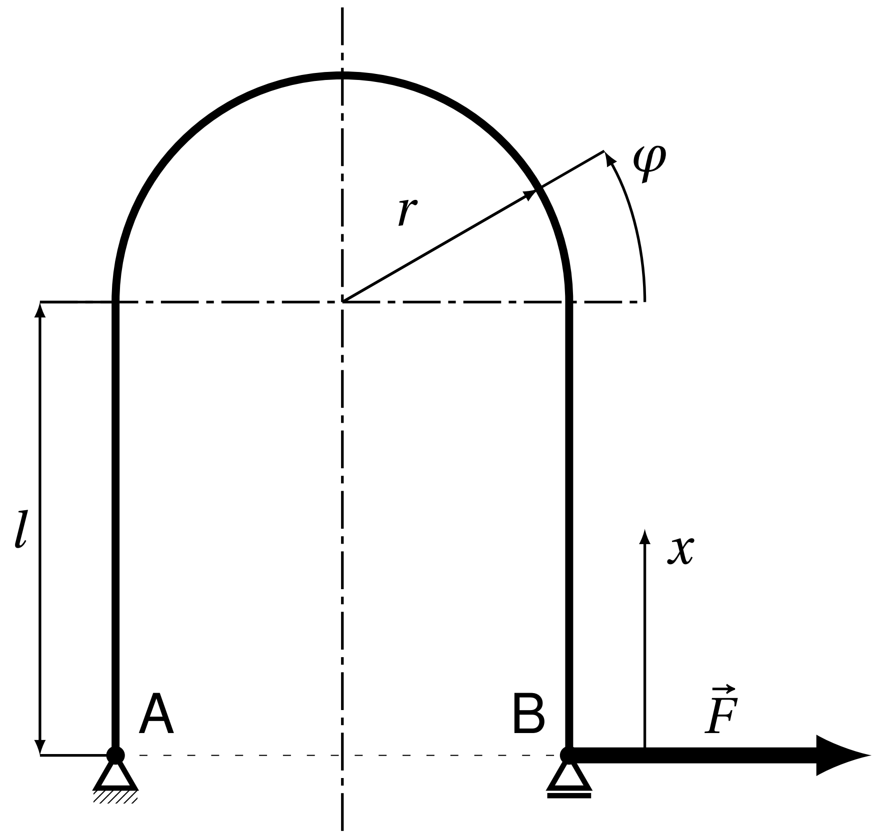
Příklad 1:



Symetrická konstrukce:

$$U = 2 \cdot U_{\frac{1}{2}}$$

$$U = 2 \cdot \int_0^l \frac{M_I^2(x)}{2 \cdot E \cdot J} dx + 2 \cdot \int_0^{\frac{\pi}{2}} \frac{M_{II}^2(\varphi)}{2 \cdot E \cdot J} \cdot r \cdot d\varphi$$

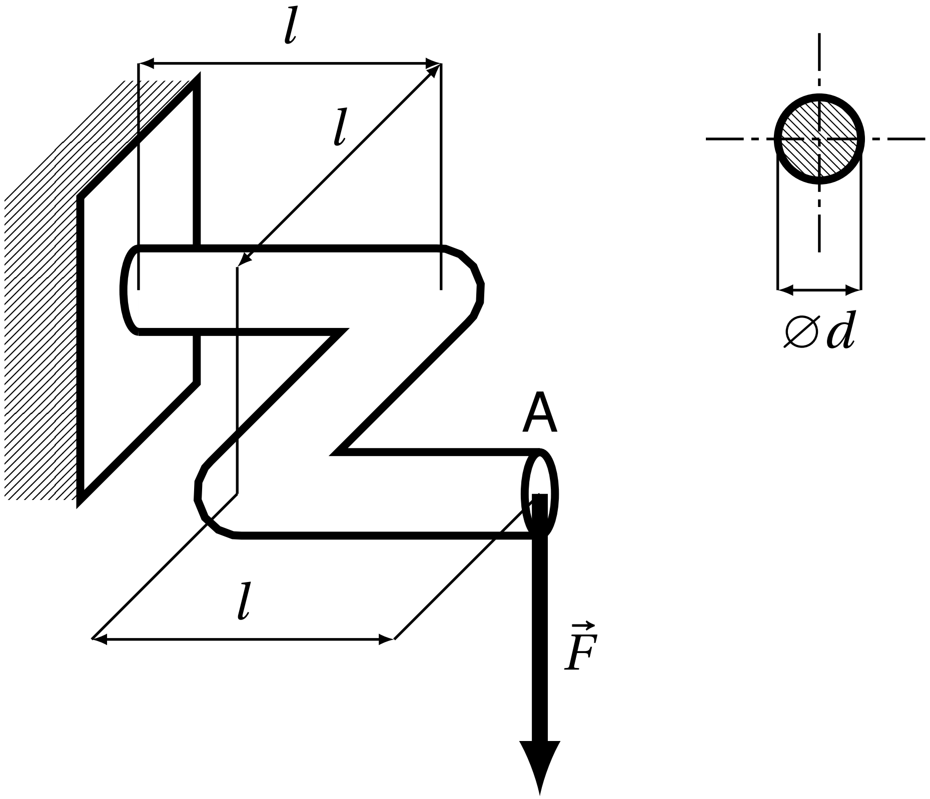
$$u_B = \frac{\partial U}{\partial F} = \frac{2}{E \cdot J} \cdot \left(\int_0^l M_I(x) \cdot \frac{\partial M_I}{\partial F} \cdot dx \right) + \frac{2}{E \cdot J} \cdot \left(\int_0^{\frac{\pi}{2}} M_{II}(\varphi) \cdot \frac{\partial M_{II}}{\partial F} \cdot r \cdot d\varphi \right)$$

$$M_I = F \cdot x \qquad \frac{\partial M_I}{\partial F} = x$$

$$M_{II} = F \cdot (l + r \cdot \sin\varphi) \qquad \frac{\partial M_{II}}{\partial F} = l + r \cdot \sin\varphi$$

$$u_B = \frac{2 \cdot F}{E \cdot J} \cdot \left(\frac{l^3}{3} + \frac{\pi}{2} \cdot r \cdot l^2 + \frac{\pi}{4} \cdot r^3 \right)$$

Příklad 2:



Silové účinky:

$$M_{o_I} = M_{o_{II}} = F \cdot x$$
$$M_{o_{III}} = F \cdot (l + x)$$
$$M_{k_I} = 0$$
$$M_{k_{II}} = F \cdot l = M_{k_{III}}$$

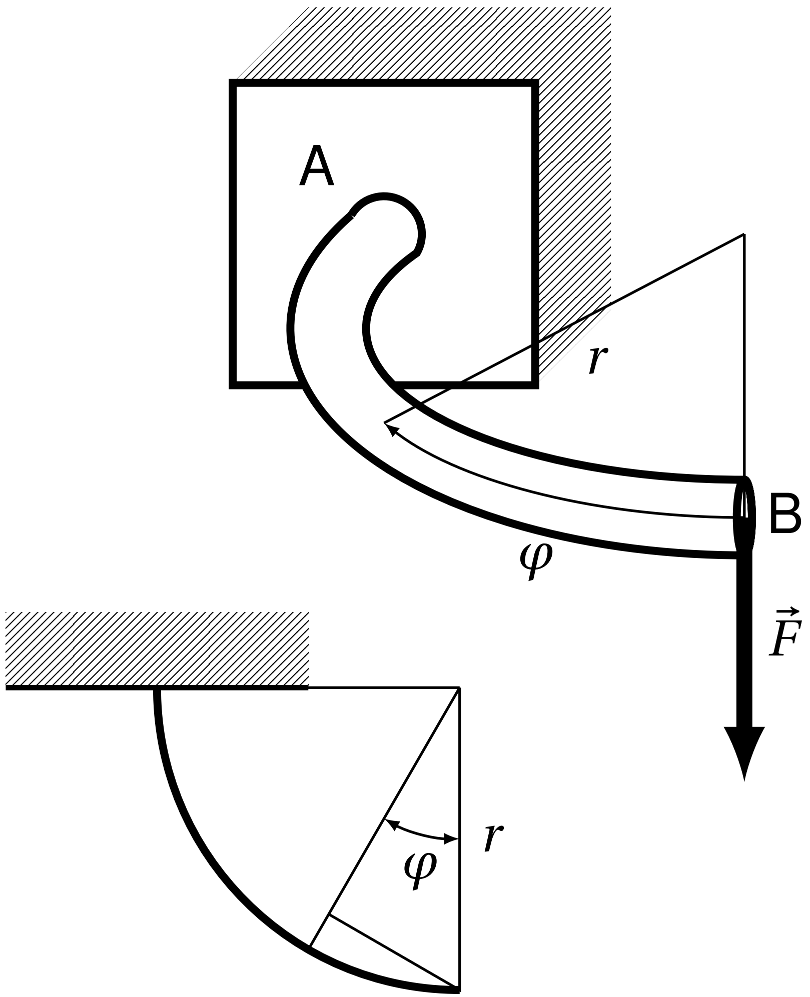
Svislý posuv bodu A:

$$v_A = \frac{\partial U}{\partial F} =$$
$$= \frac{\partial}{\partial F}\left(2 \cdot \int_0^l \frac{(F \cdot x)^2 \cdot dx}{2 \cdot E \cdot J}\right) +$$
$$+ \frac{\partial}{\partial F}\left(\int_0^l \frac{(F \cdot (l + x))^2 \cdot dx}{2 \cdot E \cdot J}\right) +$$
$$+ \frac{\partial}{\partial F}\left(2 \cdot \int_0^l \frac{(F \cdot l)^2 \cdot dx}{2 \cdot G \cdot J_k}\right)$$

Příklad 2:

$$v = F \cdot l^3 \cdot \left(\frac{3}{E \cdot J} + \frac{2}{G \cdot J_k} \right)$$

Příklad 3:



Svislý posuv bodu B:

$$w_B = \frac{\partial U}{\partial F}$$

Silové účinky v myšleném řezu φ:

- posouvací síla $T_z = F$
- ohybový moment $M_y = F \cdot r \cdot \sin\varphi$
- kroutící moment $M_x = F \cdot r \cdot (1 - \cos\varphi)$

$$U = \frac{1}{2} \cdot \int_0^{\frac{\pi}{2}} \left(\frac{M_y^2}{E \cdot J_y} + \frac{M_x^2}{G \cdot J_p} \right) \cdot r \cdot d\varphi$$

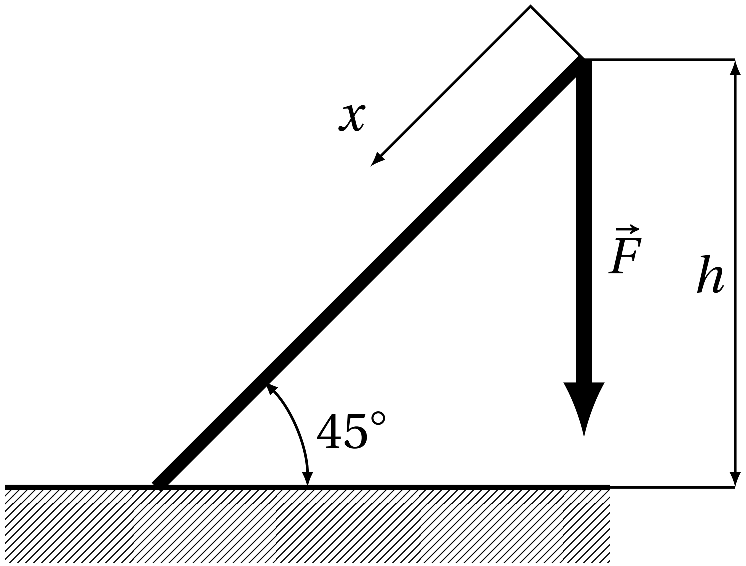
$$w_B = \int_0^{\frac{\pi}{2}} \frac{F \cdot (r \cdot \sin\varphi)^2}{E \cdot J_y} \cdot r \cdot d\varphi +$$

$$+ \int_0^{\frac{\pi}{2}} \frac{F \cdot r^2 \cdot (1 - \cos\varphi)^2}{G \cdot J_p} \cdot r \cdot d\varphi$$

Příklad 3:

$$w_B = \frac{F \cdot r^3}{E \cdot J_y} \cdot \frac{\pi}{4} + \frac{F \cdot r^3}{G \cdot J_p} \cdot \left(\frac{3}{4}\pi - 2 \right)$$

Příklad 4:



$$M_o = F_o \cdot x = \frac{F}{\sqrt{2}} \cdot x$$

$$F_t = \frac{F}{\sqrt{2}}$$

$$U = \frac{1}{2} \cdot \left(\int_L \frac{M_o^2}{E \cdot J} \cdot dx + \int_L \frac{F_t^2}{E \cdot S} \cdot dx \right)$$

$$= \int_0^L \left(\frac{\left(\frac{F}{\sqrt{2}} \cdot x\right)^2}{2 \cdot E \cdot J} + \frac{\left(\frac{F}{\sqrt{2}}\right)^2}{2 \cdot E \cdot S} \right) \cdot dx$$

$$= \frac{F^2}{4 \cdot E \cdot J} \cdot \int_0^{h \cdot \sqrt{2}} x^2 \cdot dx + \frac{F^2}{4 \cdot E \cdot S} \cdot \int_0^{h \cdot \sqrt{2}} dx$$

$$\Delta h = \frac{\partial U}{\partial F}$$

Příklad 4:

$$\Delta h = \frac{F}{2 \cdot E \cdot J} \cdot \int_0^{h \cdot \sqrt{2}} x^2 \cdot dx + \frac{F}{2 \cdot E \cdot S} \cdot \int_0^{h \cdot \sqrt{2}} dx =$$

$$= \frac{F}{2 \cdot E \cdot J} \cdot \left[\frac{x^3}{3} \right]_0^{h \cdot \sqrt{2}} + \frac{F}{2 \cdot E \cdot S} \cdot [x]_0^{h \cdot \sqrt{2}} =$$

$$= \frac{F \cdot h^3 \cdot 2^{\frac{3}{2}}}{2 \cdot 3 \cdot E \cdot J} + \frac{F \cdot h \cdot \sqrt{2}}{2 \cdot E \cdot S} =$$

$$\Delta h = \frac{\sqrt{2} \cdot F \cdot h^3}{3 \cdot E \cdot J} + \frac{F \cdot h}{\sqrt{2} \cdot E \cdot S}$$