdot g \cdot y$$

$$\frac{1}{2} m v^2 = \frac{1}{2} m v_0^2 - m \cdot g \cdot y$$

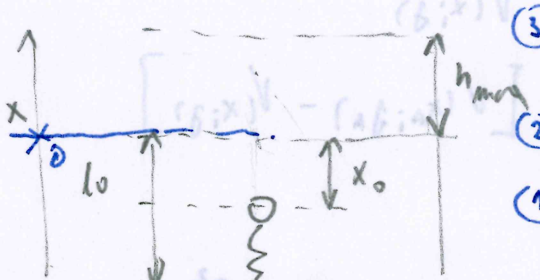
$$\underline{v^2 = v_0^2 - 2 g p x}$$

CV 3 – PŘ. 3

JAK VYSOKO VYLETÍ KULIČKA VYSTŘELENÁ PRUŽINOU

D: x_0; g; k

U: h_{max}



3.

2.

1. $V_{P_1} = \frac{1}{2} k x_0^2 - m \cdot g \cdot x_0$

$V_{P_2} = m \cdot g \cdot h_{max}$

ZZME:

$$K_1 + V_1 = K_2 + V_2 = K_3 + V_3$$

$$0 + V_1 = K_2 + 0 = V_3$$

$$V_1 = V_3$$

$$\frac{1}{2} k x_0^2 - m g x_0 = m g h_{max}$$

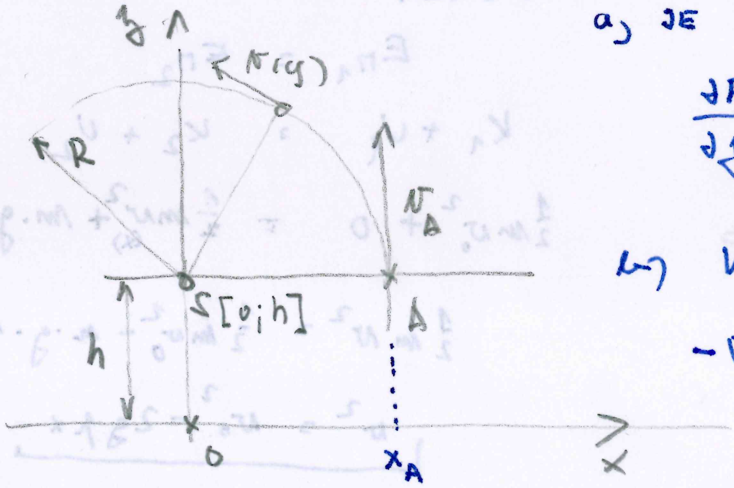
$$h_{max} = \frac{k x_0^2}{2 m g} - x_0$$

CV 3 - PŘ 4.

HMOTNÝ BOD SE POHYBUJE PO HLADKÉ KŘIVCE (KRUŽNICI) V SILOVÉM POLI $F=(F_x;F_y)$; POČÁTEČNÍ RYCHLOST v_a.

D: h; R; m; v_a; $\vec{F} = (F_x;F_y) = \left(F_0\, x y;\ F_0 \frac{x^2}{2}\right)$

U: $v(\varphi)$



a) JE $\vec{F}$ POTENCIÁLNÍ?

$$\frac{\partial F_x}{\partial y} = F_0 x = \frac{\partial F_y}{\partial x}$$

b) $V(x;y) = ?$

$$-\nabla V(x;y) = \vec{F}$$

$$-V'_x = F_0 x y \to V_x = -\int F_0 x y \, dx =$$

$$-V'_y = F_0 \frac{x^2}{2} \to V_y = -\int F_0 \frac{x^2}{2} dy =$$

$$\underline{V(x;y) = F_0 \frac{x^2}{2} y + C}$$

3. ZZME:

$$K_A + V_A = K + V$$

$$\frac{1}{2} m v_a^2 + V(x_A;y_A) = \frac{1}{2} m v^2 + V(x;y)$$

$$v^2 = v_a^2 + \frac{2}{m}\left[V(x_A;y_A) - V(x;y)\right]$$

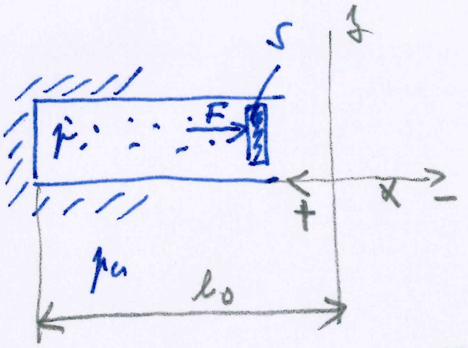
4. TRANSFORM. SOUŘADNIC

$$\left.\begin{aligned} x_A &= R\ ; & y_A &= h \\ x &= R\cos\varphi\ ; & y &= h + R\sin\varphi \end{aligned}\right\} \Rightarrow V(x_A;y_A) = -F_0 \frac{R^2}{2} h$$

$$V(x;y) = -\frac{F_0}{2}(R\cos\varphi)^2(h + R\sin\varphi)$$

5. $$\underline{v^2 = v_a^2 + \frac{R^2 F_0}{m}\left[\cos^2\varphi\,(h + R\sin\varphi)\right]}$$

Cv 3 - př. 5



$$\vec{F} = \left[-(p - p_a) S \, ; \, 0\right]$$

$\rightarrow$ není závislé na x ani y $\Rightarrow$ $V_{(x;y)}$

a) IZOTERMICKÁ ~~EXPANZE~~ EXPANZE (KOMPRESE) $\qquad x = 0$ když $p = p_a$

$$p \cdot V = \text{konst}$$

$$p_1 V_1 = p_2 V_2$$

$$p_a S l_0 = p S (l_0 - x)$$

$$\boxed{p = \frac{p_a l_0}{l_0 - x}}$$

$$-\operatorname{grad} V(x;y) = \vec{P} = \left(-p_a \left(\frac{l_0}{l_0 - x} - 1\right) \cdot S \, ; \, 0\right)$$

$$V'_x = p_a S \left(\frac{l_0}{l_0 - x} - 1\right)$$

$$V_x = p_a S \int \left(\frac{l_0}{l_0 - x} - 1\right) dx =$$

$$V_{(x;y)} = p_a S \left[-l_0 \ln(l_0 - x) - x\right]$$

b) ADIABAT. EXPANZE (KOMPRESE)

$$p_a V_1^{\kappa} = p_2 V_2^{\kappa}$$

$$p = \frac{p_a S^{\kappa} l_0^{\kappa}}{S^{\kappa} (l_0 - x)^{\kappa}} = \frac{p_a l_0^{\kappa}}{(l_0 - x)^{\kappa}}$$

$$V'_{(x)} = p_a S \left[\frac{l_0^{\kappa}}{(l_0 - x)^{\kappa}} - 1\right]$$

$$V_{(x;y)} = p_a S \left[-l_0^{\kappa} / (l_0 - x)^{\kappa - 1} \cdot \frac{1}{\kappa - 1} - x\right]$$