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Evalution of variation solution and selection the best solution

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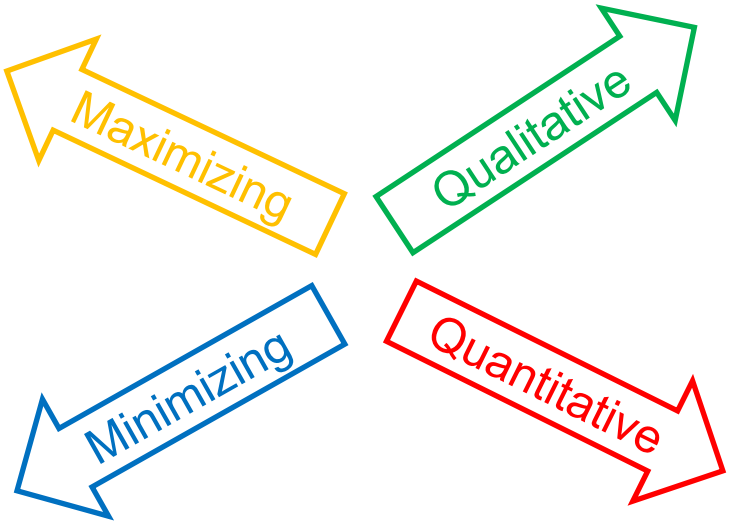
Deparment of textile and single-purpose machines

Do you know some form of procedures how evaluate variants of solutions?

Multicriterial decision

When choosing the optimum solution we encounter that the result must meet more criteria. These criteria may be of the following nature:

- **Quantitative**
- **Qualitative**
- **Maximizing**
- **Minimizing**



https://www.youtube.com/watch?v=7OoKJHvsUbo

Base terms

- **Decision** - *selection of one or more variants from the set of all proposed options*
- **Decision maker –** *the person to make the decision*
- **Variants –** *group of proposed variants (A_i)*
- **Criterions –** *terms of assessment of variants (K_j)*
- **Criterial matrix** *– the elements of this matrix express the evaluation of the i^{th} variant by the j^{th} criterion*
- **Classification of criteria –** *according to the disposition and quantifiability*
- **Preference of Criteria –** *importance of criteria*
- **Variants with special properties –** *dominant, ideal, compromise ……*
- **Properties of the compromise variant**

Methods for determining the criteria weights:

The criterion weight defines the significance of the criterion.

$\boldsymbol{v_j}$ …..$weight\ jth\ criterion$

$\boldsymbol{w_j}$ …..$norm\ weight\ of\ the\ criterion$

$$\boldsymbol{w_j} = \frac{v_j}{\sum_{k=1}^{n} v_k}, where\ j = 1,2\ \ldots\ldots, n$$

The sum of the norm weights is equal to 1

Classification of methods for determining the weight of the criteria:

- Method in order of criterias
- Fuller method
- Method of scoring
- Method of quantitative pairwise comparison **„Saaty method“**
- Method of progresive organizing weighting

Method in order of criterias:

The referee orders the criteria $K_1, K_2, \ldots K_n$ from the most important to the least significant and assigns their weight v_j to them. The following applies to the standardized scale:

$$w_j = \frac{v_j}{1+2+\cdots+n} \text{ , } j = 1,2,\ldots,n$$

Example 1:

Suggest a situation in which it is necessary to choose a solution based on a choice of several variants. For example, a product may be selected. For this example, define the criteria $K_1, K_2, \ldots .. K_n$ and their sequence. Assign weights v_1, $v_2, \ldots .. v_j$ to these criteria and calculate their normalized weight w_j.

Fuller method:

Method suitable for multiple criteria. It is based on comparing only two criteria, where we decide on the most important of this couple. We use the Fuller method for evaluation.

1	1	1	.....	1
2	3	4	.....	n
	2	2	.....	2
	3	4	.....	N
			.....	
			n-2	n-2
				n-1
				n

$$w_j = \frac{f_j}{\frac{n\cdot(n-1)}{2}} \ , \ j = 1,2,\ldots,n$$

Example 2:

According to example 1, calculate the weighting criteria using the Fuller method and compare them. Determine preferences for individual criteria (eg. $K_1 > K_3 > K_2 > K_4$). Assign weights v_1, v_2,… v_j to these criteria and calculate their normalized weight w_j.

Criterion	Number of preferences	Norm weight
K1	3	1/2
K2	1	1/6
K3	2	1/3
K4	0	0
Sum	6	1

Method of scoring:

We evaluate the importance of the criteria by the number of points. The more important the criterion, the more points it has. The range depends on the circumstances.

Example 3:

According to example 1, design the weighting criteria using the scoring method and convert to norm weights.

Criterion	Number of points	Norm weight
K1	50	0,5
K2	20	0,2
K3	25	0,25
K4	5	0,05
Sum	100	1

Saaty mehtod:

Method of quantitative pairwise comparison. The preference criterion also determines the size of this preference for each pair. Intermediate 2,4,6,8 can be used.

Expressing preferences	
Numeric	Verbal expression
1	Criteria equally significant
3	The first criterion is slightly more significant than the second
5	The first criterion is strongly more significant than the second
7	The first criterion is very much more significant than the second
9	The first criterion is absolutely more significant than the second

Example 4:

For the chosen example, assemble Saaty's matrix.

	Brand	Price	Colour	Size	Geometric mean	Norm weight
Brand						
Price						
Colour						
Size						

Example 4:

Saaty's matrix.

	Brand	Price	Colour	Size	Geometric mean	Norm weight
Brand	1	1/3	7	5	1,85	0,35
Price	3	1	5	3	2,59	0,48
Colour	1/7	1/5	1	1	0,41	0,07
Size	1/5	1/3	1	1	0,51	0,10

Gradual weighting method :

Method suitable for a large number of criteria. Individual criteria are grouped into subgroups according to kinship.

Steps:

1. Determination of standardized weight of individual groups
2. Determination of the standardized weight of each criterion in the relevant group
3. Multiplication of criteria group weights and individual criteria weights

Conclusion:

The weightings of the criteria are largely subjective. They depend on the method used and the assessor. The recommendation is to apply more methods and more evaluators for weighting, and to average these results.

Methods of definition numerical order of variants:

The aim of multicriterial evaluation methods is to determine the order of solution variants. There are different methods for which the results may vary, as this is largely a subjective method of assessment.

Conjunctive and disjunctive methods:

Information on the importance of the criteria is expressed by the expected level (aspirations). It serves to divide variants into acceptable and unacceptable variants. **Conjunctive** methods allow only variants that meet all expected levels. **Disjunctive** methods allow variants that meet at least one expected level.

Method PRIAM:

(Programe utilisant Intelligence Artificiele en Multicritere)

The method is based on a sequential search of a set of variants to find a single non-dominant solution. The basis is to determine the basic aspiration level that satisfies all variants. We gradually increase this aspiration level (for all criteria) and thus eliminate non-compliant variants. The result is the most advantageous variant.

Order method:

This method is based on converting the criterion matrix to an order matrix. We assign a serial number according to the criteria. Let's add these numbers. The first order is the variant with the lowest sum.

Example 5:

	K_1	K_2	K_3	K_4	Sum of Order	Order
A	1	3	2	1	7	1.
B	3	1	3	3	10	2.
C	2	2	1	2	7	1.

Scoring method:

In this method, the elector assigns to each element a certain number of points from the selected scale. The scoring scale is based on the assignment of points to certain value intervals evaluating the criterion. It is advisable to provide the scaling with verbal description. The result is the sum of the points of the variants.

Example 6:

Points	K1	K2	K3	K4
1	less than 2000 €	More then 50 min.	little option	within 7 o'clock
2	<2000; 3000) €	(30; 50>	medium option	(7; 8>
3	more then 3000 €	less than 30 min.	big option	after 8 o'clock

	K_1	K_2	K_3	K_4	Points	Order
A	2	3	1	1	2.05	1.
B	1	2	2	1	1.6	3.
C	1	3	1	1	1.65	2.
Norm Weight	0.4	0.3	0.25	0.05	-	-

The best option is the variant with the highest sum

Weighted sum method:

We can assign it´s value to each criterion value. Definition field of the function is interval between the best and the worst value of the relevant criterium. The range of functional values is the interval from 0 to 1. The worst values are assigned the value O, the best value is 1. We assume a linear dependence of the utility in the calculation of partial benefits.

Example 7:

$$u(A_i) = \sum_{j=1}^{n} w_j \cdot u_{ij}$$

Variant	K_1	K_2	K_3	K_4
A	20	40	5	10
B	25	35	3	8
C	30	45	2	9
h_j	30	45	5	10
d_j	20	35	2	8

	K_1	K_2	K_3	K_4	$u(A_i)$	Order
A	0	0.5	1	1	0,5	2.
B	0.5	0	0.33	0	0,33	3.
C	1	1	0	0.5	0,85	1.
Norm weight	0,5	0,3	0,25	0,1		

h_j … the best value
d_j … the worst value

Control questions:

- List at least 3 methods of determining weights of criteria.
- Explain one of these methods.
- State three methods of definition numerical order of variants.
- Explain one of these methods.

Topic of the next lecture:

„ Product lifecycle “

Thank You

Used literature and sources of information:

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