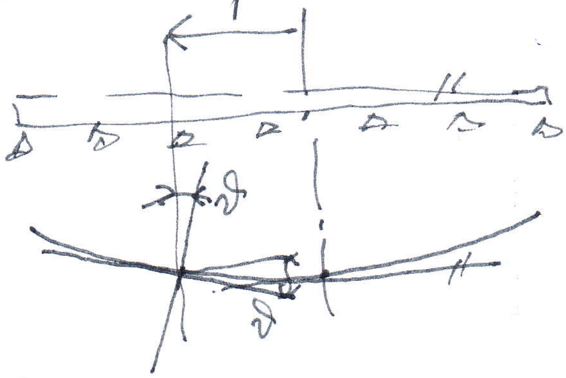
OHYB TENKÝCH KRUHOVÝCH DESEK

$\vartheta(r)$ – sklon tečny k průhybové ploše v radiálním směru



$$D = \frac{Eh^3}{12(1-\nu^2)}$$

$$\frac{d^2\vartheta(r)}{dr^2} + \frac{1}{r}\frac{d\vartheta(r)}{dr} - \frac{1}{r^2}\vartheta(r) = -\frac{T(r)}{D}$$

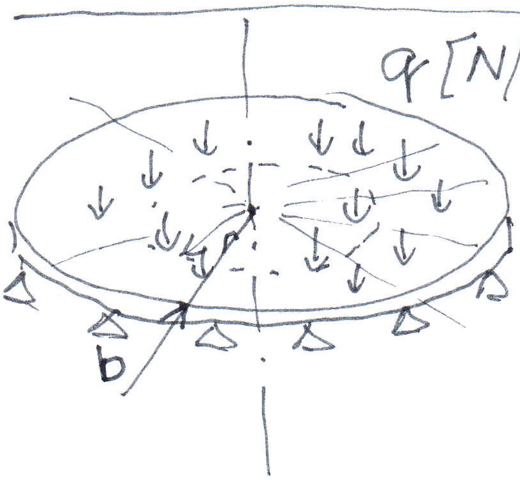
1) jestliže P.S. = 0 ~ homog. řešení

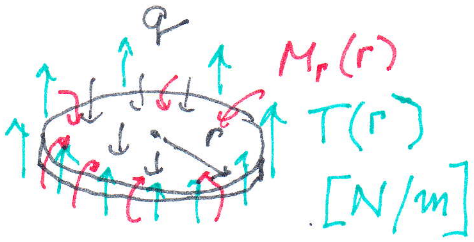
$$\vartheta_H(r) = C_1 r + C_2 \frac{1}{r}$$

2) jestliže P.S. $= K_1 \cdot r + K_2 \frac{1}{r}$

$$\vartheta_p(r) = K_1 \cdot \frac{r^3}{8} + K_2 \cdot \frac{1}{2} r \ln r$$

$r\,[m]$





$$\uparrow\ T(r)\cdot 2\pi r - q\cdot \pi r^2 = 0$$

$$T(r) = \frac{q \cdot r}{2}$$

$$M_r = D\left[\frac{d\vartheta(r)}{dr} + \nu\frac{\vartheta(r)}{r}\right]$$

$$M_t = D\left[\frac{\vartheta(r)}{r} + \nu\frac{d\vartheta(r)}{dr}\right]$$

Bezrozměrný poloměr

Vztažný poloměr – největší poloměr desky

př. b

Bezrozm. polom $\rho = \frac{r}{b}$

$\vartheta(\rho(r))$

$$\frac{d\vartheta}{dr} = \frac{d\vartheta}{d\rho}\cdot\frac{d\rho}{dr} = \frac{d\vartheta}{d\rho}\cdot\frac{1}{b}$$

$$\cancel{\frac{1}{b^2}}\frac{d^2\vartheta(\rho)}{d\rho^2} + \frac{1}{\cancel{b}\cdot\rho}\cdot\frac{1}{\cancel{b}}\frac{d\vartheta(\rho)}{d\rho} - \frac{1}{(\rho\cancel{b})^2}\cdot\vartheta(\rho) = -\frac{T(\rho)}{D}b^2$$

$$\vartheta(\rho) = C_1\rho + C_2\frac{1}{\rho} + K_1\frac{\rho^3}{8} + K_2\frac{1}{2}\rho\ln\rho$$

$$M_r = \left(\frac{d\vartheta(\rho)}{d\rho} + \nu\frac{\vartheta(\rho)}{\rho}\right)\cdot\frac{D}{b}$$

$$M_t = \left(\frac{\vartheta(\rho)}{\rho} + \nu\frac{d\vartheta(\rho)}{d\rho}\right)\cdot\frac{D}{b}$$

$$w(r) = -\int\vartheta(r)\,dr = -b\int\vartheta(\rho)\,d\rho$$

Příklad: $T(r) = \frac{qr}{2} \Rightarrow T(\rho) = \frac{q\cdot b\cdot\rho}{2}$

$$P.S. = -\frac{q b^3}{2D}\cdot\rho$$

$$\vartheta(\rho) = -\frac{qb^3}{2D}\cdot\left(C_1\rho + \cancel{C_2\frac{1}{\rho}} + \frac{\rho^3}{8}\right)$$

$r\langle 0,b\rangle \sim \rho\langle 0,1\rangle$

$\vartheta(0) = 0 \Rightarrow C_2 = 0$

$M_r\left(\begin{smallmatrix} r=b\\ \rho=1\end{smallmatrix}\right) = 0$

[Nm/m]

$$M_r(\rho) = \frac{D}{b}\left(-\frac{qb^3}{2D}\right)\left(C_1 + \frac{3\rho^2}{8} + \nu\left(C_1 + \frac{\rho^2}{8}\right)\right)$$

$$= -\frac{qb^2}{2}\left(C_1(1+\nu) + \frac{3+\nu}{8}\rho^2\right)$$

$$C_1(1+\nu) + \frac{3+\nu}{8}\cdot 1 = 0$$

$$\underline{C_1} = -\frac{3+\nu}{8(1+\nu)}$$

$$M_t = \frac{D}{b}\left(-\frac{qb^3}{2D}\right)\left(C_1 + \frac{\rho^2}{8} + \nu\left(C_1 + \frac{3\rho^2}{8}\right)\right) =$$

$$= -\frac{qb^2}{2}\left(C_1(1+\nu) + \frac{1+3\nu}{8}\rho^2\right)$$