Užitečné integrály pro křivé pruty

$\int \sin^2 x \, dx$

Take the integral:

$$\int \sin^2(x) \, dx$$

Write $\sin^2(x)$ as $\frac{1}{2} - \frac{1}{2}\cos(2x)$:

$$= \int \left(\frac{1}{2} - \frac{1}{2}\cos(2x)\right) dx$$

Integrate the sum term by term and factor out constants:

$$= -\frac{1}{2}\int \cos(2x) \, dx + \frac{1}{2}\int 1 \, dx$$

For the integrand $\cos(2x)$, substitute $u = 2x$ and $du = 2\,dx$:

$$= -\frac{1}{4}\int \cos(u) \, du + \frac{1}{2}\int 1 \, dx$$

Out[▫]=

The integral of $\cos(u)$ is $\sin(u)$:

$$= -\frac{\sin(u)}{4} + \frac{1}{2}\int 1 \, dx$$

The integral of 1 is x:

$$= \frac{x}{2} - \frac{\sin(u)}{4} + \text{constant}$$

Substitute back for $u = 2x$:

$$= \frac{x}{2} - \frac{1}{4}\sin(2x) + \text{constant}$$

Which is equal to:

Answer:

$$= \frac{1}{2}(x - \sin(x)\cos(x)) + \text{constant}$$

$\int \cos^2 x \, dx$

Take the integral:

$$\int \cos^2(x) \, dx$$

Write $\cos^2(x)$ as $\frac{1}{2} \cos(2x) + \frac{1}{2}$:

$$= \int \left(\frac{1}{2} \cos(2x) + \frac{1}{2} \right) dx$$

Integrate the sum term by term and factor out constants:

$$= \frac{1}{2} \int \cos(2x) \, dx + \frac{1}{2} \int 1 \, dx$$

For the integrand $\cos(2x)$, substitute $u = 2x$ and $du = 2\,dx$:

$$= \frac{1}{4} \int \cos(u) \, du + \frac{1}{2} \int 1 \, dx$$

Out[•]=

The integral of $\cos(u)$ is $\sin(u)$:

$$= \frac{\sin(u)}{4} + \frac{1}{2} \int 1 \, dx$$

The integral of 1 is x:

$$= \frac{\sin(u)}{4} + \frac{x}{2} + \text{constant}$$

Substitute back for $u = 2x$:

$$= \frac{x}{2} + \frac{1}{4} \sin(2x) + \text{constant}$$

Which is equal to:

Answer:

$$= \frac{1}{2} (x + \sin(x) \cos(x)) + \text{constant}$$

$\int \sin x \cos x \, dx$

Take the integral:

$$\int \sin(x)\cos(x)\, dx$$

For the integrand $\sin(x)\cos(x)$, substitute $u = \cos(x)$ and $du = -\sin(x)\, dx$:

$$= -\int u \, du$$

Out[▫]= The integral of u is $\frac{u^2}{2}$:

$$= -\frac{u^2}{2} + \text{constant}$$

Substitute back for $u = \cos(x)$:

Answer:

$$= -\frac{1}{2}\cos^2(x) + \text{constant}$$