Magnetism
Magnetic field in solids

Magnetic field of conductors with current and permanent magnets, Biot-Savart's law, electromagnetic induction, Ampère's law, energy of solenoid magnetic field. Diamagnetism, paramagnetism, ferromagnetism.

Magnetic field effects

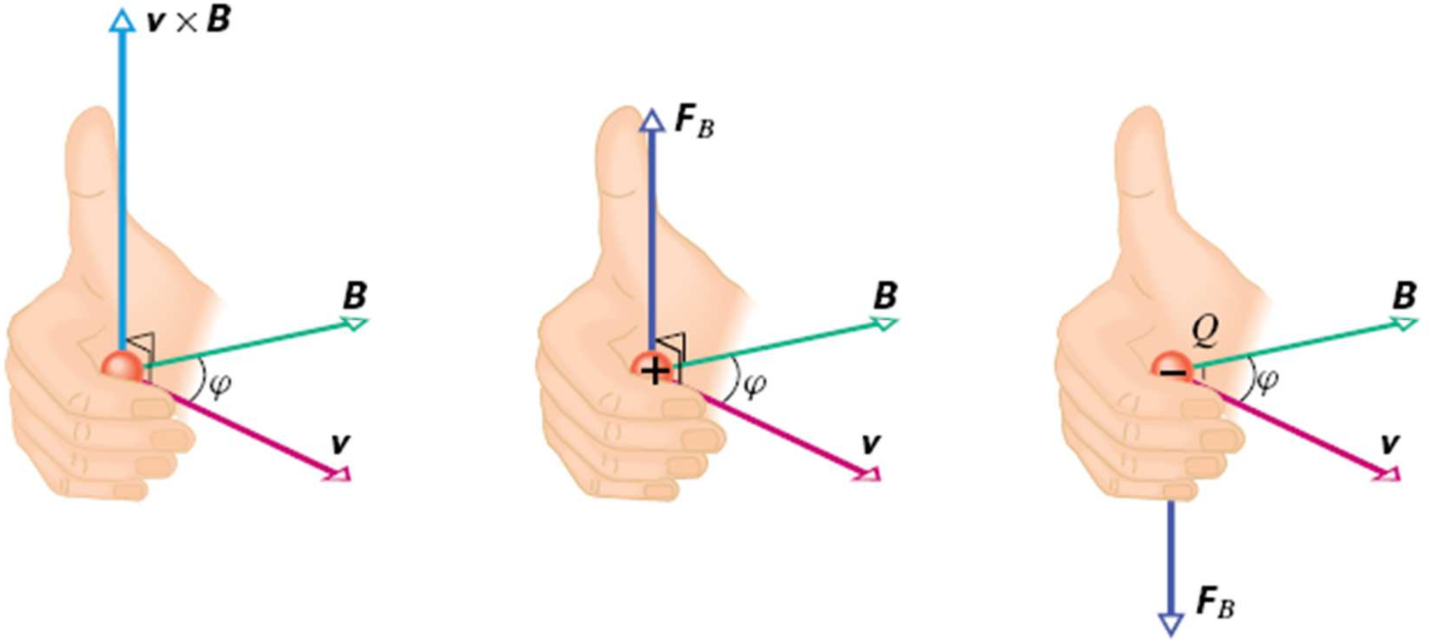
- Permanent magnets (magnetite Fe_3O_4 and maghemite Fe_2O_3 – iron ores) – compass needle
- Magnetic force on moving electric charge
- Magnetic force between two parallel wires carrying electric currents
- Electromagnetic induction
- Electromagnetic waves

Lorentz's force

Force on moving electric charge – right-hand rule

$$\mathbf{F}_B = Q\mathbf{v} \times \mathbf{B}$$

$$F_B = |Q| v B \sin\varphi.$$



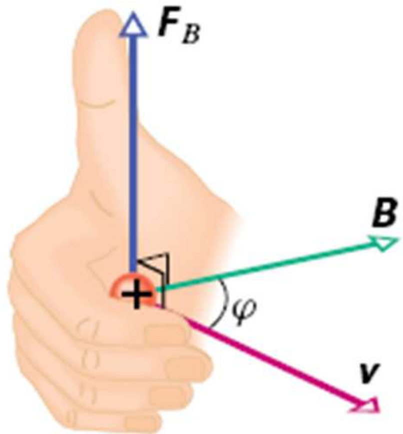
Magnetic induction

Defined by Lorentz's force F_{B}

$$B = \frac{F_{B,\max}}{|Q|v}$$

It is $\boldsymbol{F}_{\mathrm{B}} = \boldsymbol{0}$ ($\boldsymbol{v} \times \boldsymbol{B} = \boldsymbol{0}$)

in case of parallel v and B



Magnitude and unit of magnetic induction

Unit

T (Tesla)

Older unit (non SI) G (Gauss) – 1T = 10^4G

Magnitude

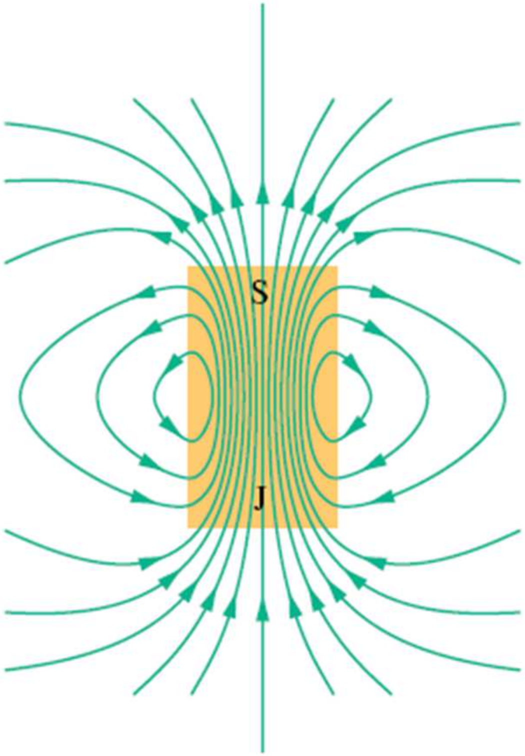
Earth magnetic field 10^{-4}T

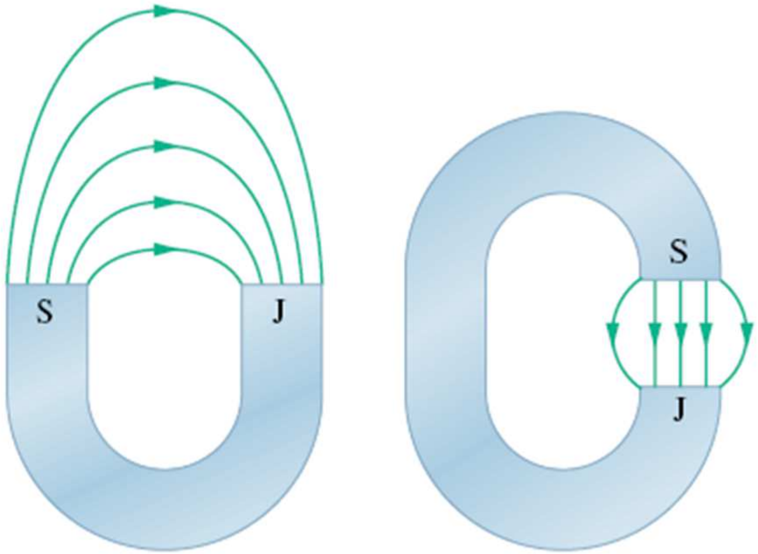
Electromagnets, NdFeB permanent magnets 1T

Superconductive magnets >1T

Magnetic field lines

Space distribution of magnetic field ***B*** directions – no direction of magnetic force!

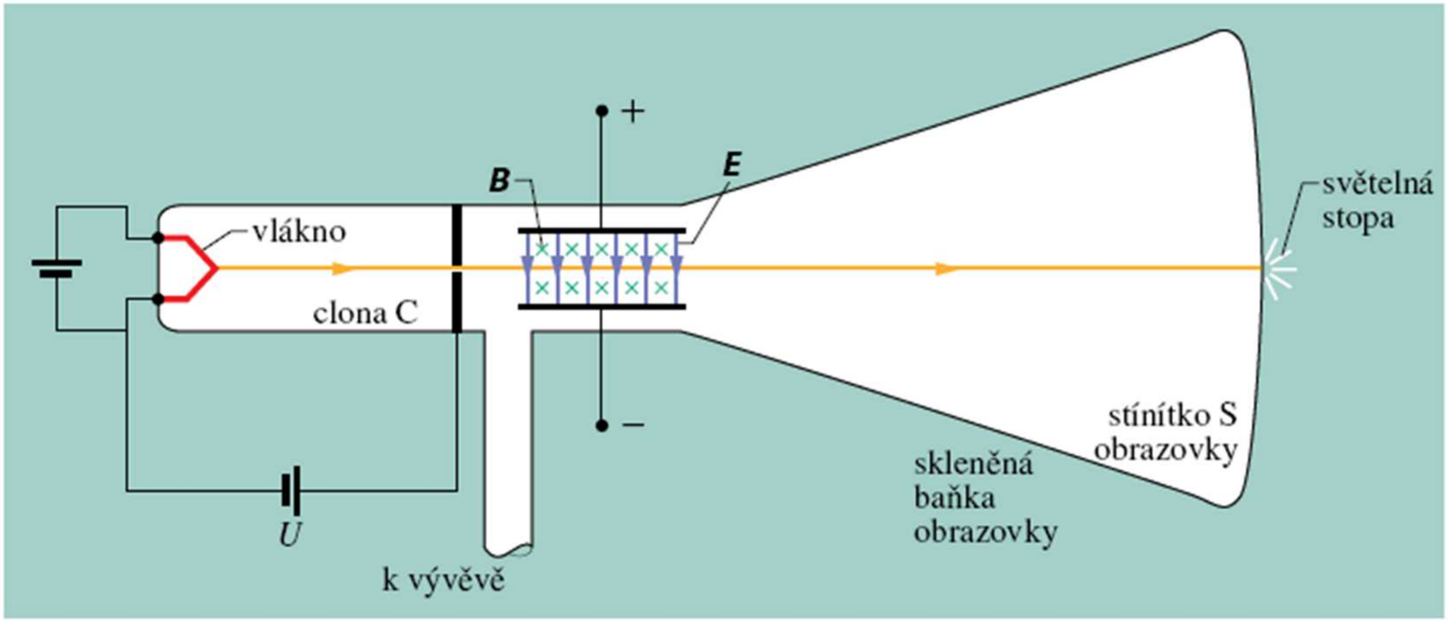




Cathode ray tube (CRT) display

Electron discovery – J.J.Thomson 1897

CRT displays – TV, oscilloscope, etc.

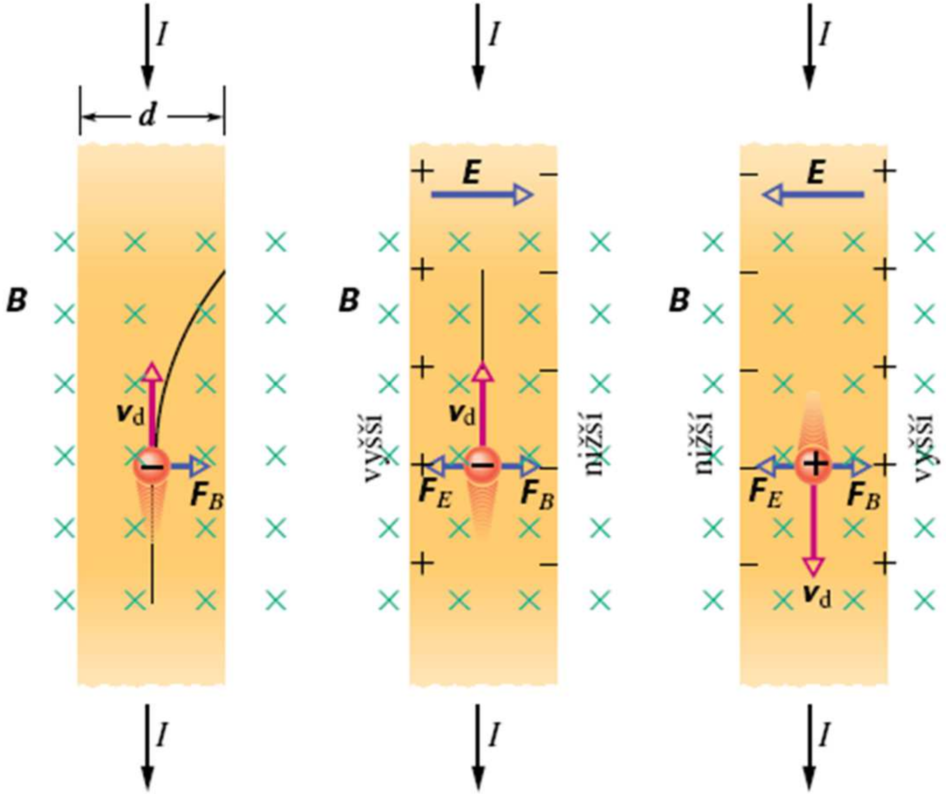


Lorentz’s force consequencies

- Hall’s effect
- Movement of charged particles in magnetic field – screw lines
- Aurora Borealis
- Accelerators for charged particles
- Simple motor
- …

Hall's effect

E.H.Hall, 1879 – measurement of charge carrier concentration



$$F_E + F_B = 0$$

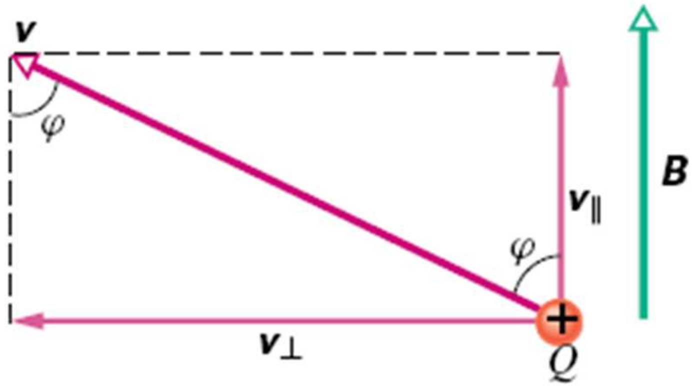
$$QE = Qv_d B$$

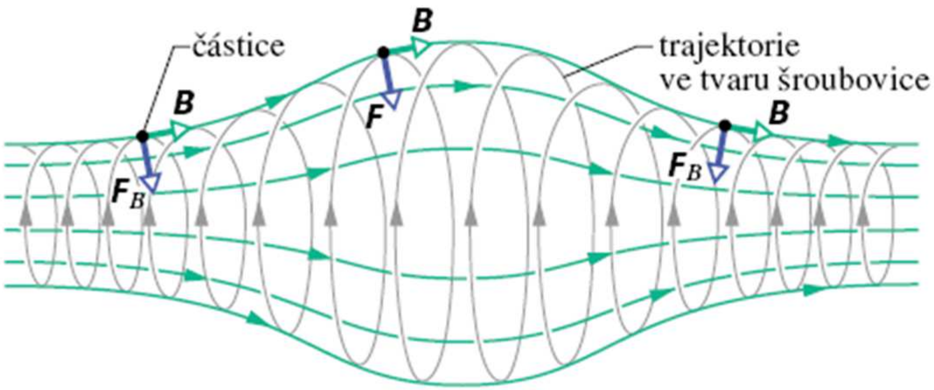
$$v_d = \frac{J}{nQ} = \frac{I}{nQS}$$

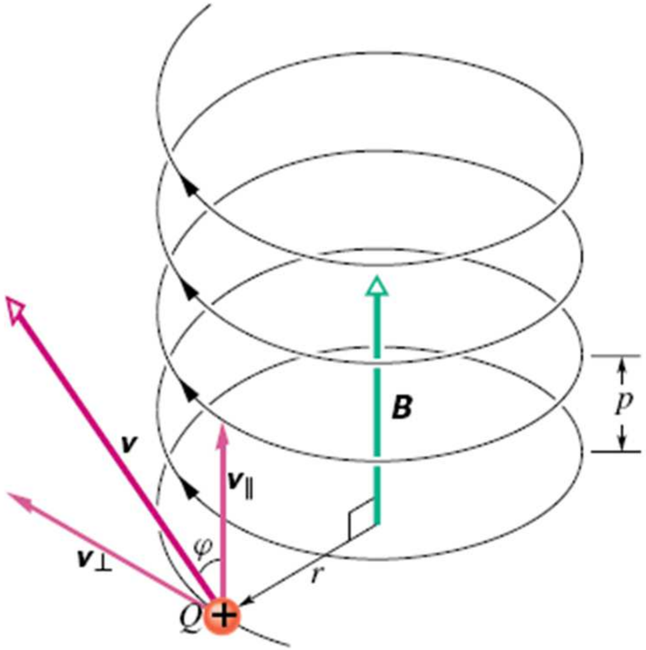
$$n = \frac{BId}{U_H S Q}$$

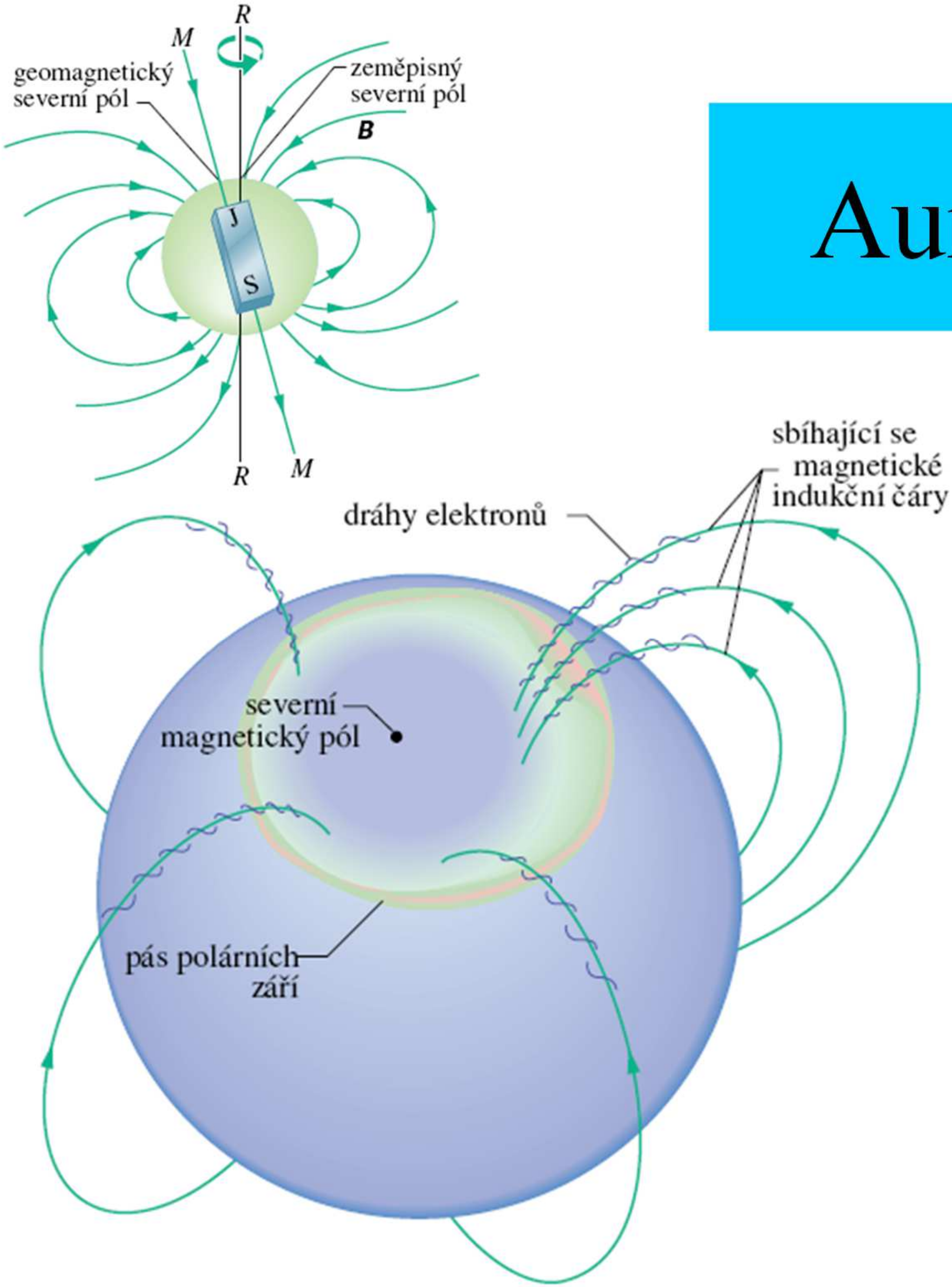
Movement of charged particles in magnetic field

Lorentz's force does not change magnitude, but the direction of velocity!









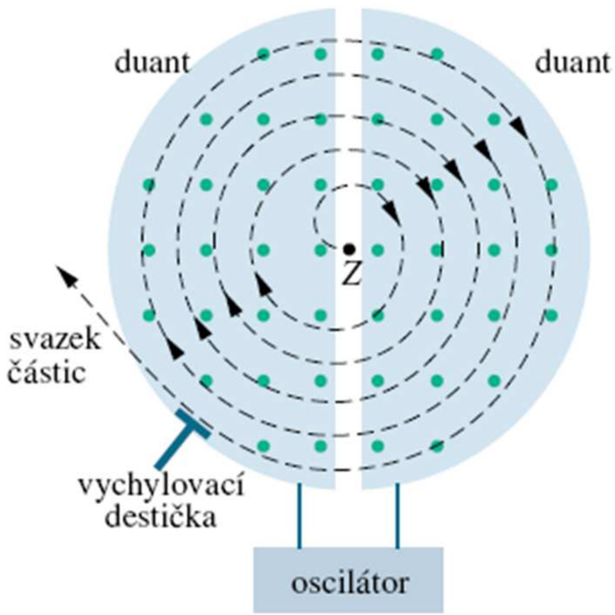
Aurora Borealis

Effect related to the concentration of electrically charges particles near the Earth magnetic poles

Collisions of oxygen (green light) and nitrogen molecules (pink light) with charged particles

Particle accelerators

Cyclotron – particle is accelerated by an electric field and its trajectory is shaped by magnetic field

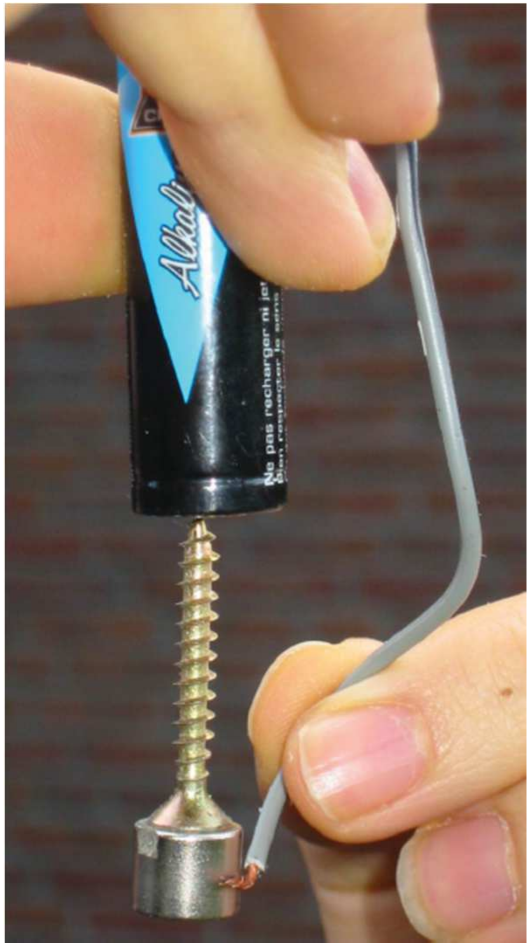


$$QvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{QB} \qquad f = \frac{1}{T} = \frac{QB}{2\pi m}$$

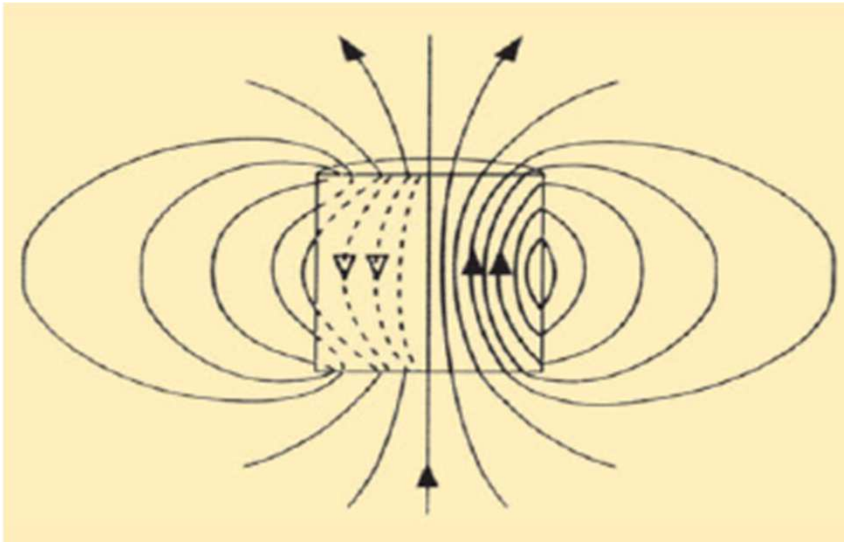
$$QB = 2\pi m f_{osc}$$

Simple motor



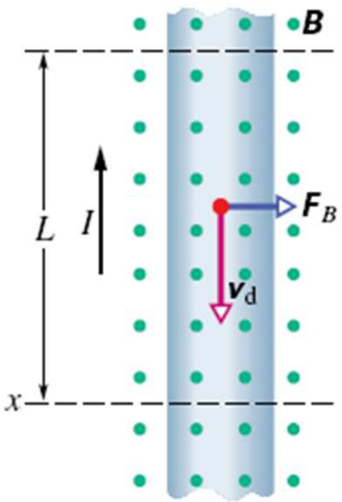
Movement due to the reaction of Lorentz's force on moving charge carriers

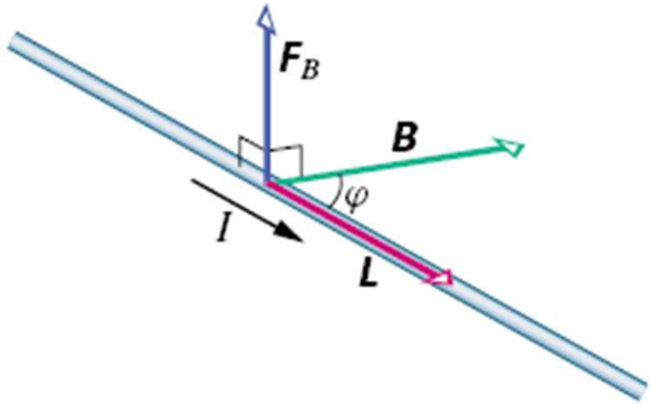
Phys. Unserer Zeit **35**, 6 (2004) 272-273



Force on current carrying wire in magnetic field

Force on moving charges





$$Q = It = \frac{IL}{v_d}$$

$$F_B = Q v_d B \sin\varphi = \\ = \frac{IL}{v_d} v_d B \sin 90^\circ$$

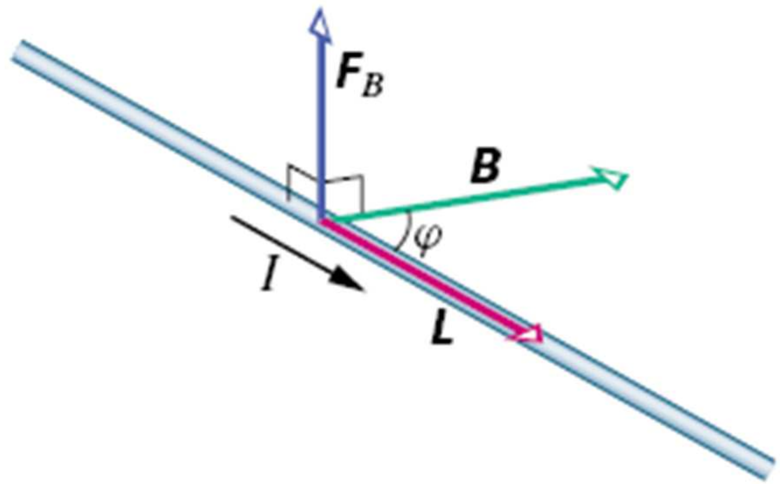
$$F_B = ILB$$

$$\mathbf{F}_B = I\mathbf{L} \times \mathbf{B}$$

$$d\mathbf{F}_B = I\, d\mathbf{s} \times \mathbf{B}$$

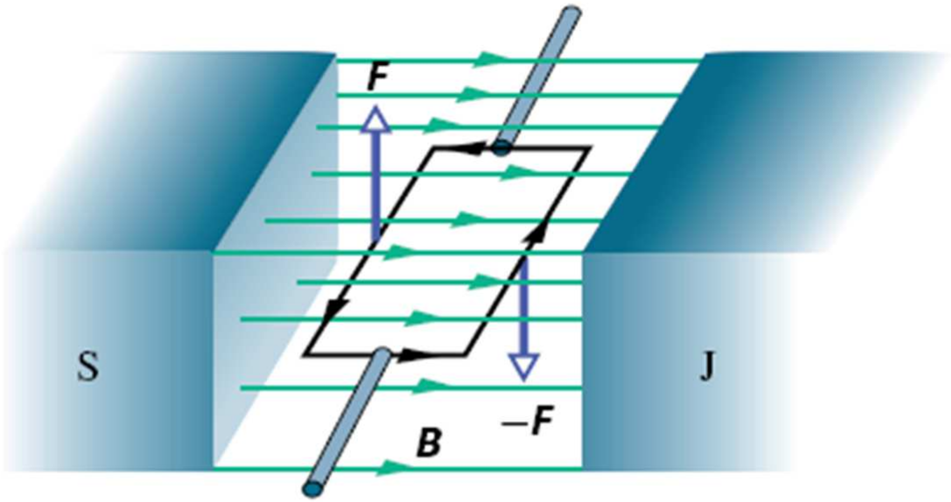
Fleming's rule

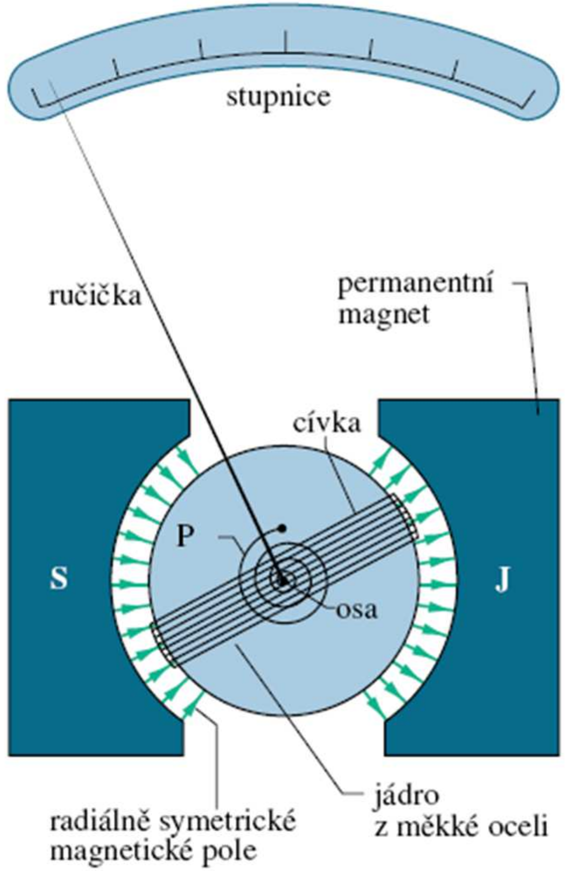
When current flows through a conducting wire, and an external magnetic field is applied across that flow, the conducting wire experiences a force perpendicular both to that field and to the direction of the current flow (i.e they are mutually perpendicular).



Torque on a current loop in magnetic field

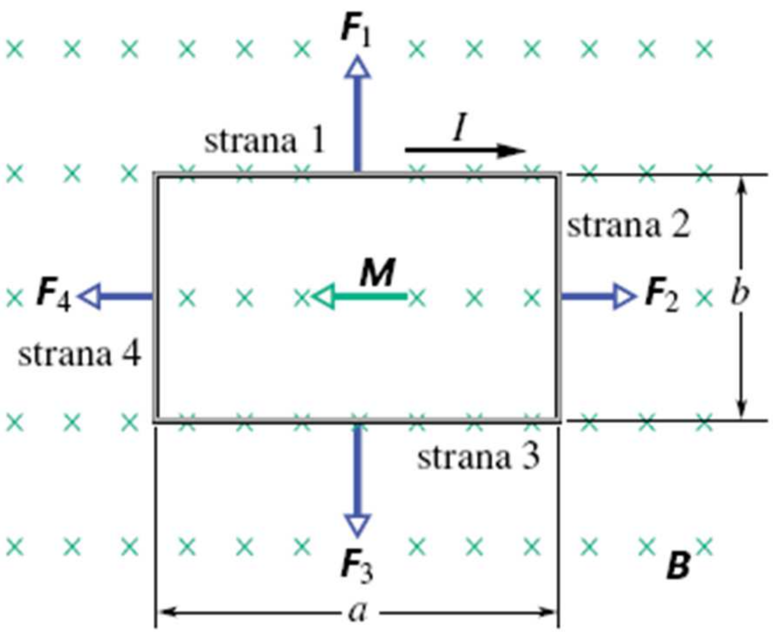
Torque – bais for motors and measurement systems (Galvanometer)

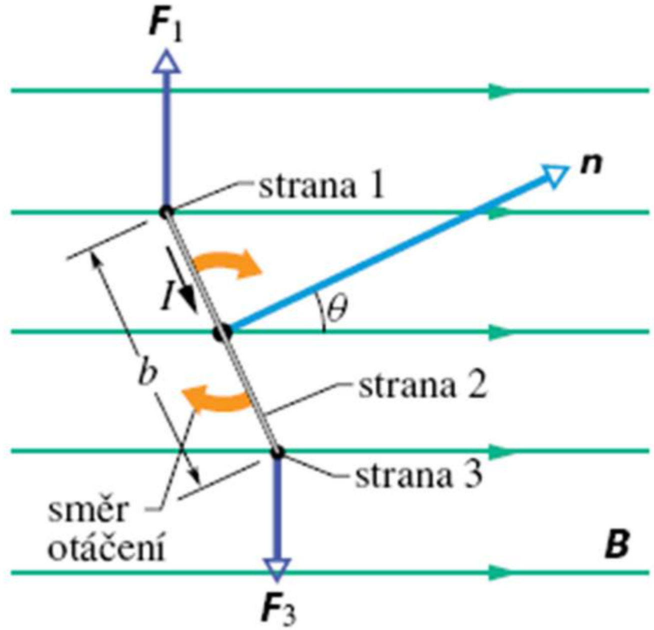




Current loop in magnetic field

Pair of forces





$$M' = \left(IaB\frac{b}{2}\sin\theta\right) + \left(IaB\frac{b}{2}\sin\theta\right) =$$

$$= IabB\sin\theta.$$

$$M = NM' = NIabB\sin\theta = (NIS)B\sin\theta$$

Magnetic dipole moment of loop

Dipole moment $\mu = NIS$

Normal to the loop area

Torque on the loop $M = \mu B \sin\theta$

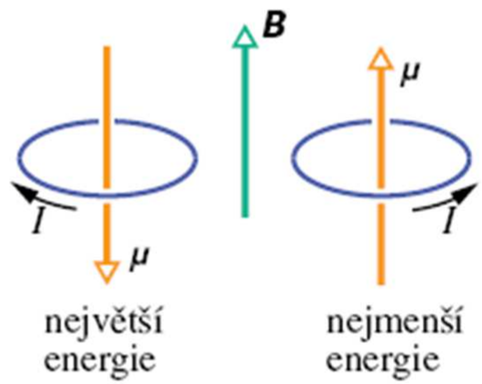
$$\boldsymbol{M} = \boldsymbol{\mu} \times \boldsymbol{B}$$

Energy of the dipole in magnetic field

$$E_p(\theta) = -\boldsymbol{\mu} \cdot \boldsymbol{B}$$

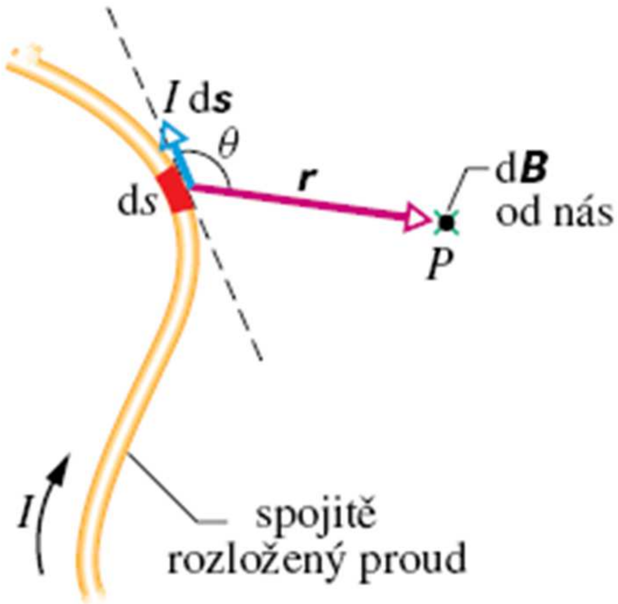
Maximum x minimum change

$$\Delta E_p = (+\mu B) - (-\mu B) = 2\mu B$$



Magnetic field of current carrying wire

Magnetic field generated by the current in the wire



Biot-Savart's law

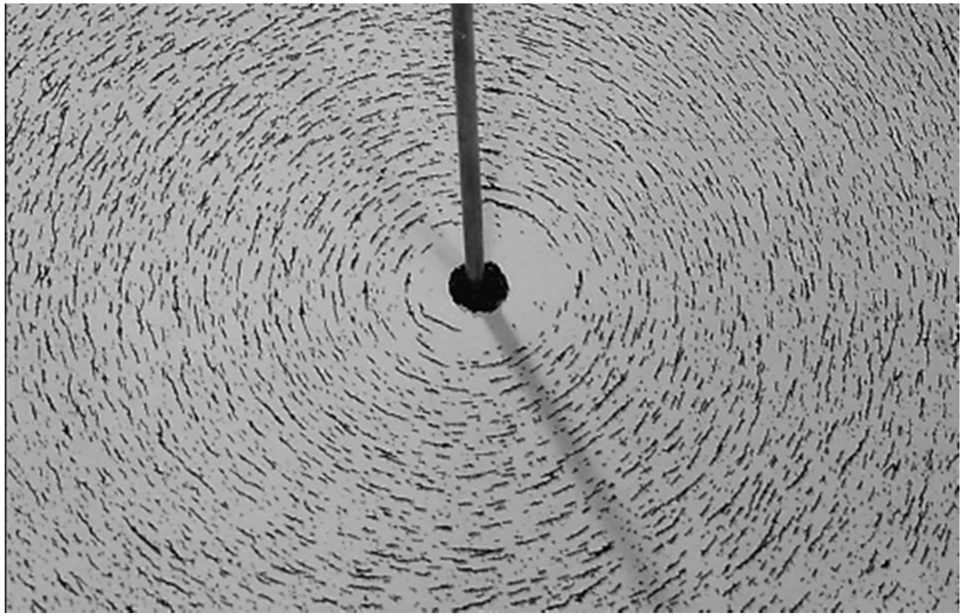
$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\mathbf{s}\times\mathbf{r}}{r^3}$$

permeability of free space

$\mu_0 = 4\pi\cdot 10^{-7}\,\mathrm{Hm}^{-1}$

Magnetic field of long straight wire

Magnetic field in the vicinity of current carrying wire



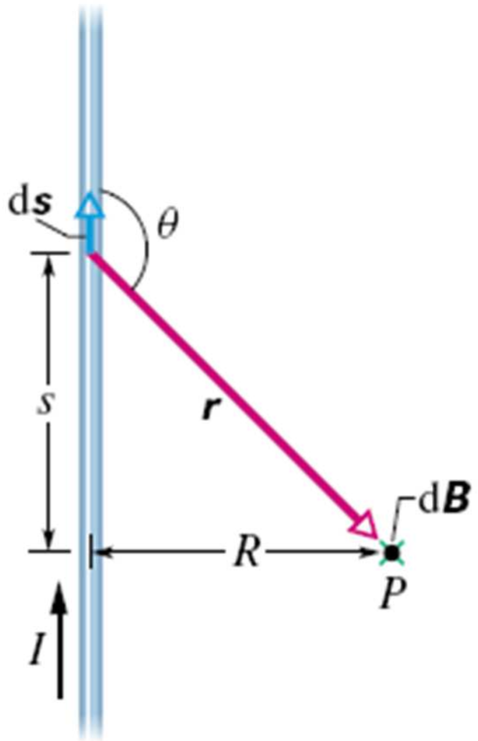
integration from Biot-Savart's law

$$B = \frac{\mu_0 I}{2\pi R}$$

Calculation of magnetic field for the long straight wire

Biot-Savart's law

$$dB = \frac{\mu_0}{4\pi}\frac{I\,ds\,\sin\theta}{r^2}$$



$$r = \sqrt{s^2 + R^2},$$

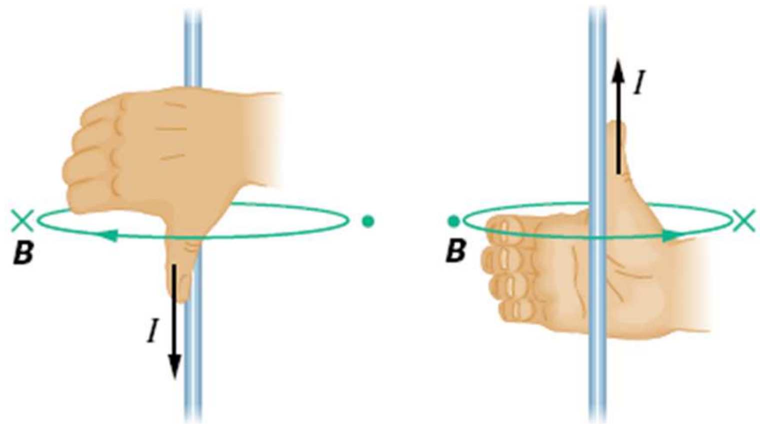
$$\sin\theta = \sin(\pi - \theta) = \frac{R}{\sqrt{s^2 + R^2}}$$

$$B = \frac{\mu_0 I}{2\pi}\int_0^\infty \frac{R\,ds}{(s^2 + R^2)^{3/2}} =$$

$$= \frac{\mu_0 I}{2\pi R}\left[\frac{s}{(s^2 + R^2)^{1/2}}\right]_0^\infty = \frac{\mu_0 I}{2\pi R}$$

Orientation of magnetic field

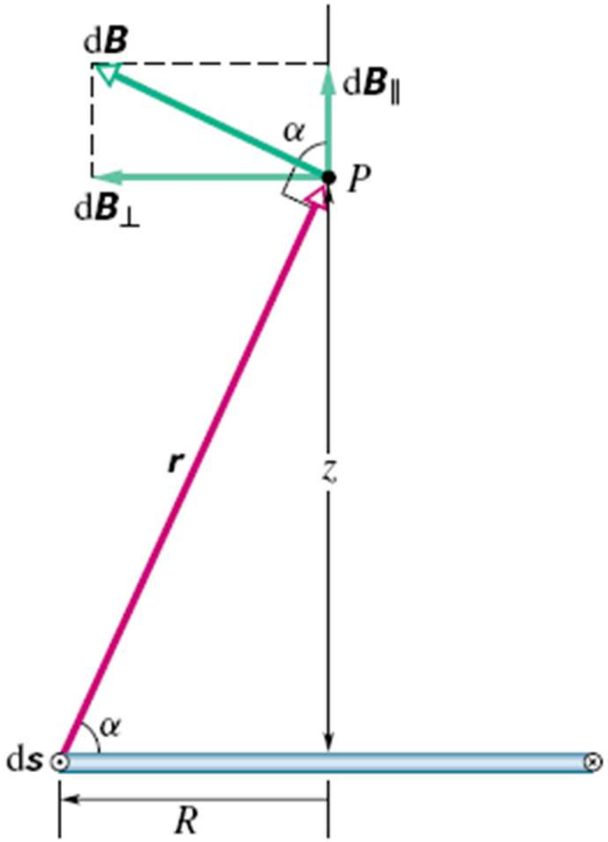
Orientation by right hand rule



Wrap fingers around wire with thumb pointing in direction of current – fingers curl in direction of magnetic field.

Magnetic field of loop

On-axis field of the loop with current



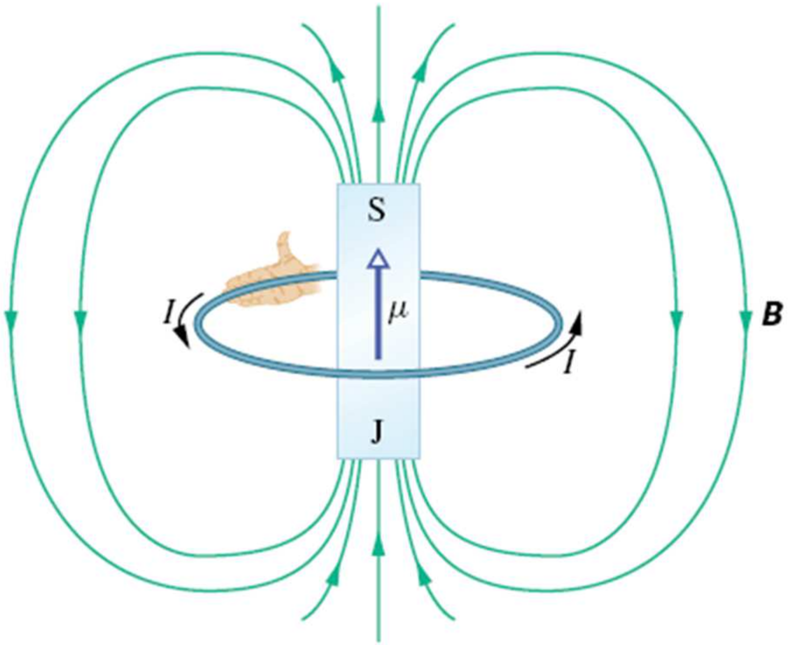
$$B(z) = \frac{\mu_0 I R^2}{2(R^2 + z^2)^{3/2}}$$

at the center of loop

$$B = \frac{\mu_0 I}{2R}$$

Magnetic field of dipole

Magnetic field of solenoid is the same as magnetic field of dipole



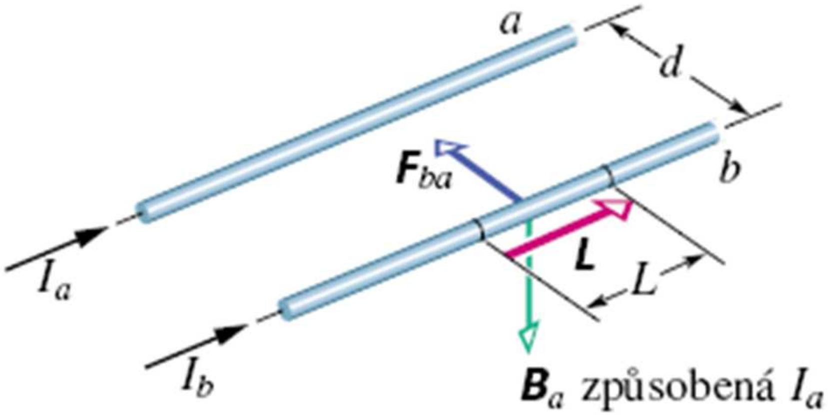
$$\mathbf{B}(z) = \frac{\mu_0}{2\pi} \frac{\boldsymbol{\mu}}{z^3}$$

magnetic dipole moment of loop

$$\mu = NIS$$

Magnetic force between two parallel wires

Magnetic field of the first wire



$$B_a = \frac{\mu_0 I_a}{2\pi d}$$

$$\mathbf{F}_{ba} = I_b \mathbf{L} \times \mathbf{B}_a$$

$$F_{ba} = I_b L B_a \sin 90^\circ = \frac{\mu_0 L I_a I_b}{2\pi d}$$

Parallel currents exert an attractive force, antiparallel currents repulsive one.

Unit Ampér in SI

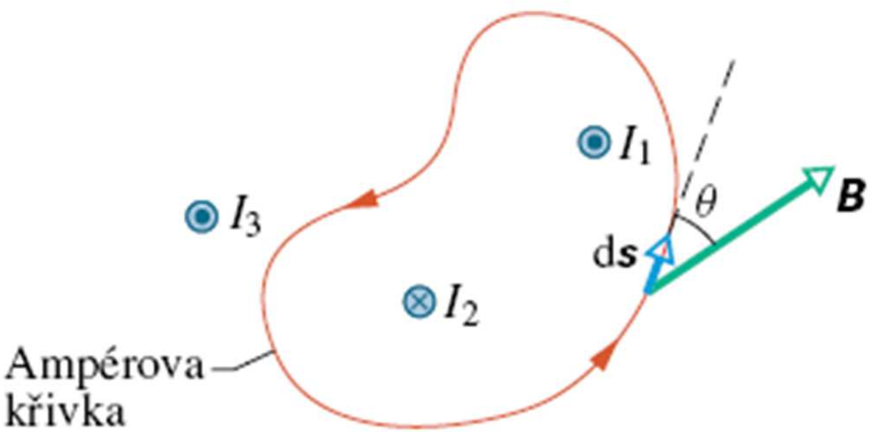
Magnetic force might be used in definition of 1 Ampère unit for electric current.

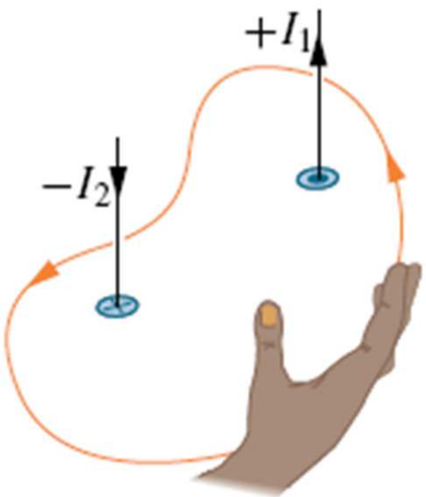
1 Ampère is defined as that current flowing in each of two long parallel wires, 1m apart, which results in a force of exactly $2 \cdot 10^{-7}$ N per meter of length of each wire.

Ampère's law

$$\oint \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{s} = \mu_0 I_c$$

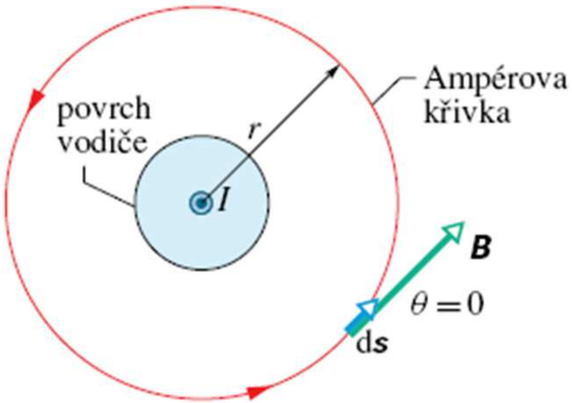
Sign of flowing currents – right hand rule





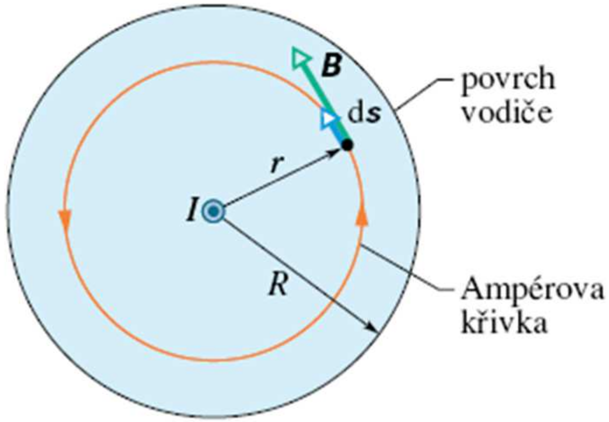
Magnetic field
inside and outside of wire

- Inside



$$B = \frac{\mu_0 I}{2\pi r}$$

- Outside



$$I_c = I\frac{\pi r^2}{\pi R^2}$$

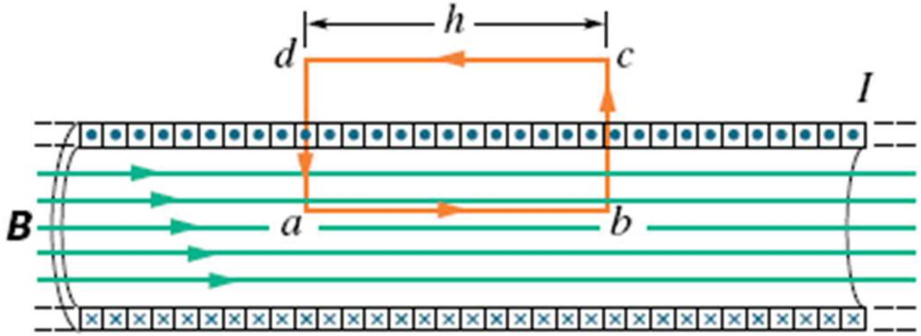
$$B = \left(\frac{\mu_0 I}{2\pi R^2}\right) r$$

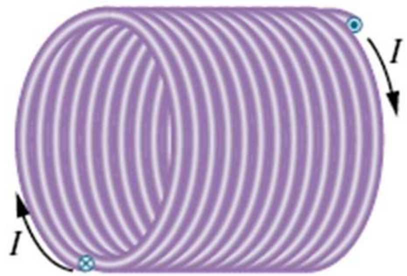
Magnetic field of solenoid

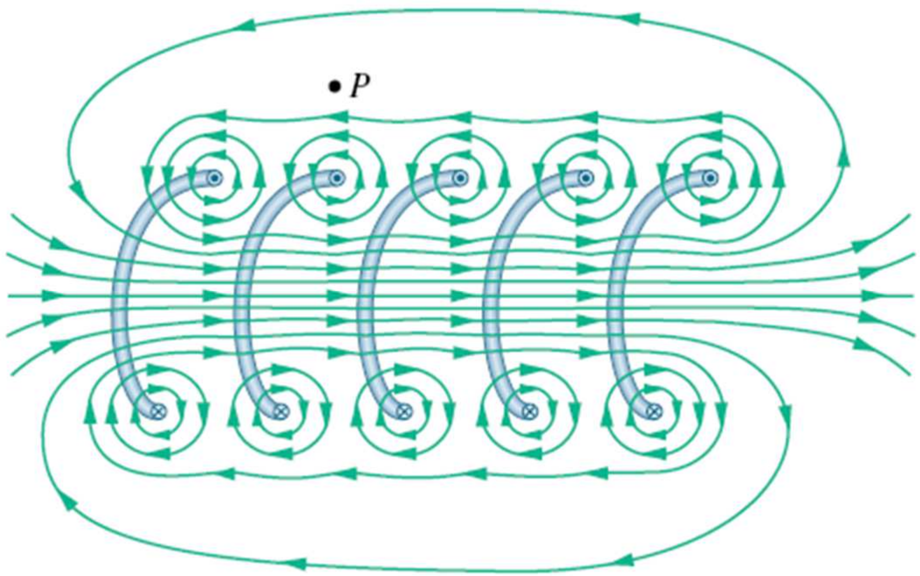
Solenoid = coil made from isolated wire

Ampère’s law

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_c$$







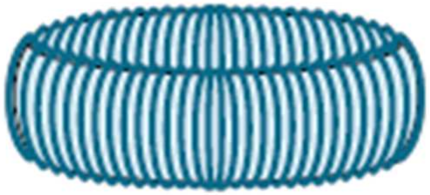
$$Bh = \mu_0 I n h$$

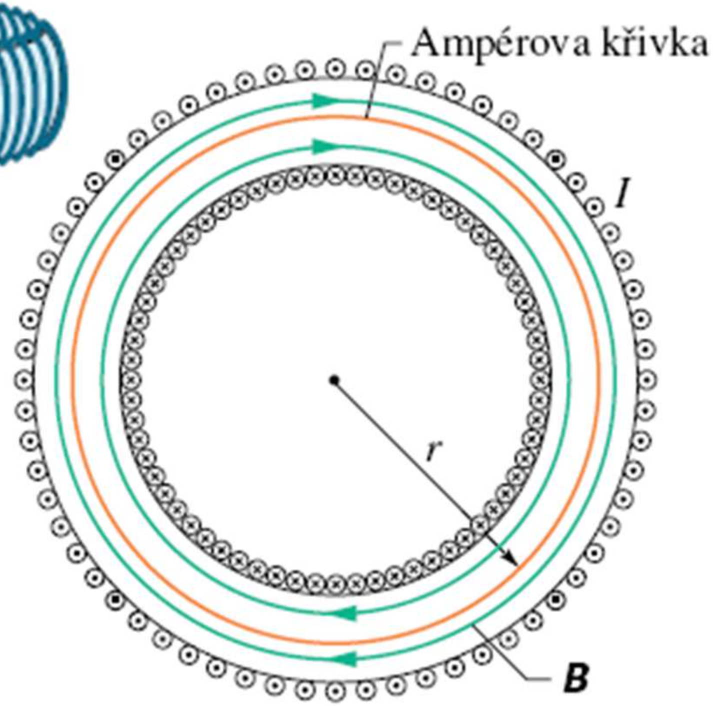
$$B = \mu_0 I n$$

Magnetic feld of toroidal coil

Toroidal coil = wires wound on the ring core

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_c$$



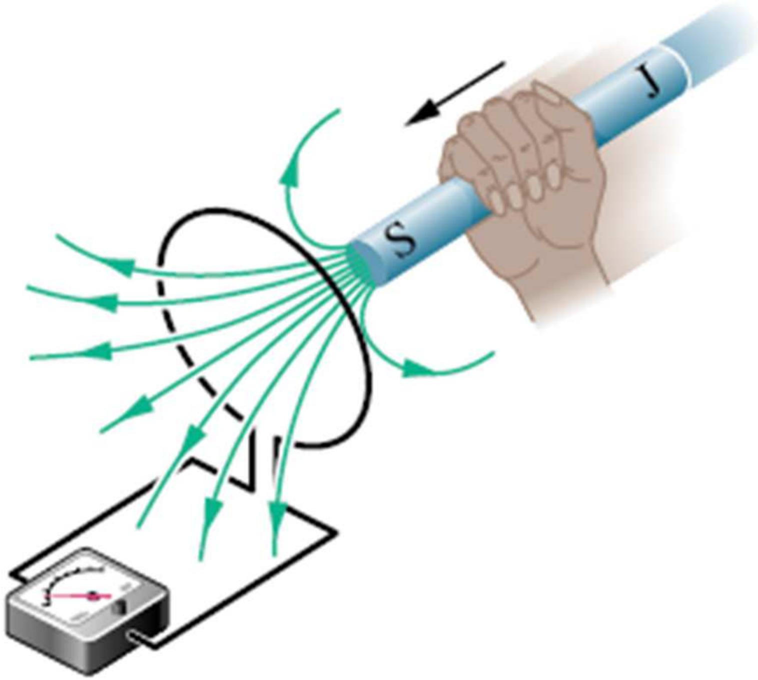


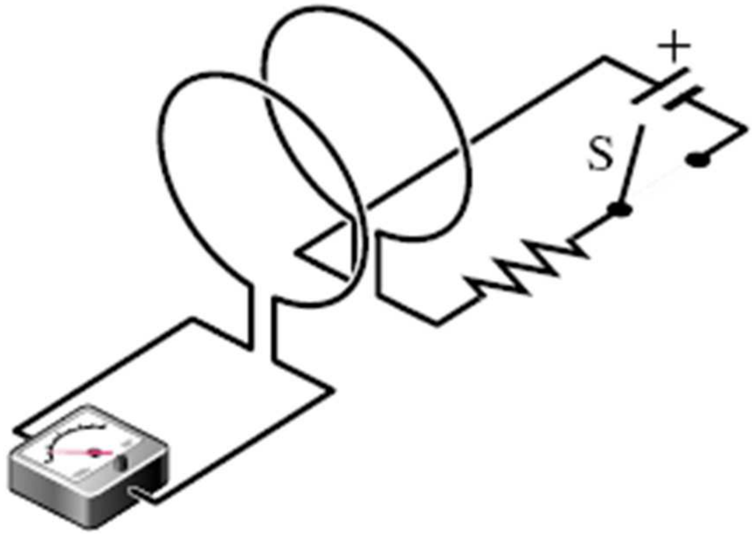
$$B(2\pi r) = \mu_0 I N.$$

$$B = \frac{\mu_0 I N}{2\pi} \frac{1}{r}$$

Electromagnetic induction

M.Faraday – change of magnetic field → EMF





Faraday's law of induction

Magnetic flux

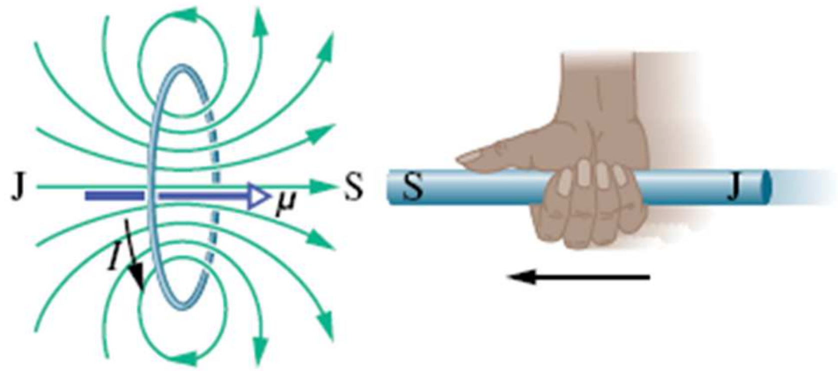
$$\Phi_B = \int_{\mathscr{S}} \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{S}$$

Induced EMF is proportional to the change of magnetic flux

$$\mathscr{E} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$$

Lenz’s rule

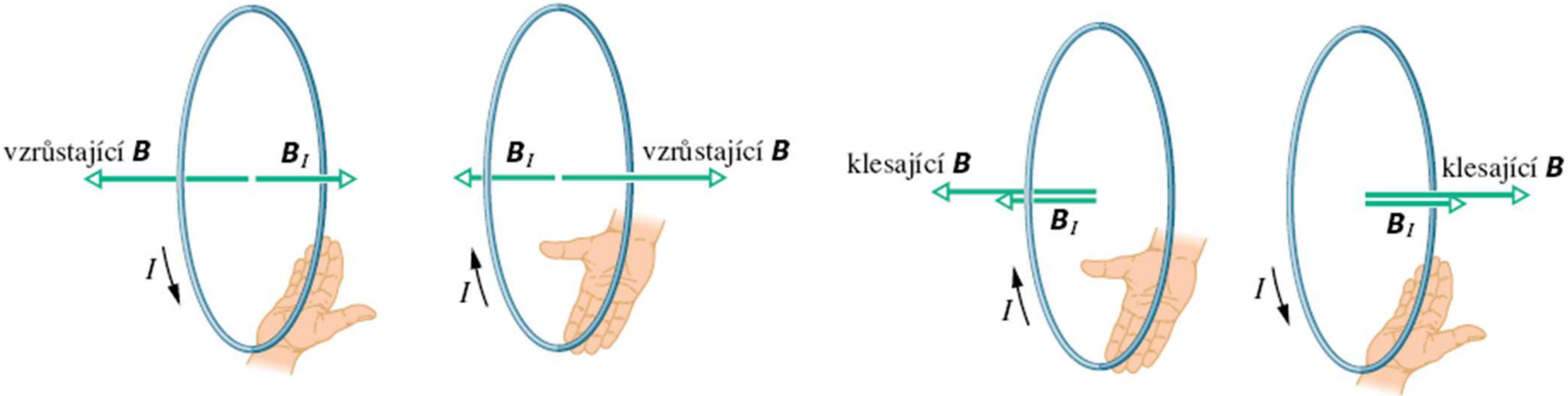
Orientation of induced current



An induced emf is always in a direction that opposes the original change in flux that caused it.

Lenz's rule

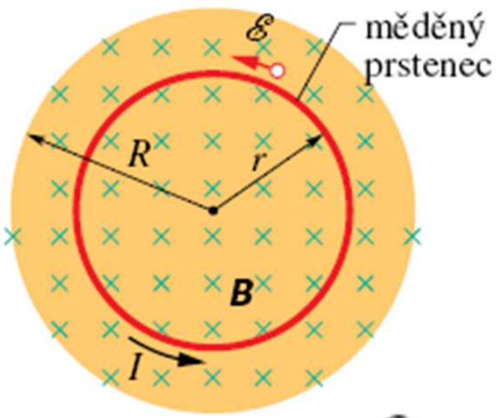
Change of magnetic field

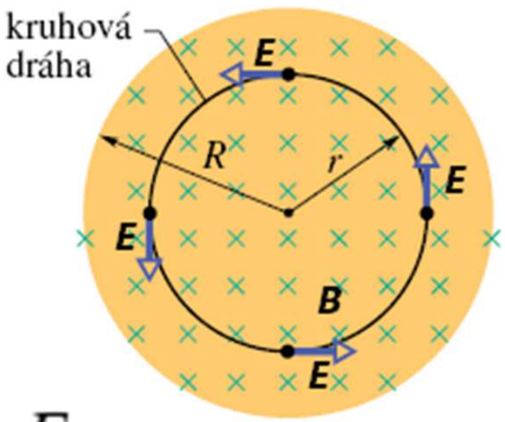


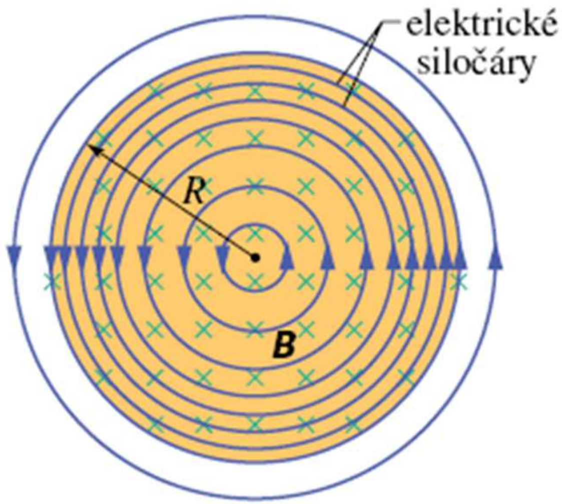
Current produced by an induced emf moves in a direction so that the magnetic field created by that current opposes the original change in flux.

Induced electric field

Electromagnetic induction gives rise to electric field with closed field lines!







$$\mathscr{E} = 2\pi r E$$

Faraday's law and induced voltage

Induced electric voltage

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{s}$$

No potential exists for an induced electric field!

Inductance of coil

Magnetic flux in coil

$$\Phi = N\Phi_{\mathrm{B}}$$

(Self) inductance of coil L[H] – unit Henry

$$N\Phi_B = LI$$

$$L = \frac{N\Phi_B}{I}$$

Inductance of solenoid

Tightly wound turns of wire

$n = N/l$ … density of turns

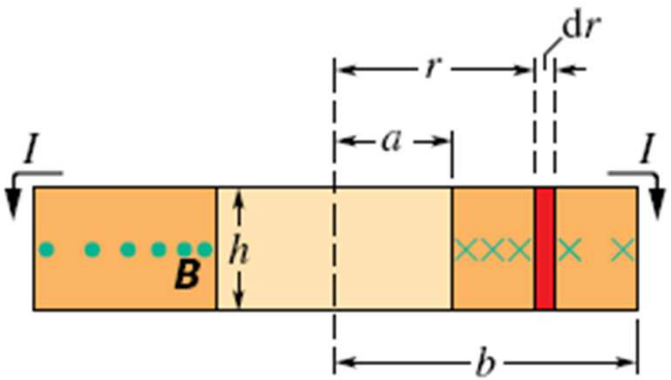
$$N\Phi_B = (nl)(BS)$$

$$L = \frac{N\Phi_B}{I} \qquad B = \mu_0 I n$$

$$\frac{L}{l} = \mu_0 n^2 S$$

Inductance of toroidal coil

Integration through the cross-section



$$\Phi_B = \int_a^b Bh\,\mathrm{d}r = \int_a^b \frac{\mu_0 I N}{2\pi r} h\,\mathrm{d}r =$$

$$= \frac{\mu_0 I N h}{2\pi} \int_a^b \frac{\mathrm{d}r}{r} = \frac{\mu_0 I N h}{2\pi} \ln\frac{b}{a}$$

$$L = \frac{N\Phi_B}{I} = \frac{N}{I}\frac{\mu_0 I N h}{2\pi} \ln\frac{b}{a}$$

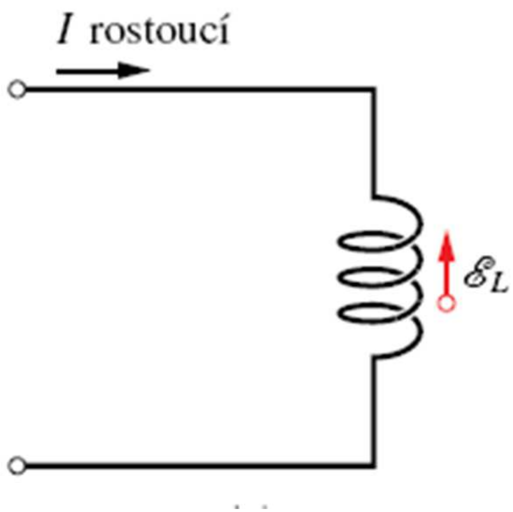
$$L = \frac{\mu_0 N^2 h}{2\pi} \ln\frac{b}{a}$$

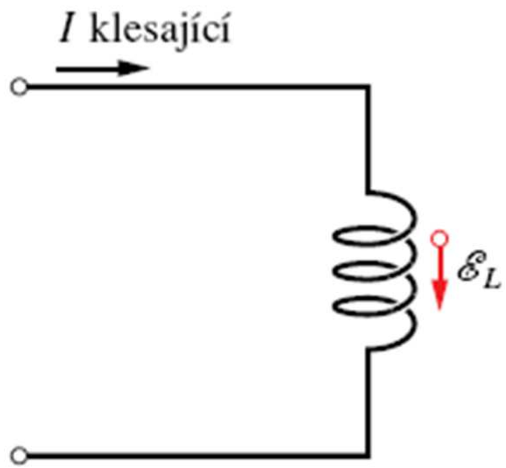
Induced EMF in coil

Any coil with changing current

$$\mathcal{E}_L = -\frac{\mathrm{d}(N\Phi_B)}{\mathrm{d}t}$$

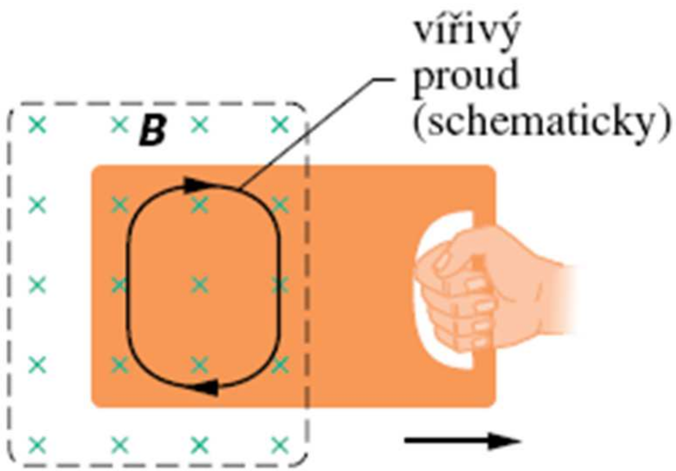
$$\mathcal{E}_L = -L\frac{\mathrm{d}I}{\mathrm{d}t}$$





Eddy currents

Induced by the change of magnetic field in solids – e.g. by the movement of permanent magnet in the vicinity of conductor



Aplication to electromagnetic brakes, induction heating etc.

Energy of magnetic field

Voltage in coil $$\mathscr{E} = L\frac{dI}{dt} + IR$$

Power in coil

$$\mathscr{E}I = LI\frac{dI}{dt} + I^2R$$

Magnetic field energy

$$\frac{dE_{mg}}{dt} = LI\frac{dI}{dt}$$

$$E_{mg} = \frac{1}{2}LI^2$$

Density of magnetic field energy

Magnetic energy per volume unit – solenoid with cross-section *S*

$$w_{mg} = \frac{E_{mg}}{Sl}$$

$$E_{mg} = \tfrac{1}{2} L I^2$$

$$w_{mg} = \frac{LI^2}{2Sl} = \frac{L}{l}\frac{I^2}{2S}$$

$$\frac{L}{l} = \mu_0 n^2 S \qquad B = \mu_0 I n$$

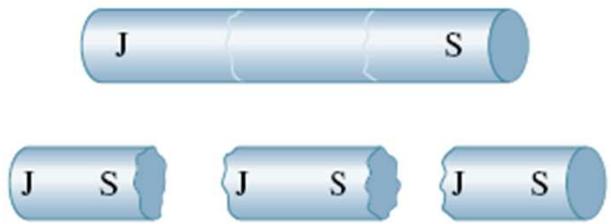
$$w_{mg} = \tfrac{1}{2}\mu_0 n^2 I^2$$

$$w_{mg} = \frac{1}{2}\frac{B^2}{\mu_0}$$

Magnetic field in matter

Magnetite Fe_3O_4, maghemite Fe_2O_3 – natural magnetic minerals

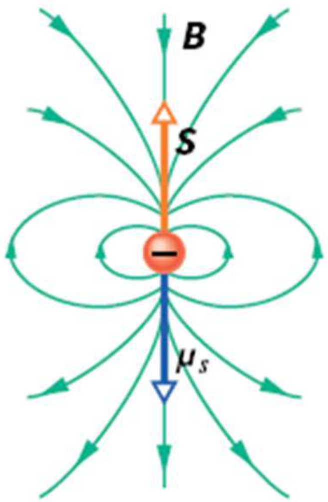
Dipole structure of permanent magnets



Magnetic dipole is basic structure in magnetism, monopoles do not exist

Magnetism in matter

- Paramagnetism
- Diamagnetism
- Ferromagnetism



spin magnetic moment of electron

$$\mu_s = -\frac{e}{m}\mathbf{S}.$$

$$S_z = m_s \hbar \quad \text{pro } m_s = \pm\tfrac{1}{2}$$

$$\mu_{s,z} = \pm\frac{e\hbar}{2m}$$

Diamagnetism

No restriction to the type of material, no internal dipole moments

Dipole moments in matter are generated by external magnetic field, they are antiparallel to external field. Diamagnetic solid is attracted from stronger to weaker external magnetic field (expelled from field).

Paramagnetism

No restriction to the type of material, weak internal dipole moments

Weak dipole moments are generated in paramagnetic matter by external field, they allign parallel to the external magnetic field.

Paramagnetic solid is attracted from weaker to stronger external magnetic field.

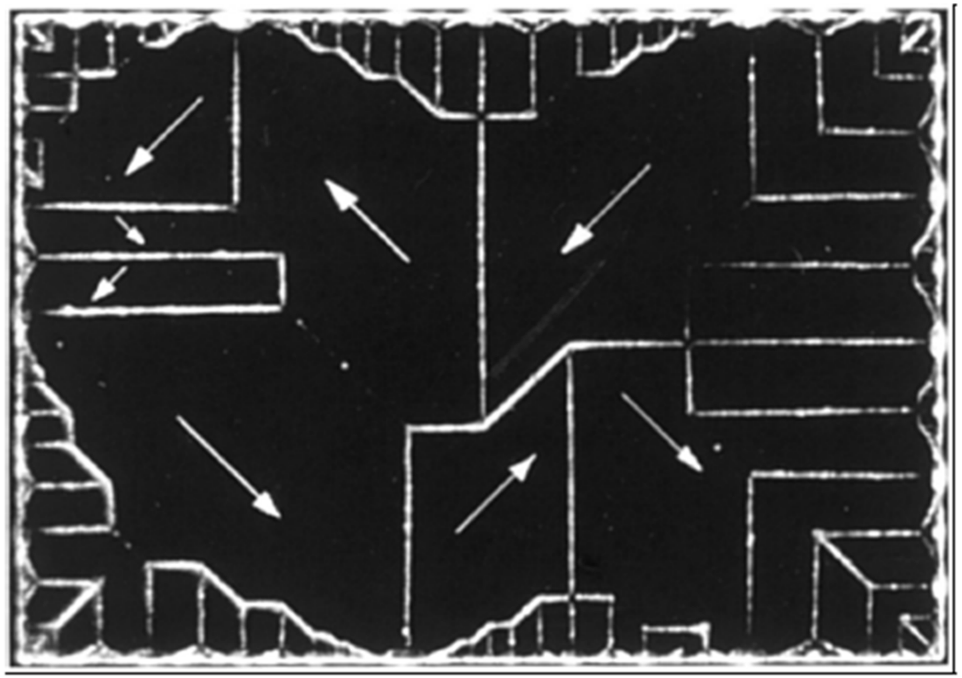
Ferromagnetism

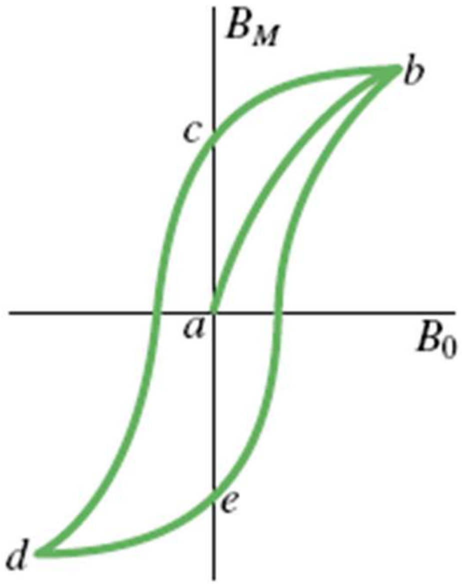
Limited to ferromagnetic materials, strong internal magnetic dipole moments

Strong magnetic dipole moments exist in ferromagnetic matter, strong alignment of dipole moments in external magnetic field. Ferromagnetic solid is attracted from weaker to stronger external magnetic field.

Properties of ferromagnetic materials

- Hysteresis – domain structure
- Curie temperature





Magnetic polarisation

Magnetic field in matter

$$B = B_0 + B_M$$

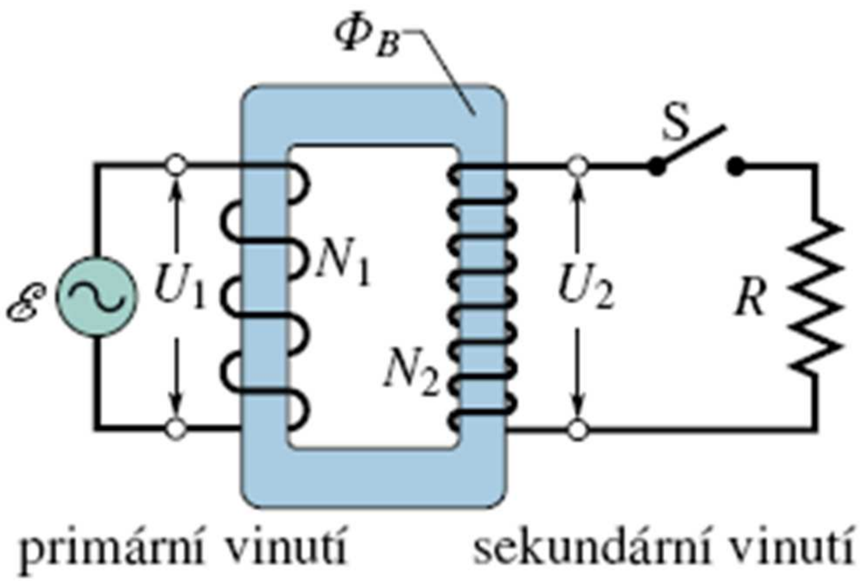
Magnetic intensity H [Am^{-1}]

$$B = \mu_0 H + B_M = \mu_r \mu_0 H$$

Relative permeability μ_r

Transformer

Transformation of voltage and current – minimization of Joule's heat losses in electric energy distribution!



electromagnetic induction – common magnetic flux in transformer core

$$\frac{U_2}{U_1} = \frac{N_2}{N_1}$$

$$\frac{I_2}{I_1} = \frac{N_1}{N_2}$$

Magnetoelectric induction

Symmetrical effect to electromagnetic induction

Faraday's law – electromagnetic induction

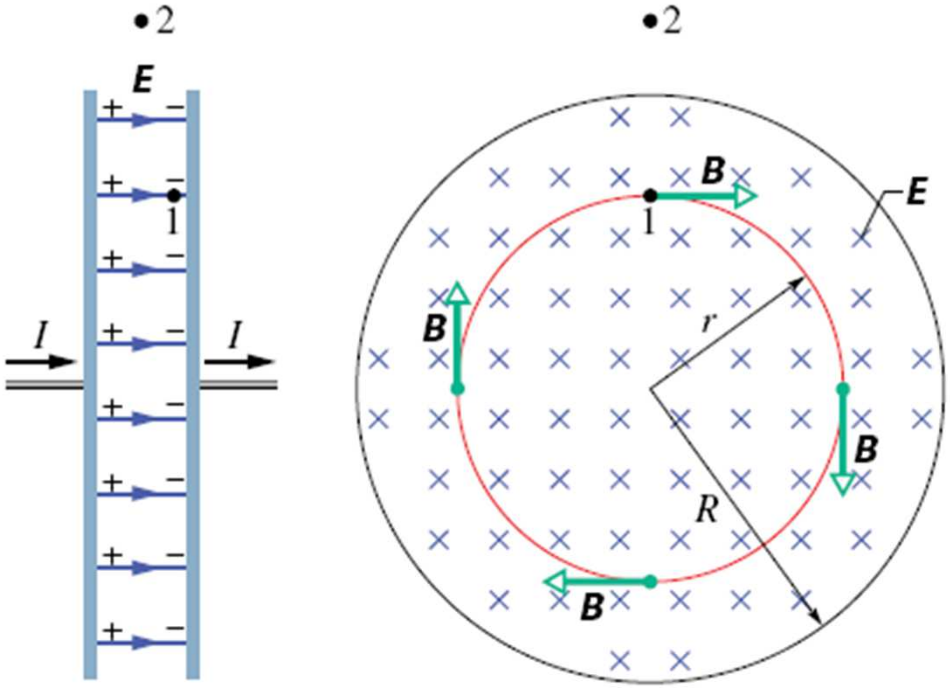
$$\oint E \cdot ds = -\frac{d\Phi_B}{dt}$$

Maxwell's law – magnetoelectric induction

$$\oint B \cdot ds = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Magnetoelectric induction

Ampère-Maxwell law – induction of magn. field



$$\frac{1}{\mu_0}\oint \boldsymbol{B}\cdot \mathrm{d}\boldsymbol{s} = \varepsilon_0 \frac{\mathrm{d}\Phi_E}{\mathrm{d}t} + I_\mathrm{c}$$

Maxwell's current

Electric flux $\Phi_E = \oint E \cdot dS$

Maxwell's current

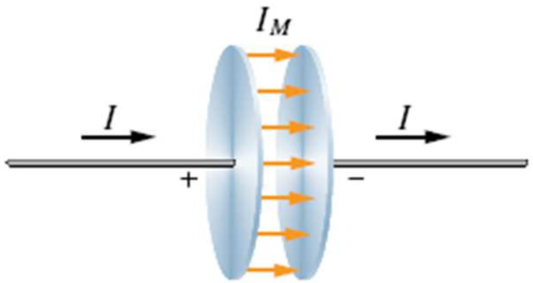
$$I_M = \varepsilon_0 \frac{\mathrm{d}\Phi_E}{\mathrm{d}t}$$

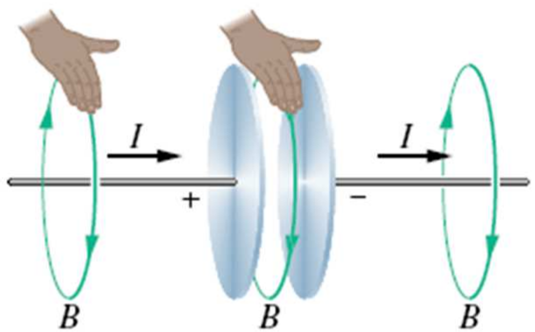
Ampère-Maxwell law

$$\oint \mathbf{B} \cdot \mathrm{d}\mathbf{s} = \mu_0 (I_{M,\mathrm{c}} + I_\mathrm{c})$$

Orientation of Maxwell's current

In homogeneous field in capacitor





$$I_M = \varepsilon_0 \frac{\mathrm{d}\Phi_E}{\mathrm{d}t} = \varepsilon_0 \frac{\mathrm{d}(ES)}{\mathrm{d}t} = \varepsilon_0 S \frac{\mathrm{d}E}{\mathrm{d}t}$$

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = I = \varepsilon_0 S \frac{\mathrm{d}E}{\mathrm{d}t}$$

$$I_M = I$$

Maxwell's current might be understood as a continuation of current *I* inside capacitor

Maxwell equations
- integral form

Gauss law

$$\oint \mathbf{E} \cdot \mathrm{d}\mathbf{S} = \frac{Q}{\varepsilon_0}$$

$$\oint \mathbf{B} \cdot \mathrm{d}\mathbf{S} = 0$$

Faraday's law

$$\oint \mathbf{E} \cdot \mathrm{d}\mathbf{s} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} \qquad \Phi_B = \int \mathbf{B} \cdot \mathrm{d}\mathbf{S}$$

Ampère-Maxwell law

$$\oint \mathbf{B} \cdot \mathrm{d}\mathbf{s} = \mu_0 \left(\varepsilon_0 \frac{\mathrm{d}\Phi_E}{\mathrm{d}t} + I_c \right) \qquad \Phi_E = \int \mathbf{E} \cdot \mathrm{d}\mathbf{S}$$

Maxwell equations - differential form

$$div\,E = \frac{\rho}{\varepsilon_0}$$

$$div\,B = 0$$

$$rot\,E + \frac{\partial B}{\partial t} = 0$$

$$rot\,B - \varepsilon_0 \mu_0 \frac{\partial E}{\partial t} = \mu_0 j$$

density of free charge ρ

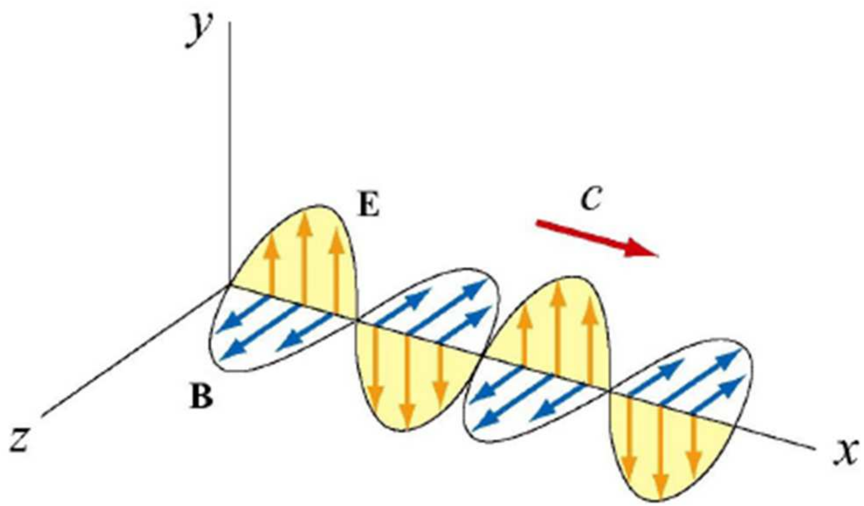
density of conductivity current j

$$divE = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$rotE = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}, \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}, \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$$

Electromagnetic waves

Maxwell equations allow for transversal planar wave



$$E_y(x,t) = E_0 \cos\left[k(x - vt)\right]$$

$$B_z(x,t) = B_0 \cos\left[k(x - vt)\right]$$

$$k = \frac{2\pi}{\lambda}$$

$$\omega = kv = 2\pi\frac{v}{\lambda} = 2\pi f$$

$$\boxed{\frac{E_0}{B_0} = \frac{\omega}{k} = c}$$

Vacuum - phase velocity equal to *c*

Properties of electromagnetic waves

Waves are transversal, *E* and *B* are perpendicular vectors, wave is propagated in the direction of *E*x*B* vector

Ratio of amplitudes

$$\frac{E}{B} = \frac{E_0}{B_0} = \frac{\omega}{k} = c$$

Wave phase velocity $c = 1/\sqrt{\varepsilon_0 \mu_0}$

Superposition principle.

Literature

Pictures used from the book:

HALLIDAY, D., RESNICK, R., WALKER, J.: Fyzika (část 3 – Elektřina a magnetismus), Vutium, Brno 2000