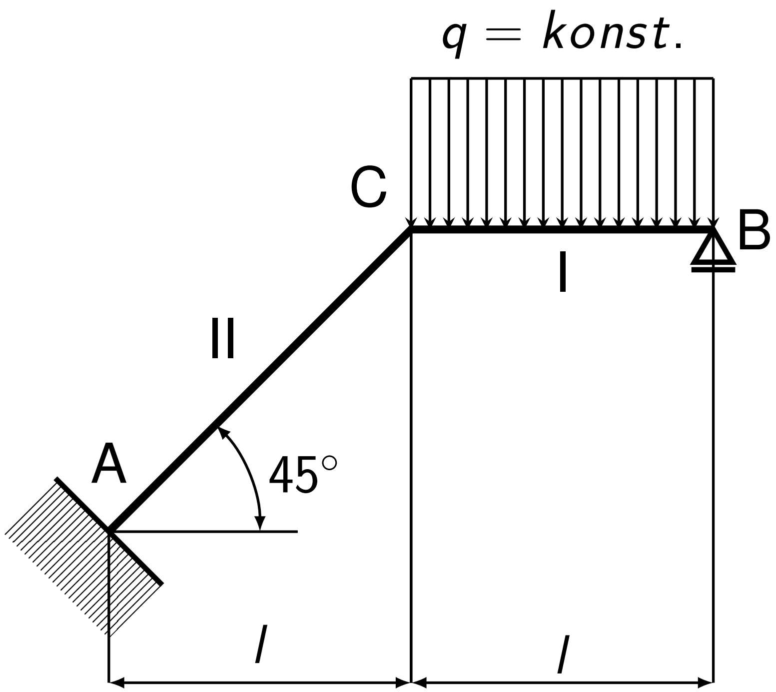
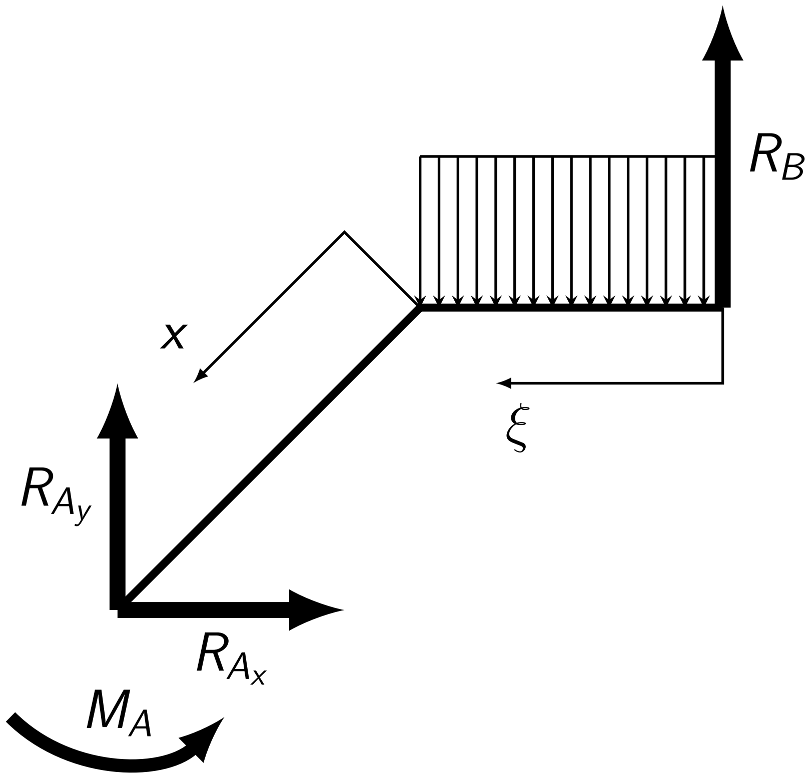
Příklad 1:

Dáno: l, E, J, q

Určete maximální ohybový moment v tyči.



Příklad 1:



Rovnice rovnováhy: 3 rovnice pro 4 neznámé $\rightarrow$1x staticky neurčitá úloha

Vnitřní silové účinky:

$$M_I = R_B \cdot \xi - \frac{q \cdot \xi^2}{2}$$

$$M_{II} = R_B \cdot \left(l + \frac{x}{\sqrt{2}} \right) - q \cdot l \cdot \left(\frac{l}{2} + \frac{x}{\sqrt{2}} \right)$$

Deformační podmínka:

$$\frac{\partial U}{\partial R_B} = \sum_{i=1}^{2} \int \frac{M_i}{E \cdot J} \cdot \frac{\partial M_i}{\partial R_B} \cdot d\xi = 0$$

Příklad 1:

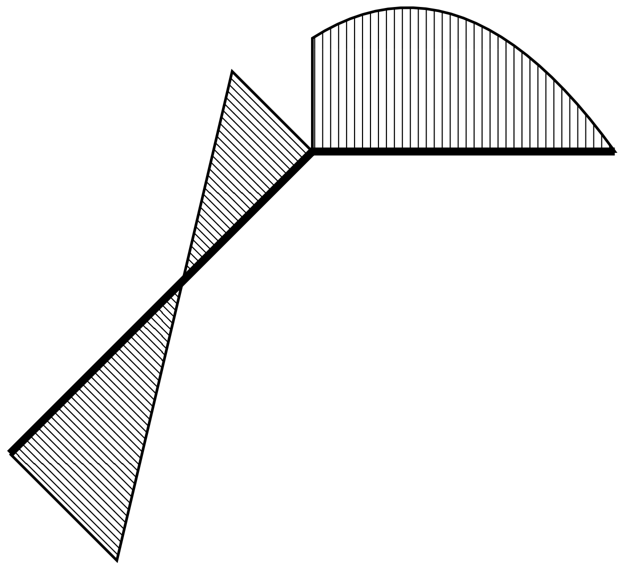
i	Meze	M_i	$\frac{\partial M_i}{\partial R_B}$
1	$0 \le \xi \le l$	$R_B \cdot \xi - \frac{q \cdot \xi^2}{2}$	ξ
2	$0 \le x \le l\sqrt{2}$	$R_B \left(l + \frac{x}{\sqrt{2}}\right) - q \cdot l \left(\frac{l}{2} + \frac{x}{\sqrt{2}}\right)$	$l + \frac{x}{\sqrt{2}}$

$$\frac{\partial U}{\partial R_B} = \frac{1}{E \cdot J} \cdot \left(\int_0^l \left(R_B \cdot \xi - \frac{q \cdot \xi^2}{2} \right) \cdot \xi \cdot d\xi + \right.$$

$$\left. + \int_0^{l\sqrt{2}} \left(R_B \cdot \left(l + \frac{x}{\sqrt{2}} \right) - q \cdot l \cdot \left(\frac{l}{2} + \frac{x}{\sqrt{2}} \right) \right) \cdot \left(l + \frac{x}{\sqrt{2}} \right) \cdot dx \right) = 0$$

$$R_B \approx 0,65 \cdot q \cdot l$$

Příklad 1:



Vetknutí:

$$M_A = R_B \cdot 2 \cdot l - q \cdot l \frac{3 \cdot l}{2} = -0,2 \cdot q \cdot l^2$$

Bod zalomení:

$$M_C = R_B \cdot l - q \cdot \frac{l^2}{2} = 0,15 \cdot q \cdot l^2$$

Maximální moment na úseku I:

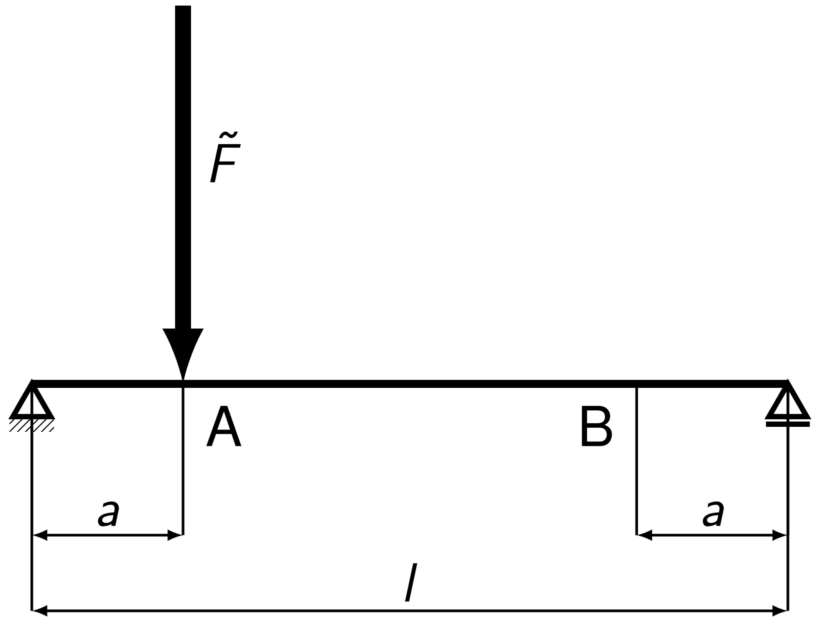
$$\frac{dM_I}{d\xi} = T(\xi) = R_B - q \cdot \overline{\xi} = 0 \rightarrow \overline{\xi} = 0,65 \cdot l$$

$$\overline{M} = \frac{1}{2} \cdot q \cdot \overline{\xi}^2 = 0,19 \cdot q \cdot l^2$$

Maximální ohybový moment:

$$M_{max} = 0,2 \cdot q \cdot l^2$$

Příklad 2:



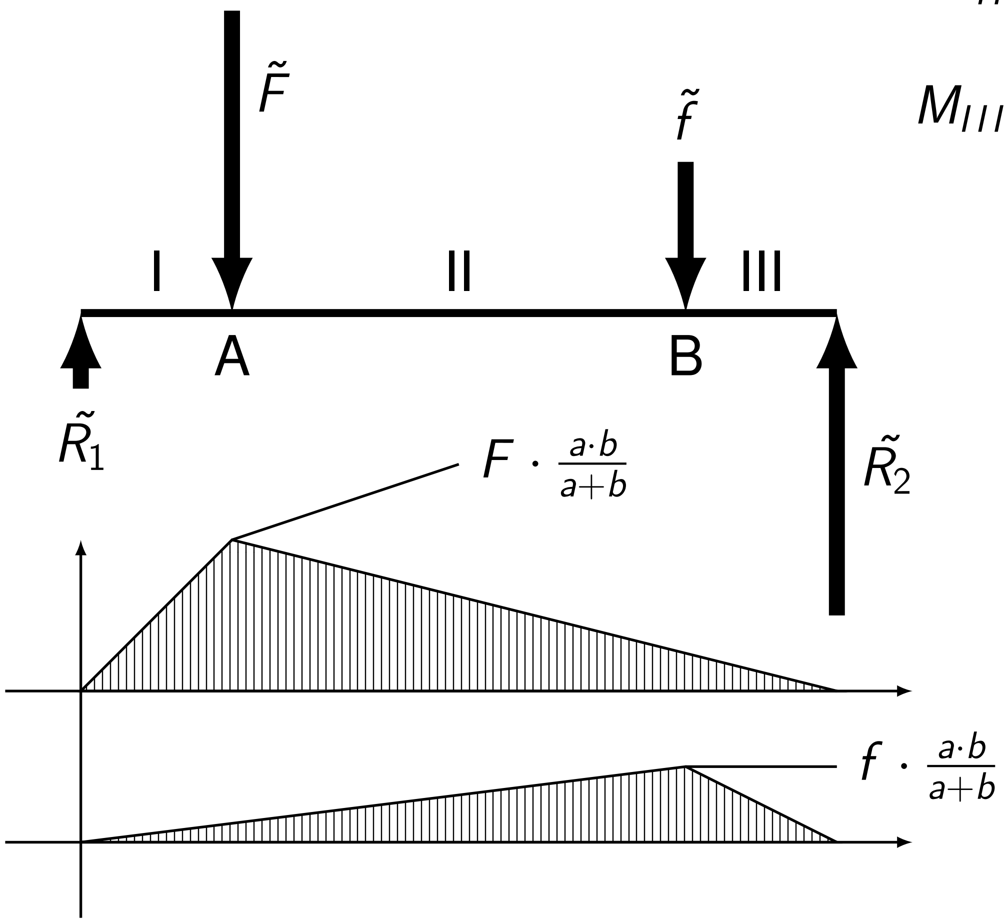
Dáno: l, a, F, E, J

Určete průhyb v bodě B.

Řešení pomocí principu minima energie:

$$u_B = \left.\frac{\partial U}{\partial f}\right|_{f=0}$$

Příklad 2:



$$M_I = F \cdot \frac{b}{a+b} \cdot x + f \cdot \frac{a}{a+b} \cdot x$$
$$M_{II} = F \cdot \frac{a}{a+b} \cdot (l-x) + f \cdot \frac{a}{a+b} \cdot x$$
$$M_{III} = F \cdot \frac{a}{a+b} \cdot (l-x) + f \cdot \frac{b}{a+b} \cdot (l-x)$$

$$m_I = \frac{\partial M_I}{\partial f} = \frac{a}{a+b} \cdot x$$
$$m_{II} = \frac{\partial M_{II}}{\partial f} = \frac{a}{a+b} \cdot x$$
$$m_{III} = \frac{\partial M_{III}}{\partial f} = \frac{b}{a+b} \cdot (l-x)$$

$$u_B = \frac{1}{E \cdot J} \cdot \sum_{i=I}^{III} \int_{l_i} M_i(x) \cdot m_i(x)\, dx$$

Příklad 2:

$$u_B = \frac{1}{E \cdot J} \cdot \left(\int_0^a F \cdot \frac{b}{a+b} \cdot x \cdot \frac{a}{a+b} \cdot x \cdot dx \right.$$

$$+ \int_a^{l-a} F \cdot \frac{a}{a+b} \cdot (l-x) \cdot \frac{a}{a+b} \cdot x \cdot dx +$$

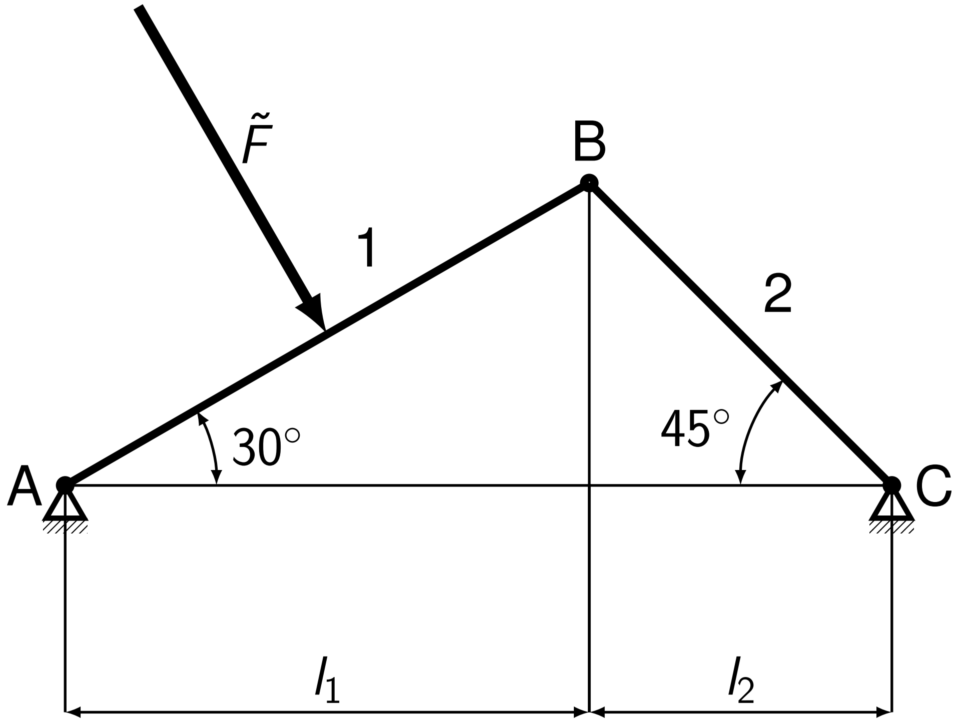
$$\left. + \int_{l-a}^{l} F \cdot \frac{a}{a+b} \cdot (l-x) \cdot \frac{b}{a+b} \cdot (l-x) \cdot dx \right)$$

$$u_B = \frac{\left(a^2 \cdot l^2 - 2 \cdot a^4\right) \cdot F}{6 \cdot l \cdot EJ}$$

Příklad 3:

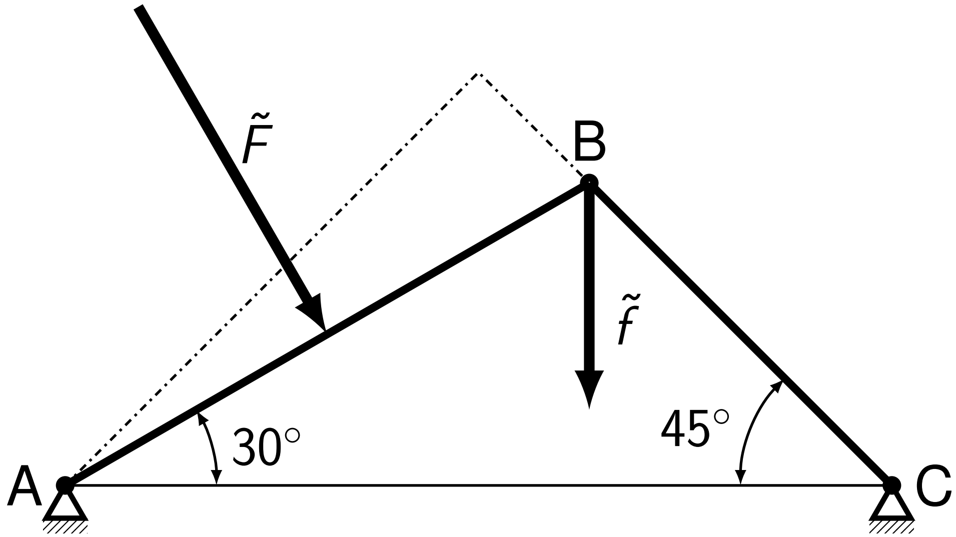
Dáno: E, S, l_1, l_2, F

Určete svislý posuv bodu B.



$$w_B = \left. \frac{\partial U}{\partial f} \right|_{f=0}$$

Příklad 3:



Síla v prutu 2:
z rovnice rovnováhy prutu 1 kolem A:

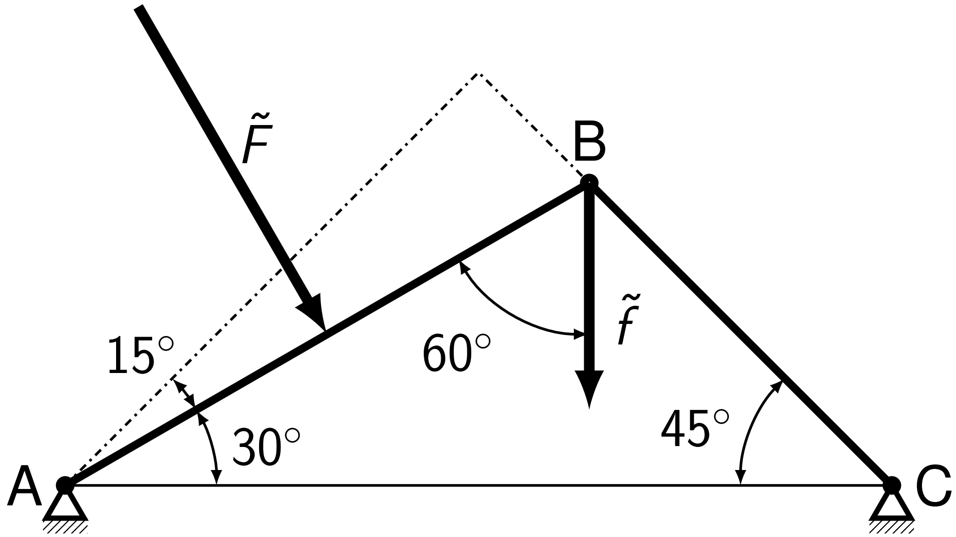
$$N_2 \cdot \frac{\sqrt{2}}{2} \cdot (l_1 + l_2) - f \cdot l_1 - F \cdot \frac{l_1}{2\cos(30^\circ)} = 0$$

$$N_2 = \frac{f \cdot l_1 + F \cdot \frac{l_1}{2 \cdot \cos(30^\circ)}}{\frac{\sqrt{2}}{2} \cdot (l_1 + l_2)} = konst.$$

$$\frac{\partial N_2}{\partial f} = \frac{l_1}{\frac{\sqrt{2}}{2} \cdot (l_1 + l_2)}$$

$$U_2 = \frac{N_2^2 \cdot \sqrt{2} \cdot l_2}{2 \cdot E \cdot S}$$

Příklad 3:



Síly v prutu 1:
osová síla:

$$N_1 = -\left(f \cdot \sin\left(30^\circ\right) + N_2 \cdot \sin\left(15^\circ\right)\right)$$

$$\frac{\partial N_1}{\partial f} = -\left(\sin\left(30^\circ\right) + \frac{\partial N_2}{\partial f} \cdot \sin\left(15^\circ\right)\right)$$

$$U_1^{(N_1)} = \frac{N_1^2 \cdot \frac{l_1}{\cos(30^\circ)}}{2 \cdot E \cdot S}$$

ohybový moment: řešíme posuv bodu B, na který nemá příčná deformace mezi body A a B vliv!

Příklad 3:

$$w_b = U_1^{(N_1)} + U_2 = \left.\frac{\partial U_1^{(N_1)}}{\partial f}\right|_{f=0} + \left.\frac{\partial U_2}{\partial f}\right|_{f=0} = \frac{N_1 \cdot \frac{l_1}{\cos(30^\circ)}}{E \cdot S} \cdot \frac{\partial N_1}{\partial f} + \frac{N_2 \cdot \sqrt{2} \cdot l_2}{E \cdot S} \cdot \frac{\partial N_2}{\partial f} =$$

$$= \frac{1}{E \cdot S} \cdot \left(\frac{F \cdot \frac{l_1^2}{2 \cdot \cos(30^\circ)}}{\frac{\sqrt{2}}{2} \cdot (l_1 + l_2)} \cdot \sin(15^\circ) \cdot \left(\sin(30^\circ) + \frac{l_1 \cdot \sin(15^\circ)}{\frac{\sqrt{2}}{2} \cdot (l_1 + l_2)} \right) + \right.$$

$$\left. + \frac{\frac{F \cdot l_1^2}{2 \cdot \cos(30^\circ)}}{\frac{\sqrt{2}}{2} \cdot (l_1 + l_2)} \cdot \frac{l_1}{\frac{\sqrt{2}}{2} \cdot (l_1 + l_2)} \right) =$$

$$= \frac{1}{E \cdot S} \cdot \frac{F \cdot \frac{l_1^2}{2 \cdot \cos(30^\circ)}}{\frac{\sqrt{2}}{2} \cdot (l_1 + l_2)} \cdot \left(\sin(15^\circ) \cdot \left(\sin(30^\circ) + \frac{l_1 \cdot \sin(15^\circ)}{\frac{\sqrt{2}}{2} \cdot (l_1 + l_2)} \right) + \frac{l_1}{\frac{\sqrt{2}}{2} \cdot (l_1 + l_2)} \right) =$$

$$= \frac{1,0858}{E \cdot S} \cdot \frac{F \cdot \frac{l_1^2}{2 \cdot \cos(30^\circ)}}{\frac{\sqrt{2}}{2} \cdot (l_1 + l_2)} = \frac{1,0858}{E \cdot S} \cdot \frac{F \cdot \frac{l_1^2}{\sqrt{3}}}{\frac{\sqrt{2}}{2} \cdot (l_1 + l_2)} = 0,887 \cdot \frac{F \cdot l_1^2}{E \cdot S \cdot (l_1 + l_2)}$$