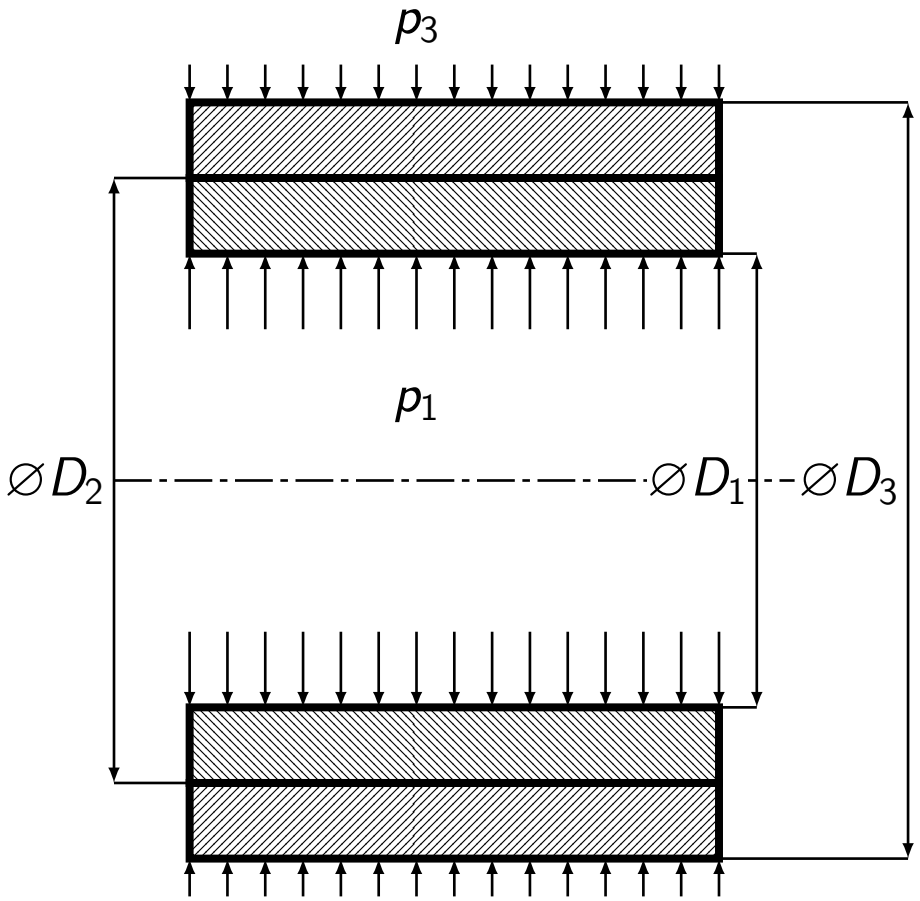
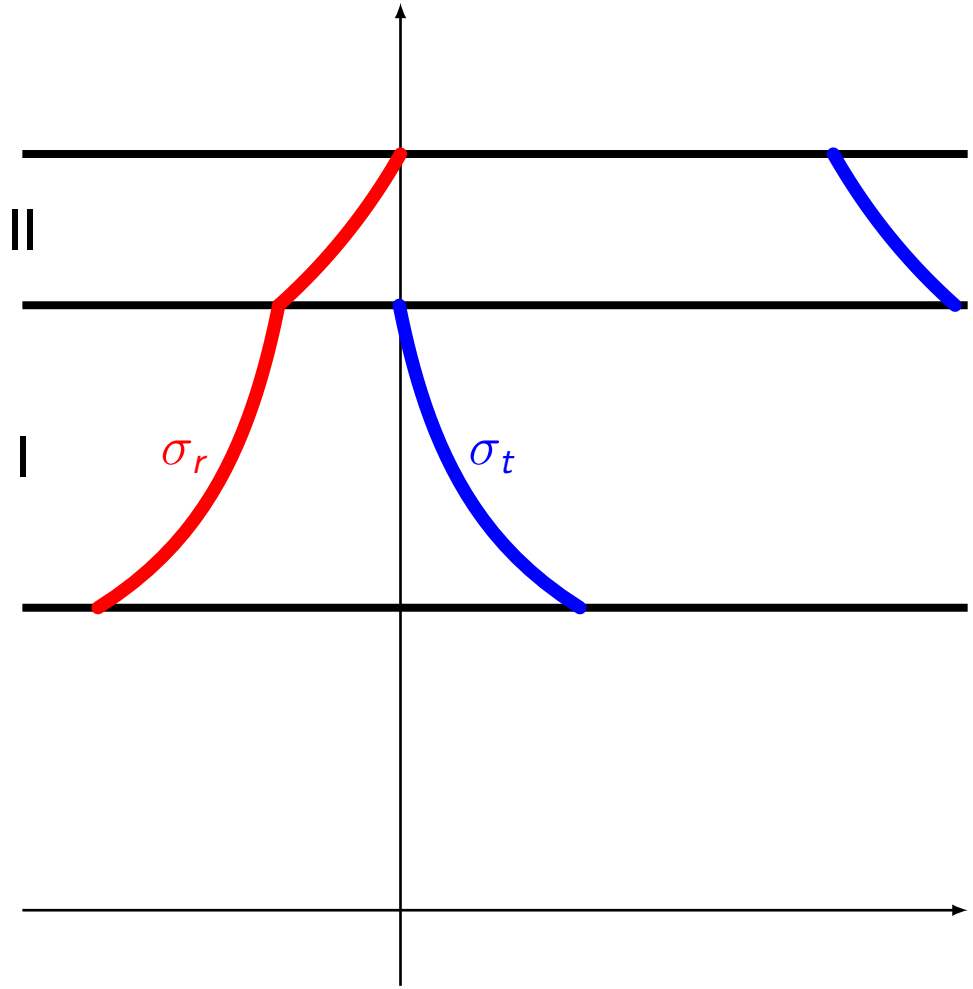
Příklad 1:

Dáno: E_1, E_2, μ_I, μ_{II}, D_1, D_2, D_3, p_1, p_3.
Určete maximální σ_{red}.



Řešení 1:



$$\sigma_t = A + B \cdot \frac{1}{r^2}$$

$$\sigma_r = A - B \cdot \frac{1}{r^2}$$

Rovnice rovnováhy:

$$p_{2I} = p_{1II} = -\sigma_{rI}\big|_{r=R_2} = -\sigma_{rII}\big|_{r=R_2} = p_k$$

$$A_I = \frac{p_1 \cdot R_1^2 - p_k \cdot R_2^2}{R_2^2 - R_1^2}$$

$$A_{II} = \frac{p_k \cdot R_2^2 - p_3 \cdot R_3^2}{R_3^2 - R_2^2}$$

$$B_I = \frac{p_1 - p_k}{R_2^2 - R_1^2} \cdot R_1^2 \cdot R_2^2$$

$$B_{II} = \frac{p_k - p_3}{R_3^2 - R_2^2} \cdot R_2^2 \cdot R_3^2$$

Řešení 1:

Deformační podmínka:

$$u_I\big|_{r=R_2} = u_{II}\big|_{r=R_2}$$

Posuv v radiálním směru:

$$u = \frac{r}{E} \cdot (\sigma_t - \mu \cdot \sigma_r)$$

$$\frac{\sigma_{tI} - \mu_I \cdot \sigma_{rI}}{E_I} = \frac{\sigma_{tII} - \mu_{II} \cdot \sigma_{tII}}{E_{II}}$$

$$\frac{1}{E_I} \cdot \left(A_I + B_I \cdot \frac{1}{R_2^2} - \mu_I \cdot \left(A_I - B_I \cdot \frac{1}{R_2^2} \right) \right) = \frac{1}{E_{II}} \cdot \left(A_{II} + B_{II} \cdot \frac{1}{R_2^2} - \mu_{II} \cdot \left(A_{II} - B_{II} \cdot \frac{1}{R_2^2} \right) \right)$$

Řešení 1:

$$\frac{1}{E_I}\cdot\left(\frac{p_1\cdot R_1^2-p_k\cdot R_2^2+p_1\cdot R_1^2-p_k\cdot R_1^2}{R_2^2-R_1^2}-\mu_I\cdot\frac{p_1\cdot R_1^2-p_k\cdot R_2^2-p_1\cdot R_1^2+p_k\cdot R_1^2}{R_2^2-R_1^2}\right)=$$
$$=\frac{1}{E_{II}}\cdot\left(\frac{p_k\cdot R_2^2-p_3\cdot R_3^2+p_k\cdot R_3^2-p_3\cdot R_3^2}{R_3^2-R_2^2}-\mu_{II}\cdot\frac{p_k\cdot R_2^2-p_3\cdot R_3^2-p_k\cdot R_3^2+p_3\cdot R_3^2}{R_3^2-R_2^2}\right)$$

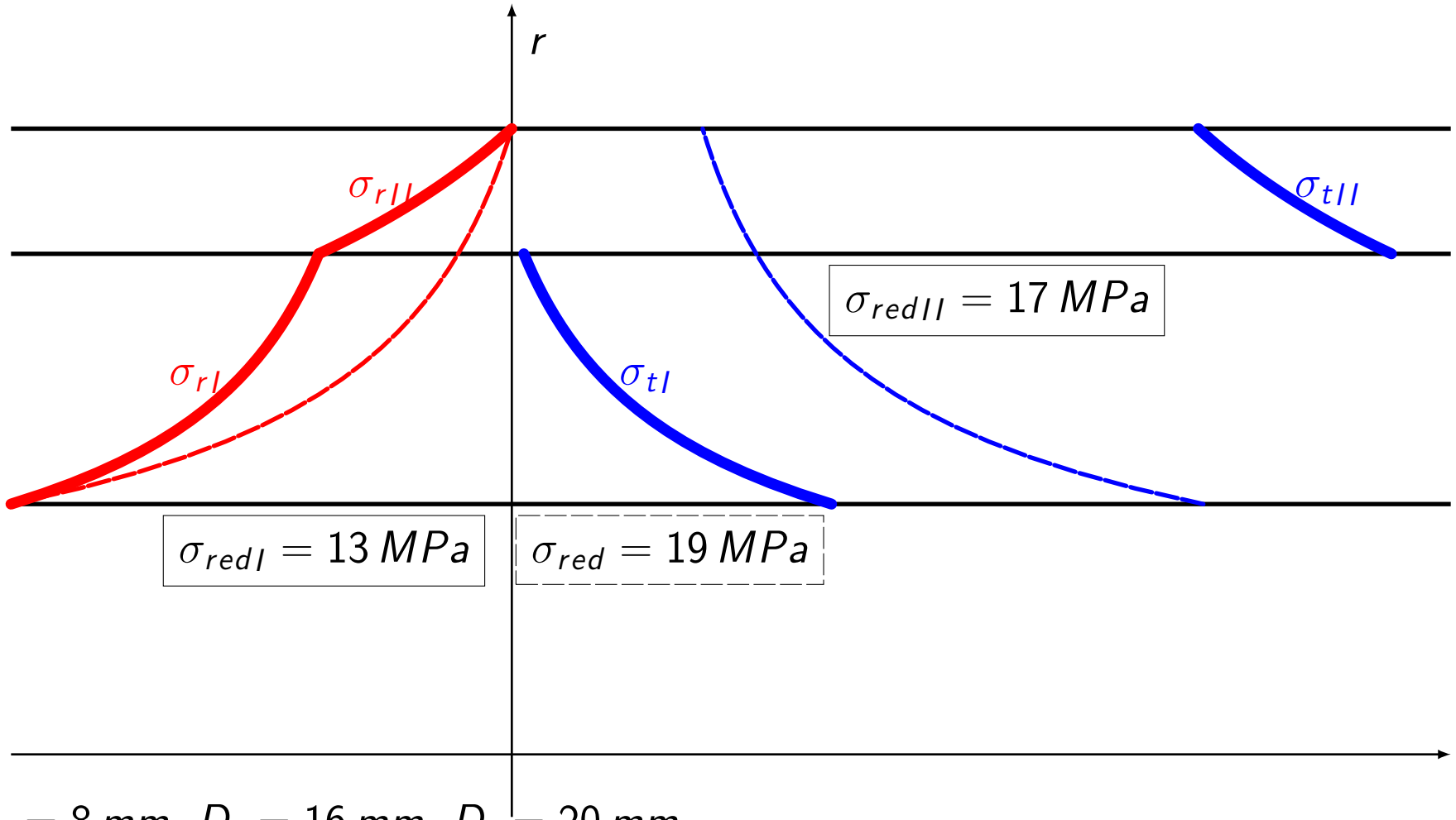
$$\frac{1}{E_I}\cdot\frac{2\cdot p_1\cdot R_1^2-p_k\cdot\left(R_1^2+R_2^2-\mu_I\cdot\left(R_2^2-R_1^2\right)\right)}{R_2^2-R_1^2}=\frac{1}{E_{II}}\cdot\frac{-2\cdot p_3\cdot R_3^2+p_k\cdot\left(R_2^2+R_3^2+\mu_{II}\cdot\left(R_3^2-R_2^2\right)\right)}{R_3^2-R_2^2}$$

$$\frac{2\cdot p_1\cdot R_1^2}{E_I\cdot\left(R_2^2-R_1^2\right)}+\frac{2\cdot p_3\cdot R_3^2}{E_{II}\cdot\left(R_3^2-R_2^2\right)}=p_k\cdot\left(\frac{R_1^2+R_2^2-\mu_I\cdot\left(R_2^2-R_1^2\right)}{E_I\cdot\left(R_2^2-R_1^2\right)}+\frac{R_2^2+R_3^2+\mu_{II}\cdot\left(R_3^2-R_2^2\right)}{E_{II}\cdot\left(R_3^2-R_2^2\right)}\right)$$

$$p_k=\ldots$$

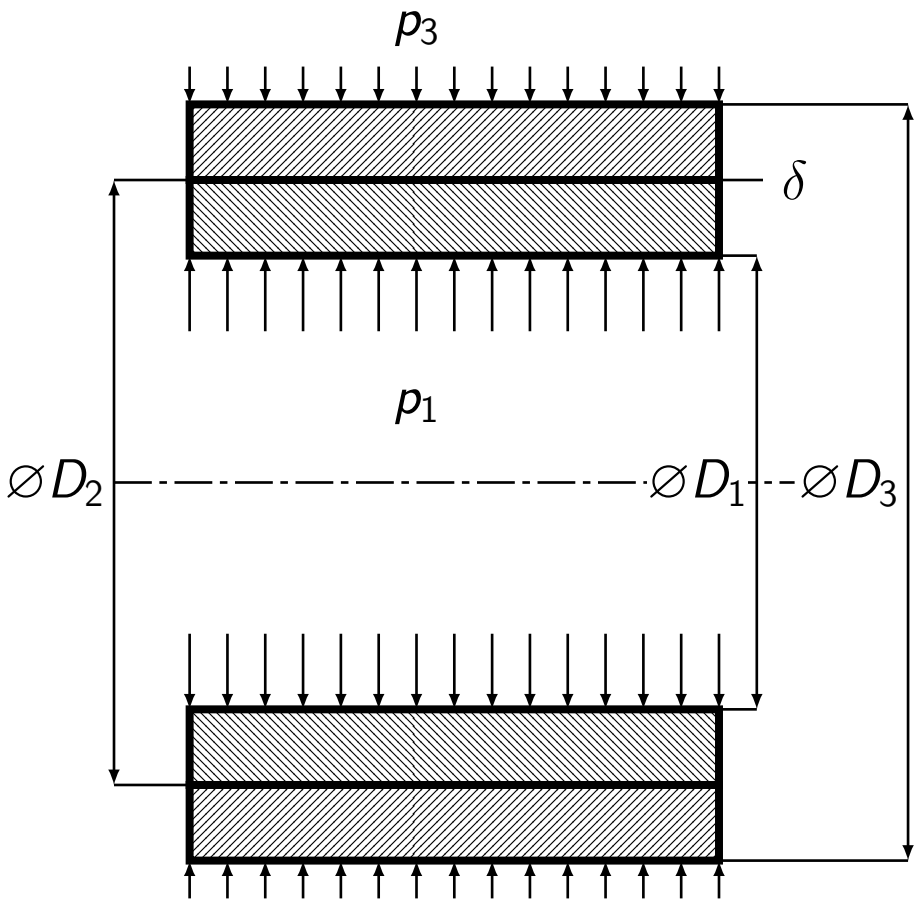
$$A_I=\ldots,\quad B_I=\ldots,\quad A_{II}=\ldots,\quad B_{II}=\ldots$$

Řešení 1:



$D_1 = 8\,mm,\ D_2 = 16\,mm,\ D_3 = 20\,mm,$
$E_I = 10\,GPa,\ E_{II} = 134\,GPa,$
$p_1 = 80\,atm,\ p_3 = 0\,atm$

Příklad 2:

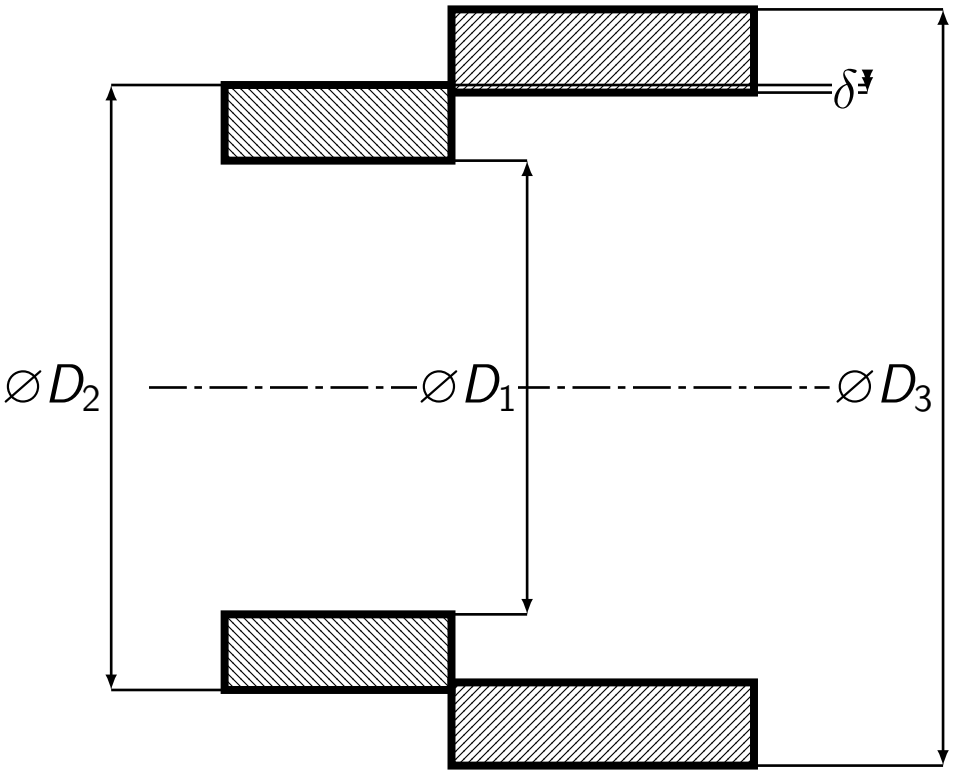


Dáno: E, D_1, D_2, D_3, $\delta \ll D_2$, p_1, p_3, α.
Určete:

1. ΔT potřebné k montáži.
2. napjatost při zatížení p_1 a p_3.

Porovnejte řešení s obyčejnou nádobou o stejné hmotnosti (tj. stejnými hodnotami vnějšího a vnitřního průměru).

Řešení 2:



$$\varepsilon_t = \frac{u}{r} = \frac{1}{E} \cdot (\sigma_t - \mu \cdot \sigma_r) + \alpha \cdot \Delta T$$

$$\varepsilon_r = \frac{du}{dr} = \frac{1}{E} \cdot (\sigma_r - \mu \cdot \sigma_t) + \alpha \cdot \Delta T$$

$$\sigma_r = A - \frac{B}{r^2}, \quad \sigma_t = A + \frac{B}{r^2}$$

$$\sigma_r|_{r=R_2} = A - \frac{B}{R_2^2} = 0$$

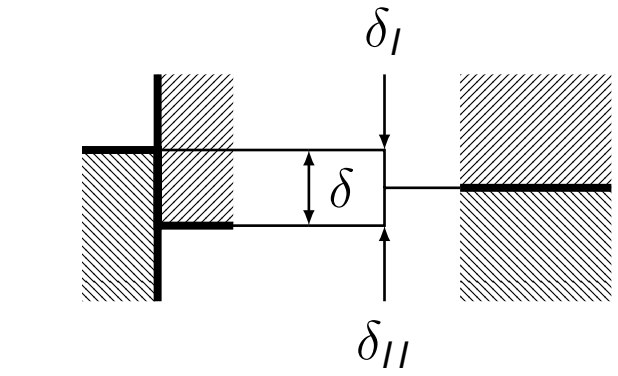
$$\sigma_r|_{r=R_3} = A - \frac{B}{R_3^2} = 0$$

$$A = 0, \quad B = 0$$

$$u|_{r=R_2} = R_2 \cdot \alpha \cdot \Delta T = \delta$$

$$\Delta T = \frac{\delta}{R_2 \cdot \alpha}$$

Řešení 2:



Radiální posuv $u = u(r)$:

$$u = \frac{r}{E} \cdot \left(\left(A + \frac{B}{r^2} \right) - \mu \cdot \left(A - \frac{B}{r^2} \right) \right)$$

$$u = \frac{r}{E} \cdot \left(A \cdot (1 - \mu) + \frac{B}{r^2} \cdot (1 + \mu) \right)$$

Rovnováha:

$$p_{2I} = p_{1II} = -\sigma_{rI}|_{r=R_2} = -\sigma_{rII}|_{r=R_2} = p_k$$

$$A_I - \frac{B_I}{R_2^2} = A_{II} - \frac{B_{II}}{R_2^2}$$

Deformační podmínka:

$$\delta_I + \delta_{II} = -u_I|_{r=R_2} + u_{II}|_{r=R_2} = \delta$$

Řešení 2:

$$\frac{R_2}{E} \cdot \left((-A_I + A_{II}) \cdot (1-\mu) + \frac{-B_I + B_{II}}{R_2^2} \cdot (1+\mu) \right) = \delta$$

Okrajové podmínky:

$$\sigma_{rI}|_{r=R_1} = A_I - \frac{B_I}{R_1^2} = -p_1, \quad \sigma_{rII}|_{r=R_3} = A_{II} - \frac{B_{II}}{R_3^2} = -p_3$$

Řešení soustavy rovnic:

$$B_I \cdot \left(\frac{1}{R_1^2} - \frac{1}{R_2^2} \right) + B_{II} \cdot \left(\frac{1}{R_2^2} - \frac{1}{R_3^2} \right) = p_1 - p_3$$

$$-B_I \cdot \left(\frac{1-\mu}{R_1^2} + \frac{1+\mu}{R_2^2} \right) + B_{II} \cdot \left(\frac{1+\mu}{R_2^2} + \frac{1-\mu}{R_3^2} \right) = \frac{\delta}{R_2} \cdot E - (p_1 - p_3) \cdot (1-\mu)$$

Řešení 2:

$$B_I = -\frac{\left(\delta \cdot E \cdot \left(R_3^2 - R_2^2\right) + \left(2 \cdot p_3 - 2 \cdot p_1\right) \cdot R_2 \cdot R_3^2\right) \cdot R_1^2}{2 \cdot R_2 \cdot \left(R_3^2 - R_1^2\right)}$$

$$B_{II} = \frac{\left(\delta \cdot E \cdot \left(R_2^2 - R_1^2\right) + \left(2 \cdot p_1 - 2 \cdot p_3\right) \cdot R_1^2 \cdot R_2\right) \cdot R_3^2}{2 \cdot R_2 \cdot \left(R_3^2 - R_1^2\right)}$$

$$A_I = \frac{B_I}{R_1^2} - p_1, \quad A_{II} = \frac{B_{II}}{R_3^2} - p_3$$

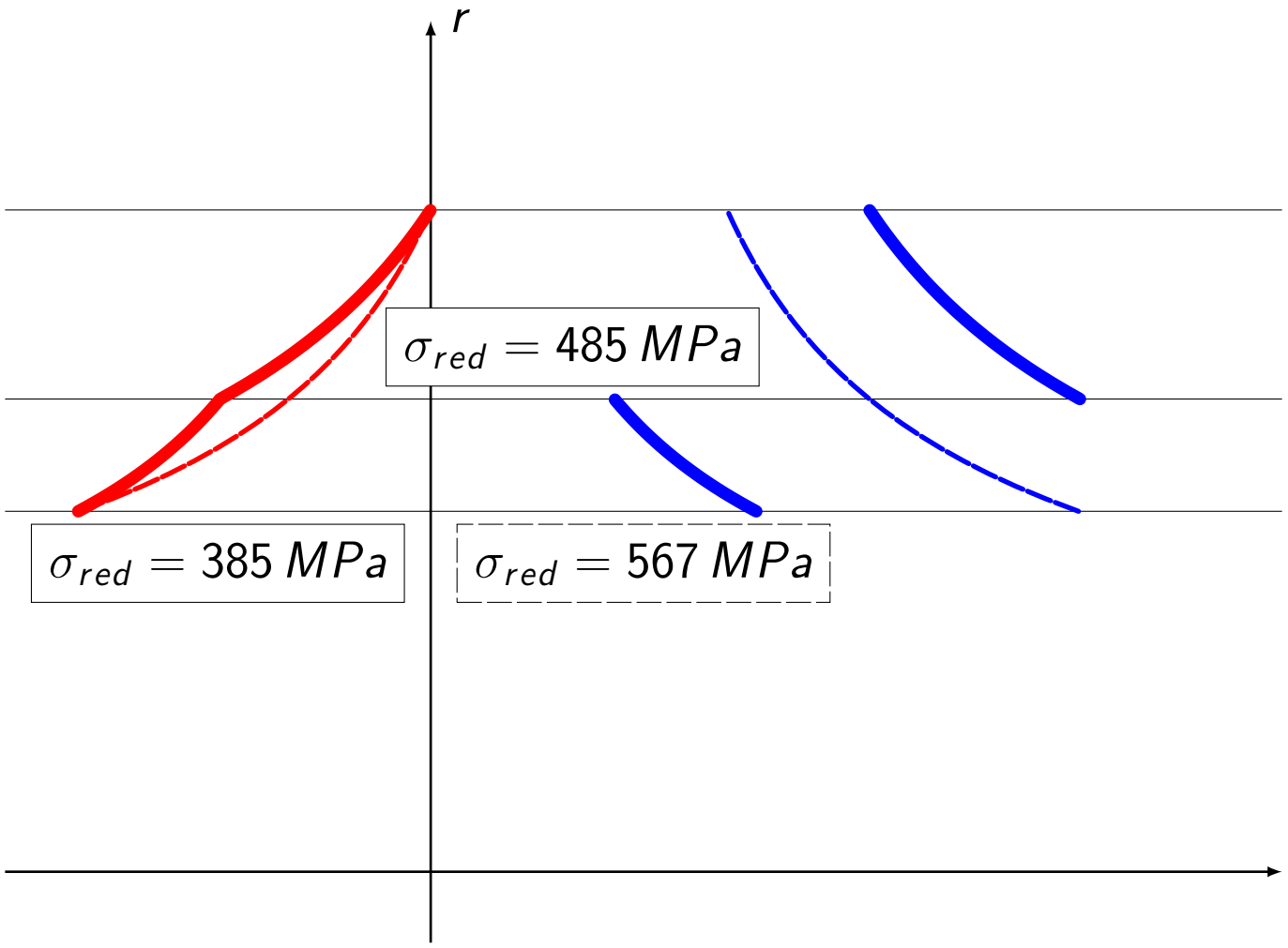
Řešte pro hodnoty:

$D_1 = 305\,mm$, $D_2 = 400\,mm$, $D_3 = 560\,mm$, $\delta = 0.25\,mm$

$p_1 = 200\,MPa$, $p_3 = 0\,Pa$,

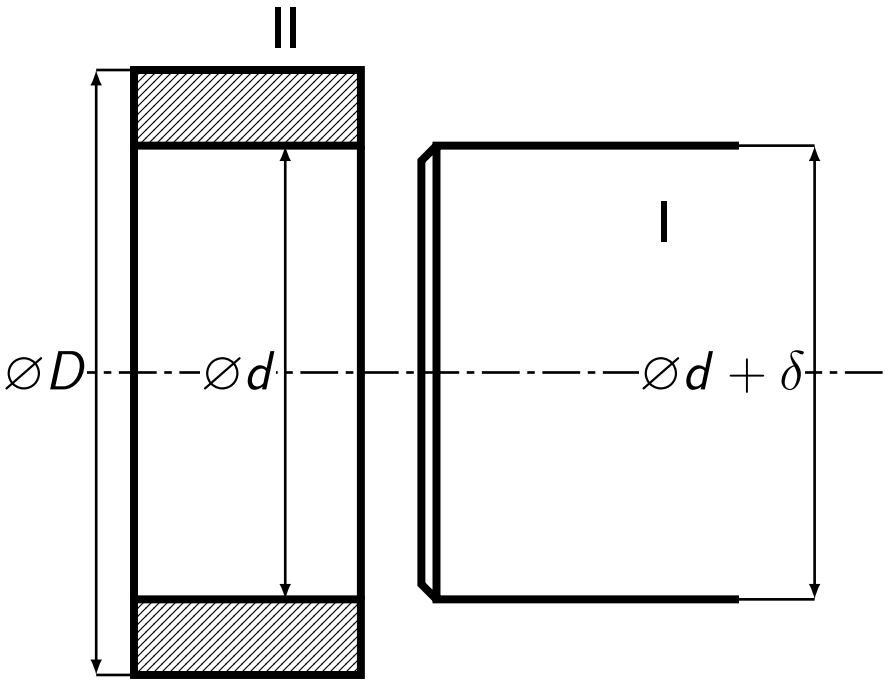
$E = 210000\,MPa$, $\alpha = 10^{-5}\frac{1}{K}$

Řešení 2:



Příklad 3:

Dáno: D,d, δ, E.
Určete kontaktní tlak.



Řešení 3:

$$r = 0 \rightarrow \sigma_{tI} = \sigma_{rI} = A_I, \quad B_I = 0$$

$$\sigma_{rII} = A_{II} - \frac{B_{II}}{r^2}, \quad \sigma_{tII} = A_{II} + \frac{B_{II}}{r^2}$$

Okrajové podmínky:

$$\sigma_{rI}\left(\frac{d}{2}\right) = -p$$

$$\sigma_{rII}\left(\frac{d}{2}\right) = -p$$

$$\sigma_{rII}\left(\frac{D}{2}\right) = 0$$

$$A_I = -p, \quad A_{II} = p \cdot \frac{\left(\frac{d}{2}\right)^2}{\left(\frac{D}{2}\right)^2 - \left(\frac{d}{2}\right)^2} = p \cdot \frac{d^2}{D^2 - d^2}, \quad B_{II} = p \cdot \frac{\left(\frac{d}{2}\right)^2 \cdot \left(\frac{D}{2}\right)^2}{\left(\frac{D}{2}\right)^2 - \left(\frac{d}{2}\right)^2} = p \cdot \frac{d^2 \cdot D^2}{4 \cdot (D^2 - d^2)}$$

Řešení 3:

$$\sigma_{rII}\left(\frac{d}{2}\right) = -p, \quad \sigma_{tII}\left(\frac{d}{2}\right) = p \cdot \frac{\left(\frac{D}{2}\right)^2 + \left(\frac{d}{2}\right)^2}{\left(\frac{D}{2}\right)^2 - \left(\frac{d}{2}\right)^2} = p \cdot \frac{D^2 + d^2}{D^2 - d^2}$$

$$\delta_I = \frac{p \cdot \frac{d}{2}}{E} \cdot (1 - \mu)$$

$$\delta_{II} = \frac{p \cdot \frac{d}{2}}{E} \cdot \left(\frac{D^2 + d^2}{D^2 - d^2} + \mu\right)$$

Deformační podmínka:

$$\delta_I + \delta_{II} = \delta$$

$$p \cdot \left(1 - \mu + \frac{D^2 + d^2}{D^2 - d^2} + \mu\right) = \frac{2 \cdot \delta}{d} \cdot E \rightarrow p = E \cdot \frac{\delta}{d} \cdot \frac{D^2 - d^2}{D^2}$$