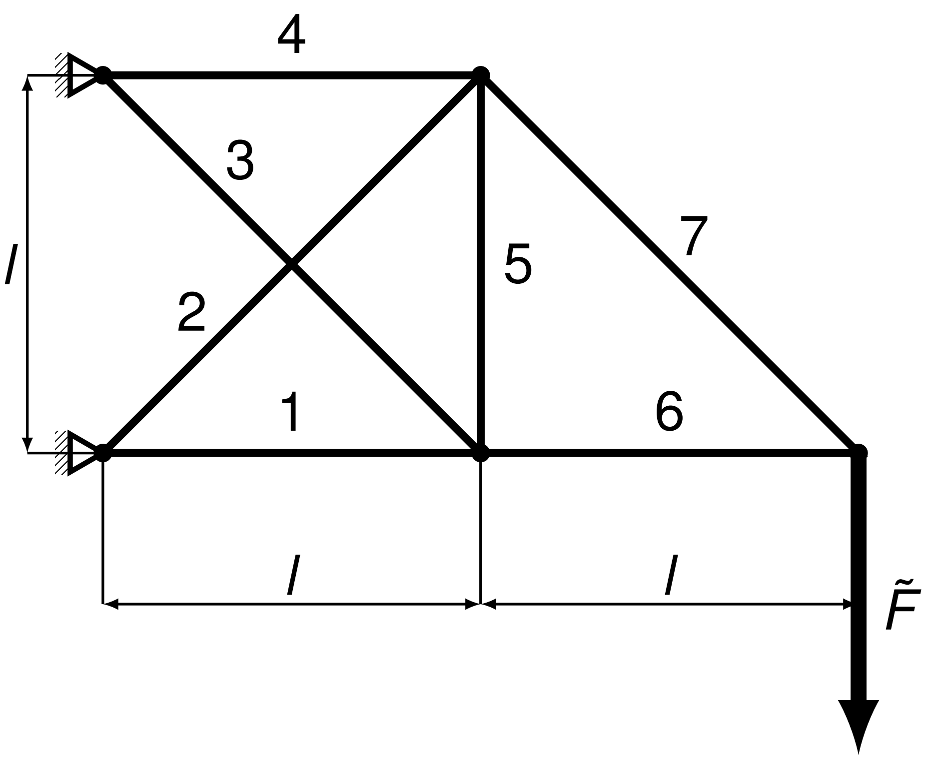
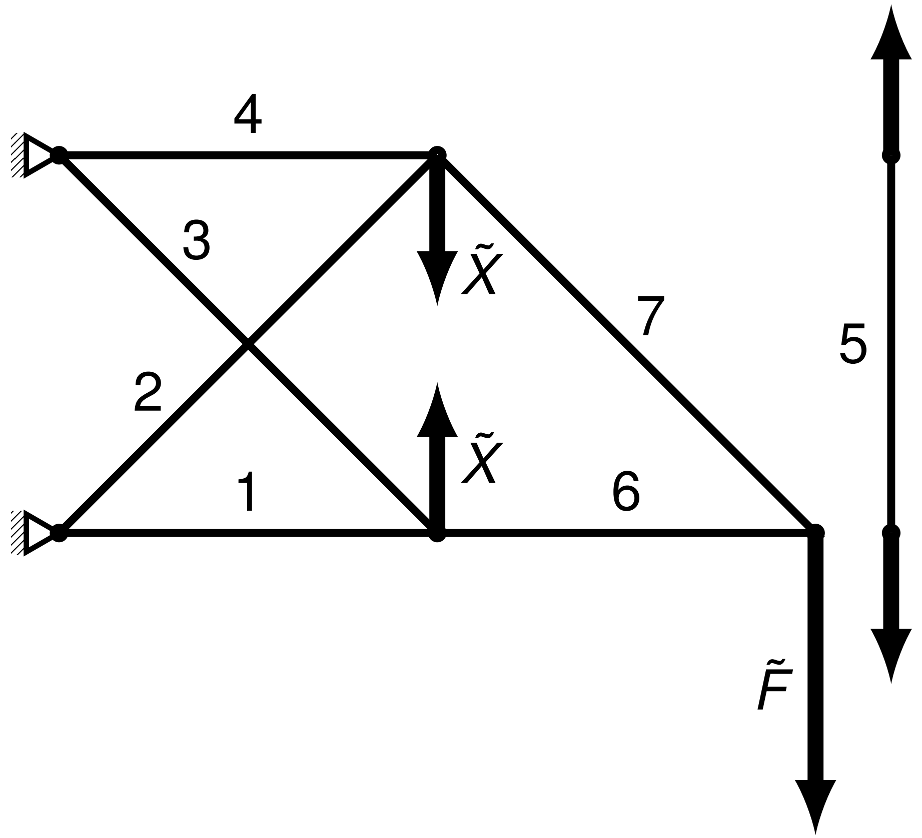
Příklad 1:

Dáno: l, E, S, F

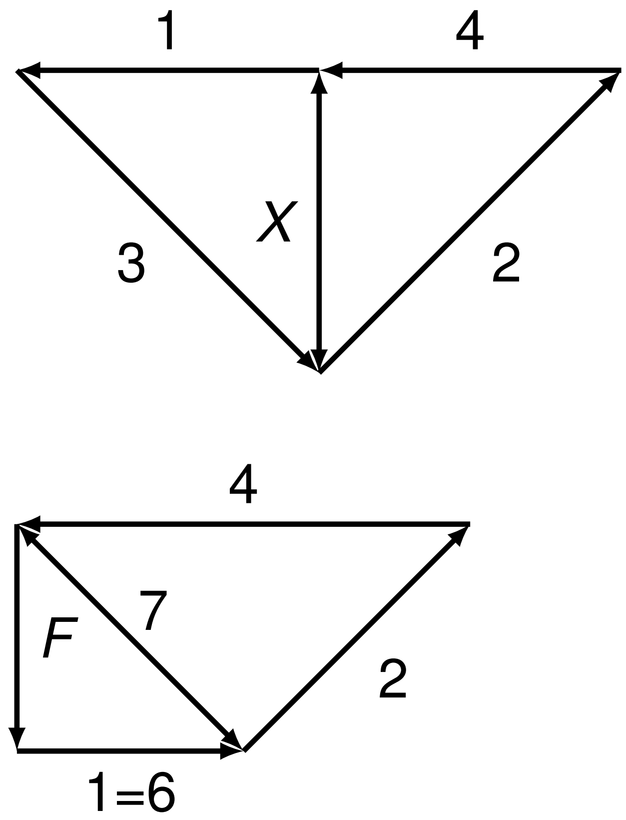
Určete: posuv bodu pod silou F.



Příklad 1:



Rovnováha styčníků:



Příklad 1:
Síly v prutech:

$$N_1 = -F + X \qquad \frac{\partial N_1}{\partial X} = 1 \qquad \frac{\partial N_1}{\partial F} = -1$$

$$N_2 = -F \cdot \sqrt{2} - X \cdot \sqrt{2} \qquad \frac{\partial N_2}{\partial X} = -\sqrt{2} \qquad \frac{\partial N_2}{\partial F} = -\sqrt{2}$$

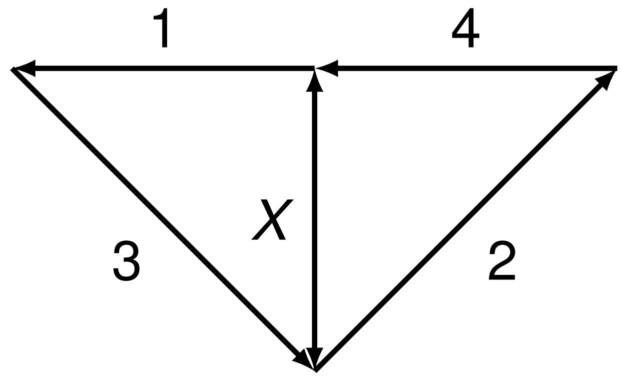
$$N_3 = -X \cdot \sqrt{2} \qquad \frac{\partial N_3}{\partial X} = -\sqrt{2} \qquad \frac{\partial N_3}{\partial F} = 0$$

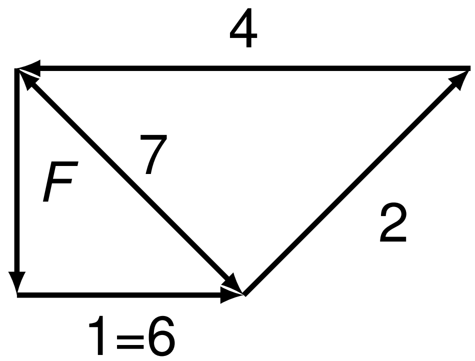
$$N_4 = 2 \cdot F + X \qquad \frac{\partial N_4}{\partial X} = 1 \qquad \frac{\partial N_4}{\partial F} = 2$$

$$N_5 = X \qquad \frac{\partial N_5}{\partial X} = 1 \qquad \frac{\partial N_5}{\partial F} = 0$$

$$N_6 = -F \qquad \frac{\partial N_6}{\partial X} = 0 \qquad \frac{\partial N_6}{\partial F} = -1$$

$$N_7 = F \cdot \sqrt{2} \qquad \frac{\partial N_7}{\partial X} = 0 \qquad \frac{\partial N_7}{\partial F} = \sqrt{2}$$





Délky prutů:

$$l_1 = l_4 = l_5 = l_6 = l$$

$$l_2 = l_3 = l_7 = l \cdot \sqrt{2}$$

Příklad 1:

$$U = \sum_{i=1}^{7} \frac{N_i^2 \cdot l_i}{2 \cdot E \cdot S}$$

Pro staticky neurčitou X musí platit:

$$\frac{\partial U}{\partial X} = \sum_{i=1}^{7} \frac{N_i \cdot l_i}{E \cdot S} \cdot \frac{\partial N_i}{\partial X} = 0$$

$$\Big((-F + X) + \left(-F \cdot \sqrt{2} - X \cdot \sqrt{2}\right) \cdot \left(-\sqrt{2}\right) \cdot \sqrt{2} +$$

$$+ \left(-X \cdot \sqrt{2}\right) \cdot \left(-\sqrt{2}\right) \cdot \sqrt{2} + (2 \cdot F + X) + X + (-F) \cdot 0 + F \cdot \sqrt{2} \cdot 0\Big) \cdot l = 0$$

$$\left(1 + 2 \cdot \sqrt{2}\right) \cdot F + \left(3 + 4 \cdot \sqrt{2}\right) \cdot X = 0$$

$$X = \frac{1 + 2 \cdot \sqrt{2}}{3 + 4 \cdot \sqrt{2}} \doteq -0,4422 \cdot F$$

Příklad 1:

$$N_1 \doteq -1,4422 \cdot F, \quad N_2 \doteq -0,7888 \cdot F, \quad N_3 \doteq 0,6254 \cdot F, \quad N_4 \doteq 1,5578 \cdot F$$

$$N_5 \doteq -0,4422 \cdot F, \quad N_6 = -F, \quad N_7 \doteq 1,4142 \cdot F$$

Posuv pod silou F:

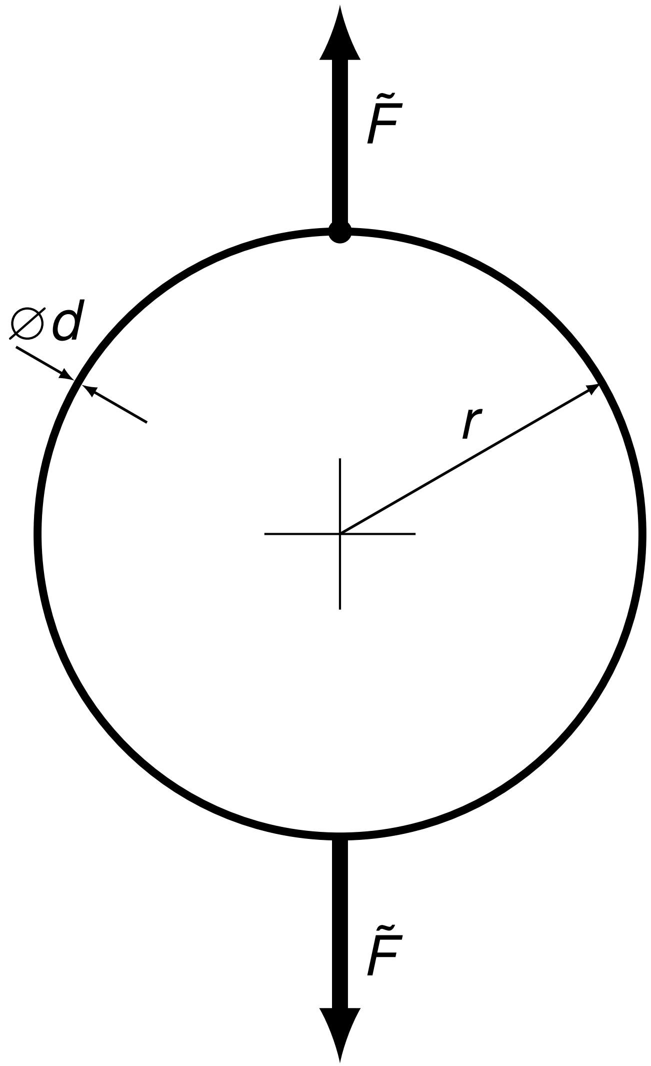
$$w = \frac{\partial U}{\partial F} = \frac{1}{E \cdot S} \cdot \sum_{i=1}^{7} N_i \cdot \frac{\partial N_i}{\partial F} \cdot l_i$$

$$w = \frac{l}{E \cdot S} \cdot \Big((-F + X) \cdot (-1) + \left(-F \cdot \sqrt{2} - X \cdot \sqrt{2}\right) \cdot \left(-\sqrt{2}\right) \cdot \sqrt{2} +$$

$$+ \left(-X \cdot \sqrt{2}\right) \cdot 0 \cdot \sqrt{2} + (2 \cdot F + X) \cdot 2 + X \cdot 0 + (-F) \cdot (-1) + F \cdot \sqrt{2} \cdot \sqrt{2} \cdot \sqrt{2} \Big) =$$

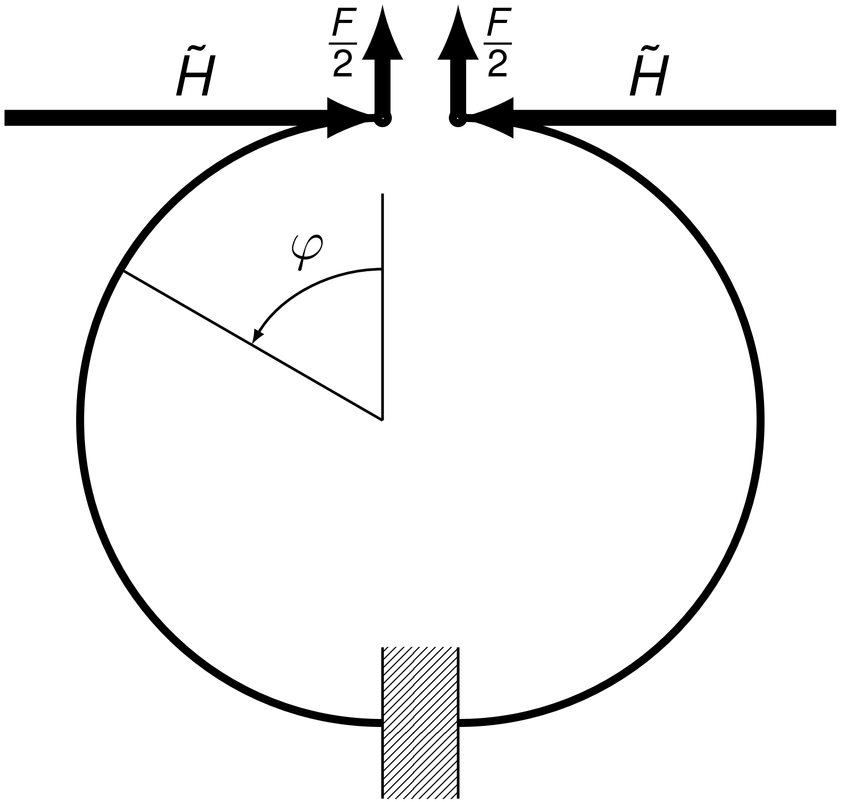
$$= \frac{l}{E \cdot S} \cdot \left(\left(6 + 4 \cdot \sqrt{2}\right) \cdot F + \left(1 + 2 \cdot \sqrt{2}\right) \cdot X \right) \doteq 9,9638 \cdot \frac{F \cdot l}{E \cdot S}$$

Příklad 2:



Dáno: r, $d \ll r$, E, F.
Určete: maximální ohybový moment.

Příklad 2:



Pro staticky neurčitou *H* musí platit:

$$\frac{\partial U}{\partial H} = 0, \quad U = \int_{\varphi} \frac{M^2(\varphi)}{2 \cdot E \cdot J} \cdot r \cdot d\varphi$$

Ohybový moment:

$$M(\varphi) = \frac{F}{2} \cdot r \cdot \sin(\varphi) - H \cdot r \cdot (1 - \cos(\varphi))$$

$$\frac{\partial M(\varphi)}{\partial H} = -r \cdot (1 - \cos(\varphi))$$

$$\frac{\partial U}{\partial H} = 2 \cdot \frac{1}{E \cdot J} \cdot \int_0^{\pi} M(\varphi) \cdot \frac{\partial M(\varphi)}{\partial H} \cdot r \cdot d\varphi =$$

$$= \frac{2 \cdot r^3}{E \cdot J} \cdot \int_0^{\pi} \left(\frac{F}{2} \cdot \sin(\varphi) - H \cdot (1 - \cos(\varphi)) \right) \cdot$$

$$\cdot (-1) \cdot (1 - \cos(\varphi))\, d\varphi = 0$$

Příklad 2:

$$\frac{\partial U}{\partial H} = -\frac{2\cdot r^3}{E\cdot J}\cdot\frac{F}{2}\cdot\int_0^{\pi}(\sin(\varphi)-\sin(\varphi)\cdot\cos(\varphi))\cdot d\varphi +$$

$$+\frac{2\cdot r^3}{E\cdot J}\cdot H\cdot\int_0^{\pi}\left(1-2\cdot\cos(\varphi)+\cos^2(\varphi)\right)\cdot d\varphi =$$

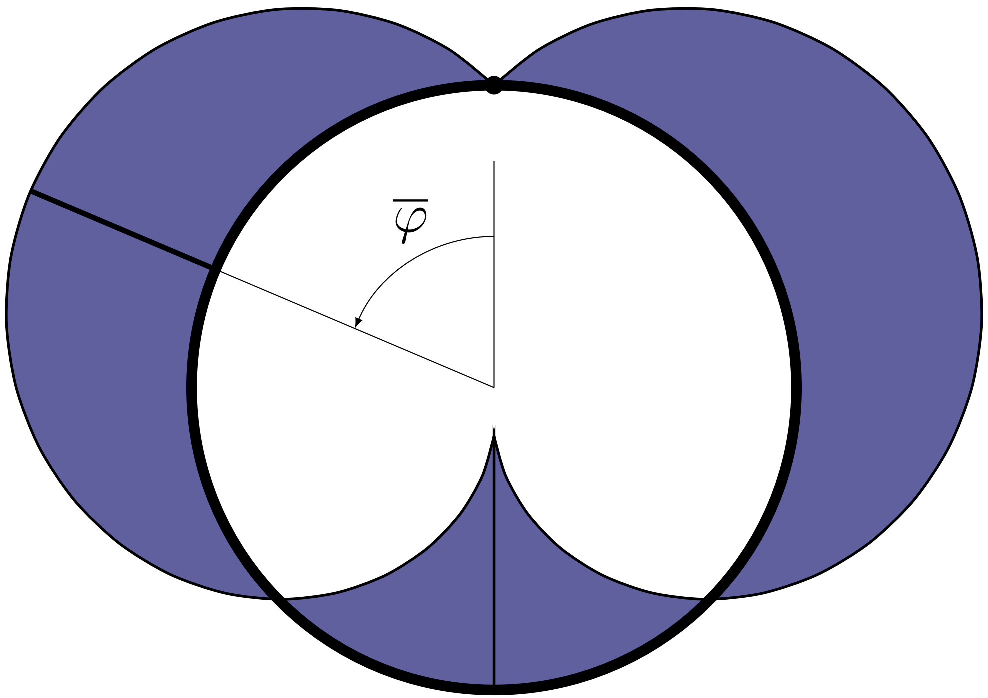
$$=\frac{2\cdot r^3}{E\cdot J}\cdot H\cdot\left[\varphi-2\cdot\sin(\varphi)+\frac{\sin(2\cdot\varphi)+2\cdot\varphi}{4}\right]_0^{\pi} -$$

$$-\frac{2\cdot r^3}{E\cdot J}\cdot\frac{F}{2}\cdot\left[-\cos(\varphi)-\frac{\cos^2(\varphi)}{2}\right]_0^{\pi} =$$

$$=\frac{2\cdot r^3}{E\cdot J}\cdot\left(H\cdot\frac{3\cdot\pi}{2}-\frac{F}{2}\cdot(1-(-1))\right)=0$$

$$H=\frac{2\cdot F}{3\cdot\pi}\doteq 0,21\cdot F$$

Příklad 2:



Příklad 2:

Maximální moment v místě $\overline{\varphi}$:

$$\frac{\partial M(\varphi)}{\partial \varphi} = F \cdot r \cdot \left(\frac{1}{2} \cdot \cos(\overline{\varphi}) - \frac{2}{3 \cdot \pi} \cdot \sin(\overline{\varphi}) \right) = 0$$

$$\frac{1}{2} - \frac{2}{3 \cdot \pi} \cdot \tan(\overline{\varphi}) = 0$$

$$\tan(\overline{\varphi}) = \frac{3 \cdot \pi}{4} \rightarrow \overline{\varphi} \doteq 67^\circ$$

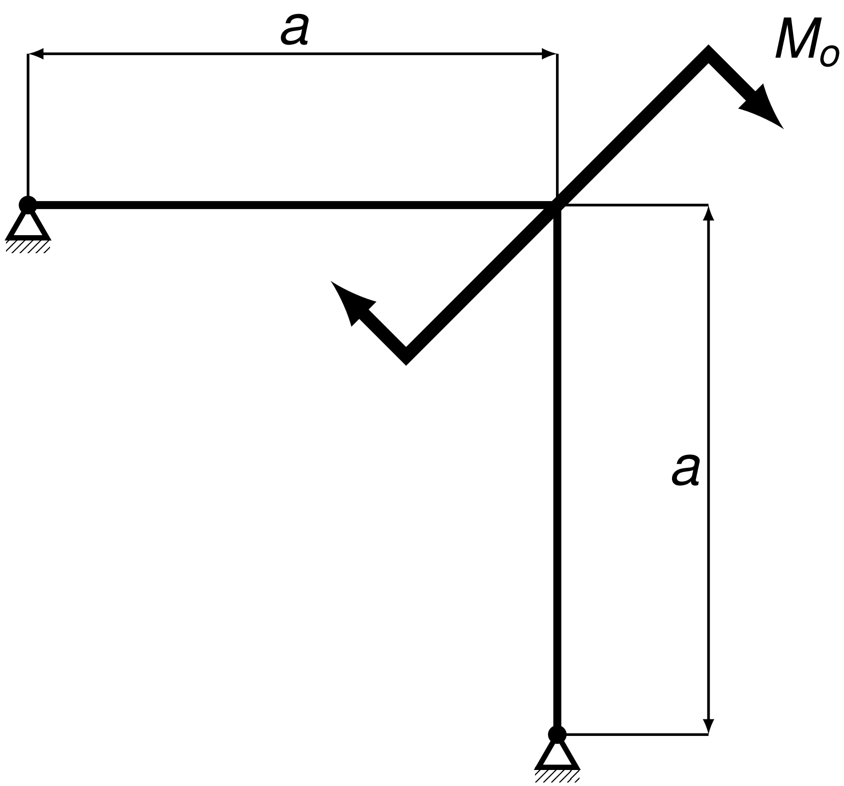
$$M(\overline{\varphi}) = \frac{F}{2} \cdot r \cdot \sin(\overline{\varphi}) - H \cdot r \cdot (1 - \cos(\overline{\varphi})) \doteq 0,33 \cdot F \cdot r$$

$$M(\pi) = -2 \cdot H \cdot r \doteq -0,42 \cdot F \cdot r$$

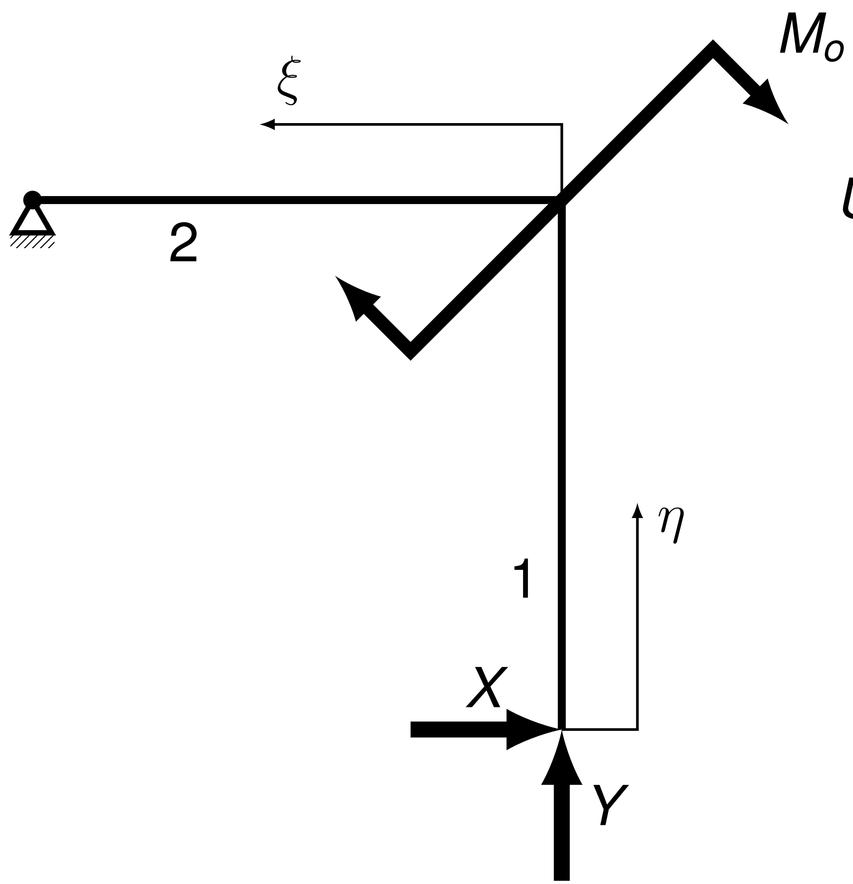
$$M_{max} = 0,42 \cdot F \cdot r$$

Příklad 3:

Dáno: a, E, J, M_o.
Určete: maximální ohybový moment.



Příklad 3:



$$M_1 = X \cdot \eta$$

$$M_2 = X \cdot a + Y \cdot \xi - M_o$$

$$U = \frac{1}{2 \cdot E \cdot J} \cdot \left(\int_0^a M_1^2 \cdot d\eta + \int_0^a M_2^2 \cdot d\xi \right)$$

$$\frac{\partial U}{\partial X} = 0, \quad \frac{\partial U}{\partial Y} = 0$$

$$\int_0^a M_1 \cdot \frac{\partial M_1}{\partial X} \cdot d\eta + \int_0^a M_2 \cdot \frac{\partial M_2}{\partial X} \cdot d\eta = 0$$

$$\int_0^a M_1 \cdot \frac{\partial M_1}{\partial Y} \cdot d\xi + \int_0^a M_2 \cdot \frac{\partial M_2}{\partial Y} \cdot d\xi = 0$$

Příklad 3:

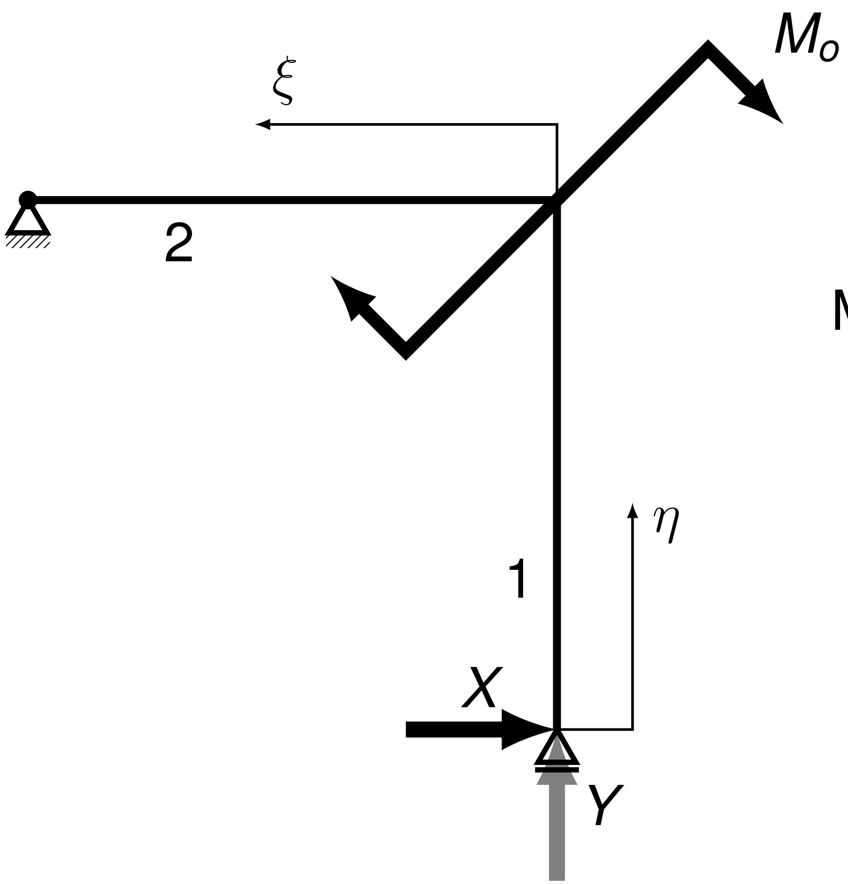
$$8 \cdot X \cdot a + 3 \cdot Y \cdot a = 6 \cdot M_o \quad \rightarrow \quad X = \frac{3 \cdot M_o}{7 \cdot a}$$

$$3 \cdot X \cdot a + 2 \cdot Y \cdot a = 3 \cdot M_o \quad \rightarrow \quad Y = \frac{6 \cdot M_o}{7 \cdot a}$$

$$M_{max} = \ldots$$

?

Příklad 3:



Rovnice rovnováhy:

$$X \cdot a + Y \cdot a - M_o = 0$$

$$Y = \frac{M_o}{a} - X$$

$$\frac{\partial U}{\partial X} + \frac{\partial U}{\partial Y} \cdot \frac{\partial Y}{\partial X} = 0$$

Momenty:

$$M_1 = X \cdot \eta$$

$$M_2 = X \cdot a + Y \cdot \xi - M_o$$

$$\frac{\partial M_1}{\partial X} = \eta, \quad \frac{\partial M_2}{\partial X} = a$$

$$\frac{\partial M_1}{\partial Y} = 0, \quad \frac{\partial M_2}{\partial Y} = \xi$$

$$\frac{\partial Y}{\partial X} = -1$$

Příklad 3:

$$\left(\int_0^a M_1 \cdot \frac{\partial M_1}{\partial X} \cdot d\eta + \int_0^a M_2 \cdot \frac{\partial M_2}{\partial X} \cdot d\xi \right) +$$

$$+ \left(\int_0^a M_1 \cdot \frac{\partial M_1}{\partial Y} \frac{\partial Y}{\partial X} \cdot d\eta + \int_0^a M_2 \cdot \frac{\partial M_2}{\partial Y} \frac{\partial Y}{\partial X} \cdot d\xi \right) = 0$$

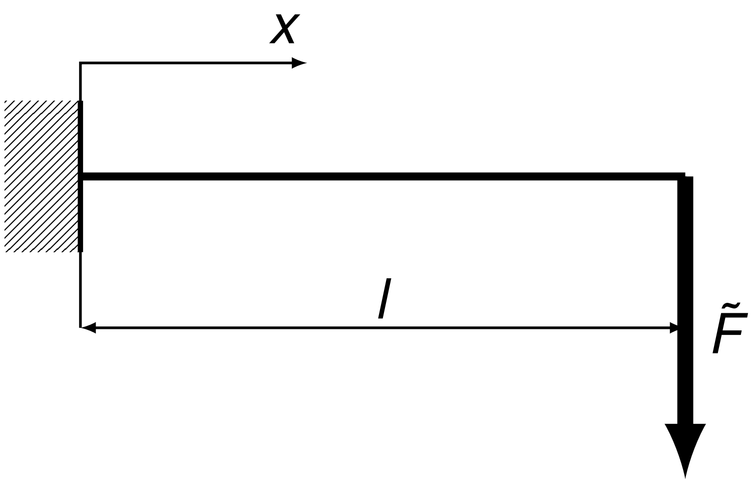
$$\int_0^a X \cdot \eta^2 \cdot d\eta + a \cdot \int_0^a (X \cdot a + Y \cdot \xi - M_o) \cdot d\xi - \int_0^a (X \cdot a + Y \cdot \xi - M_o) \cdot \xi \cdot d\xi = 0$$

$$\frac{5}{6} \cdot X \cdot a + \frac{1}{6} \cdot Y \cdot a = \frac{1}{2} \cdot M_o$$

$$X \cdot a + Y \cdot a = M_o$$

$$X = Y = \frac{M_o}{2 \cdot a} \quad \rightarrow \quad M_{max} = \ldots$$

Příklad 4:



Dány všechny rozměry a F.
Určete průhyb na konci.

Princip minima celkové potenciální energie:

$$\delta W = 0, \quad W = U + V$$

$$U = \int_{(l)} \frac{M^2(x)}{2 \cdot E \cdot J} \cdot dx$$

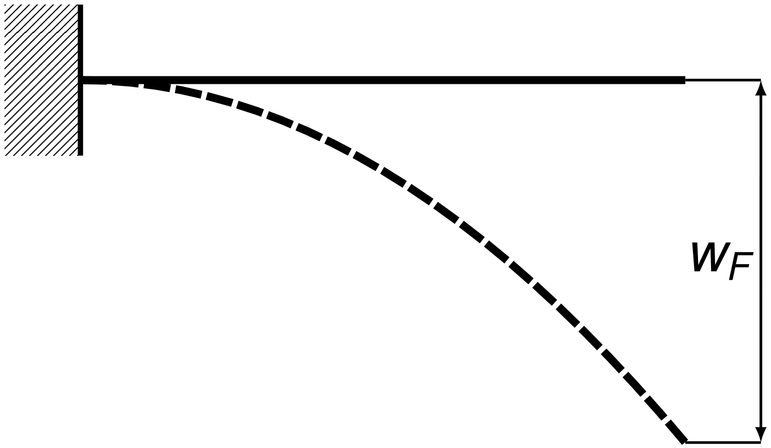
$$w''(x) = -\frac{M(x)}{E \cdot J}$$

$$U = \int_{(l)} \frac{E \cdot J}{2} \cdot \left(w''\right)^2 \cdot dx$$

Hledáme přibližné řešení; musí splňovat geometrické okrajové podmínky.

Příklad 4:

Předpokládejme např. $w = c \cdot x^2$



$$w''(x) = 2 \cdot c$$

$$U = \int_0^l \frac{E \cdot J}{2} \cdot (2 \cdot c)^2 \cdot dx$$

$$V = -F \cdot c \cdot l^2$$

$$W = U + V = \frac{E \cdot J}{2} \cdot 4 \cdot c^2 \cdot l - F \cdot c \cdot l^2$$

Variace c o δc dává δW:

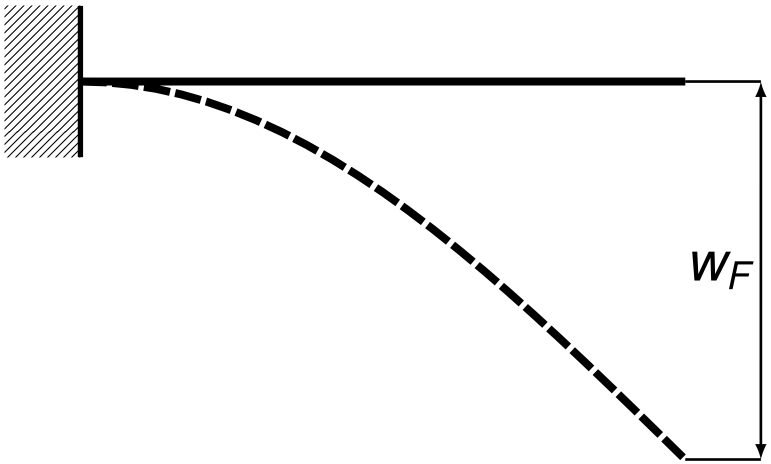
$$\delta W = 4 \cdot E \cdot J \cdot c \cdot l \cdot \delta c - F \cdot l^2 \cdot \delta c = 0$$

$$c = \frac{F \cdot l}{4 \cdot E \cdot J} \quad \rightarrow \quad w_F = \frac{F \cdot l^3}{4 \cdot E \cdot J}$$

Předpokládejme

$$w = w_F \cdot \left(1 - \cos\frac{\pi \cdot x}{2 \cdot l}\right)$$

$$w''(x) = w_F \cdot \left(\frac{\pi}{2 \cdot l}\right)^2 \cdot \cos\frac{\pi \cdot x}{2 \cdot l}$$



$$U = \frac{E \cdot J}{2} \cdot w_F^2 \cdot \left(\frac{\pi}{2 \cdot l}\right)^4 \int_0^l \left(\cos\frac{\pi \cdot x}{2 \cdot l}\right)^2 \cdot dx$$

$$= \frac{E \cdot J}{2} \cdot w_F^2 \left(\frac{\pi}{2 \cdot l}\right)^4 \cdot \frac{l}{2}$$

$$W = U + V = \frac{E \cdot J}{2} \cdot w_F^2 \cdot \left(\frac{\pi}{2 \cdot l}\right)^4 \cdot \frac{l}{2} - F \cdot w_F$$

Variace w_F o δw_F dává δW:

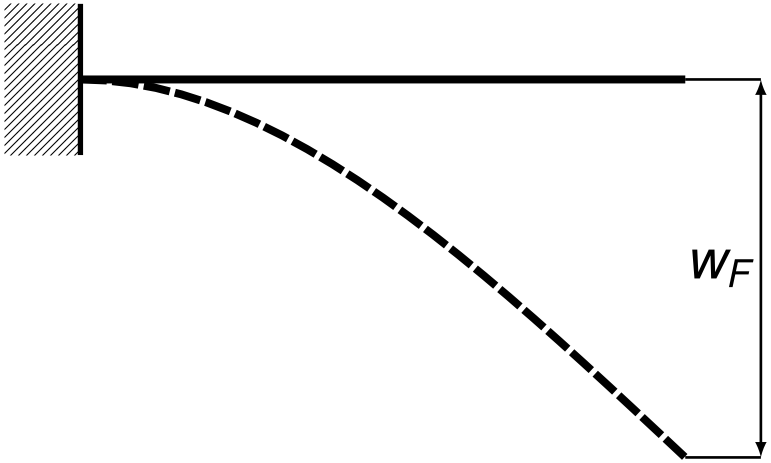
$$\delta W = E \cdot J \cdot w_F \cdot \left(\frac{\pi}{2 \cdot l}\right)^4 \cdot \frac{l}{2} \cdot \delta w_F - F \cdot \delta w_F = 0$$

$$w_F = \frac{32}{\pi^4} \cdot \frac{F \cdot l^3}{E \cdot J} \doteq 0,3285 \cdot \frac{F \cdot l^3}{E \cdot J}$$

Předpokládejme

$$w = c \cdot x^2 + d \cdot x^3$$

$$w''(x) = 2 \cdot c + 6 \cdot d \cdot x$$



$$U = \int_0^l \frac{E \cdot J}{2} \cdot (2 \cdot c + 6 \cdot d \cdot x)^2 \cdot dx$$

$$= \frac{E \cdot J}{2} \cdot \left(4 \cdot c^2 \cdot l + 24 \cdot c \cdot d \cdot \frac{l^2}{2} + 12 \cdot d^2 \cdot l^3\right)$$

$$W = U + V =$$

$$= E \cdot J \cdot \left(2 \cdot c^2 \cdot l + 6 \cdot c \cdot d \cdot l^2 + 6 \cdot d^2 \cdot l^3\right)$$

$$- F \cdot \left(c \cdot l^2 + d \cdot l^3\right)$$

Variace c o δc při $d = konst$, $\delta d = 0$:

$$E \cdot J \cdot \left(4 \cdot c \cdot l \cdot \delta c + 6 \cdot d \cdot l^2 \cdot \delta c\right) - F \cdot l^2 \cdot \delta c = 0$$

Variace d o δd při $c = konst$, $\delta c = 0$:

$$E \cdot J \cdot \left(6 \cdot c \cdot l^2 \cdot \delta d + 12 \cdot d \cdot l^3 \cdot \delta d\right) - F \cdot l^3 \cdot \delta d = 0$$

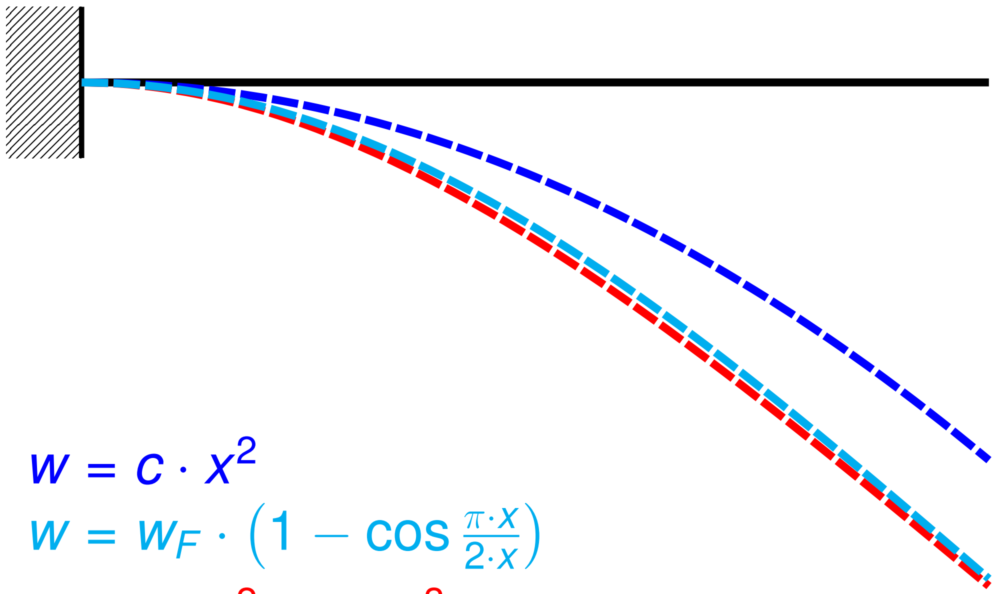
Soustava 2 rovnic o neznámých *c* a *d*:

$$4 \cdot c \cdot l + 6 \cdot d \cdot l^2 = \frac{F \cdot l^2}{E \cdot J}$$
$$6 \cdot c \cdot l^2 + 12 \cdot d \cdot l^3 = \frac{F \cdot l^3}{E \cdot J}$$

Její řešení vede na:

$$c = \frac{F \cdot l}{2 \cdot E \cdot J}, \quad d = -\frac{F}{6 \cdot E \cdot J}$$
$$w(x) = \frac{F}{E \cdot J} \cdot \left(\frac{l}{2} \cdot x^2 - \frac{x^3}{6}\right) \quad \rightarrow \quad w_F = w(l) = \frac{F \cdot l^3}{3 \cdot E \cdot J}$$

Srovnání jednotlivých aproximací:



$$w = c \cdot x^2$$

$$w = w_F \cdot \left(1 - \cos \frac{\pi \cdot x}{2 \cdot x}\right)$$

$$w = c \cdot x^2 + d \cdot x^3$$