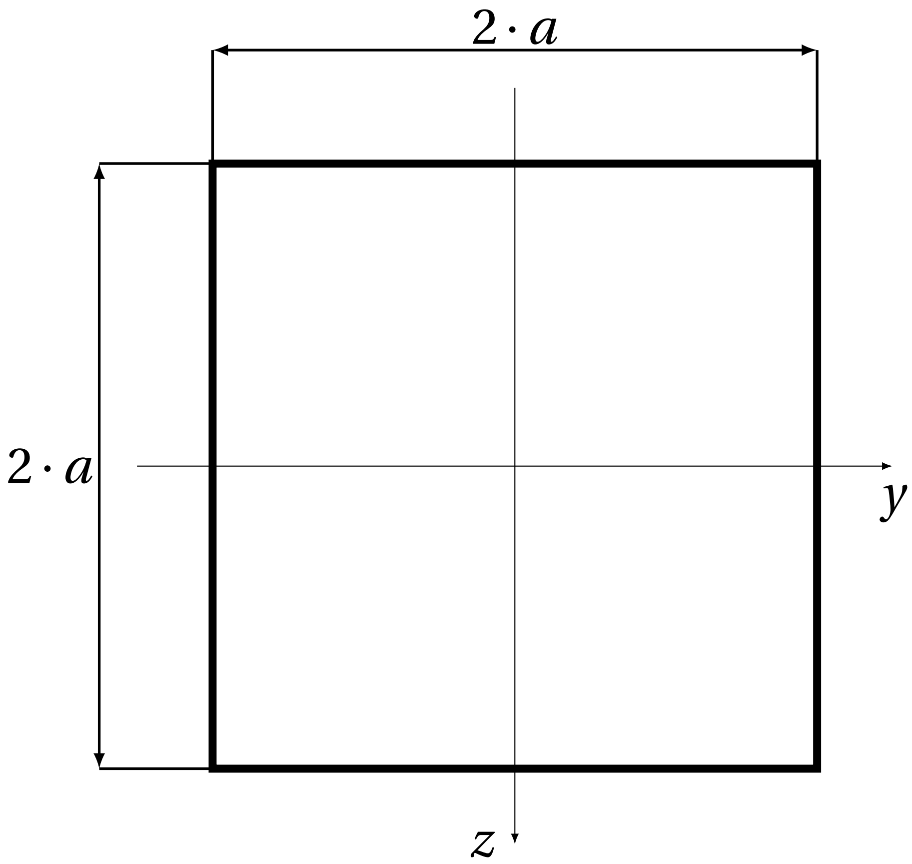
Příklad 1:

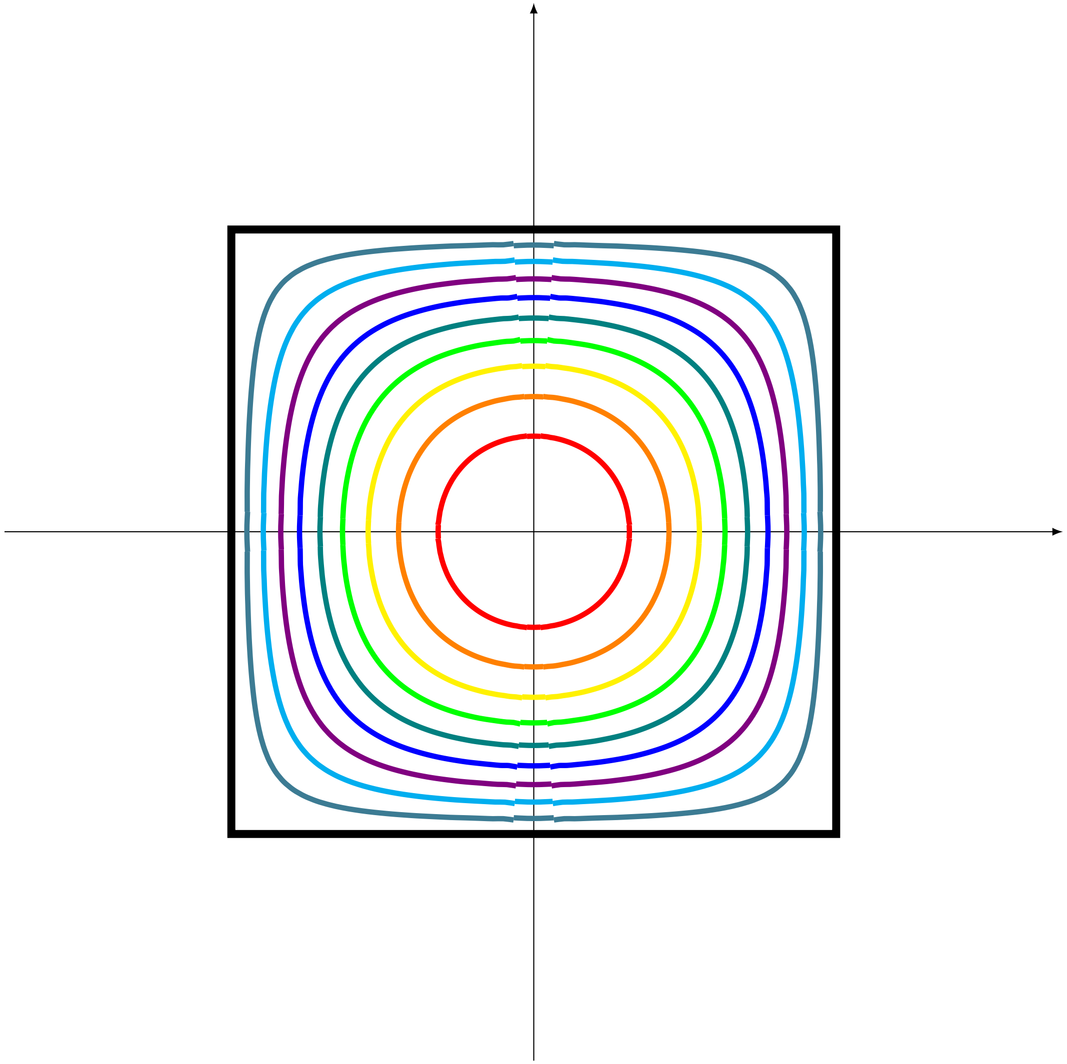


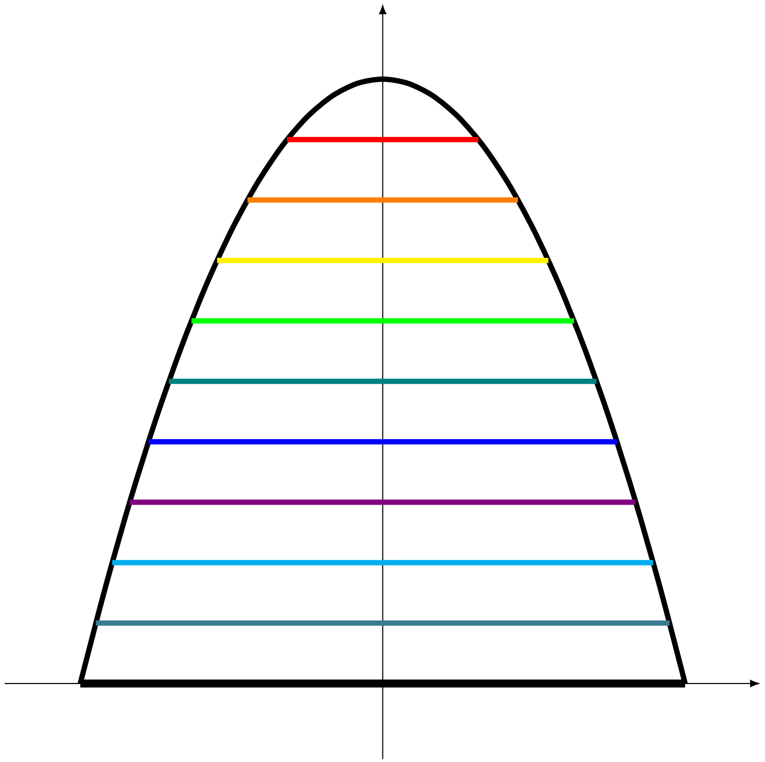
Dáno: a
Určete J_k a W_k.

Průhyb membrány nad čtvercovým otvorem:

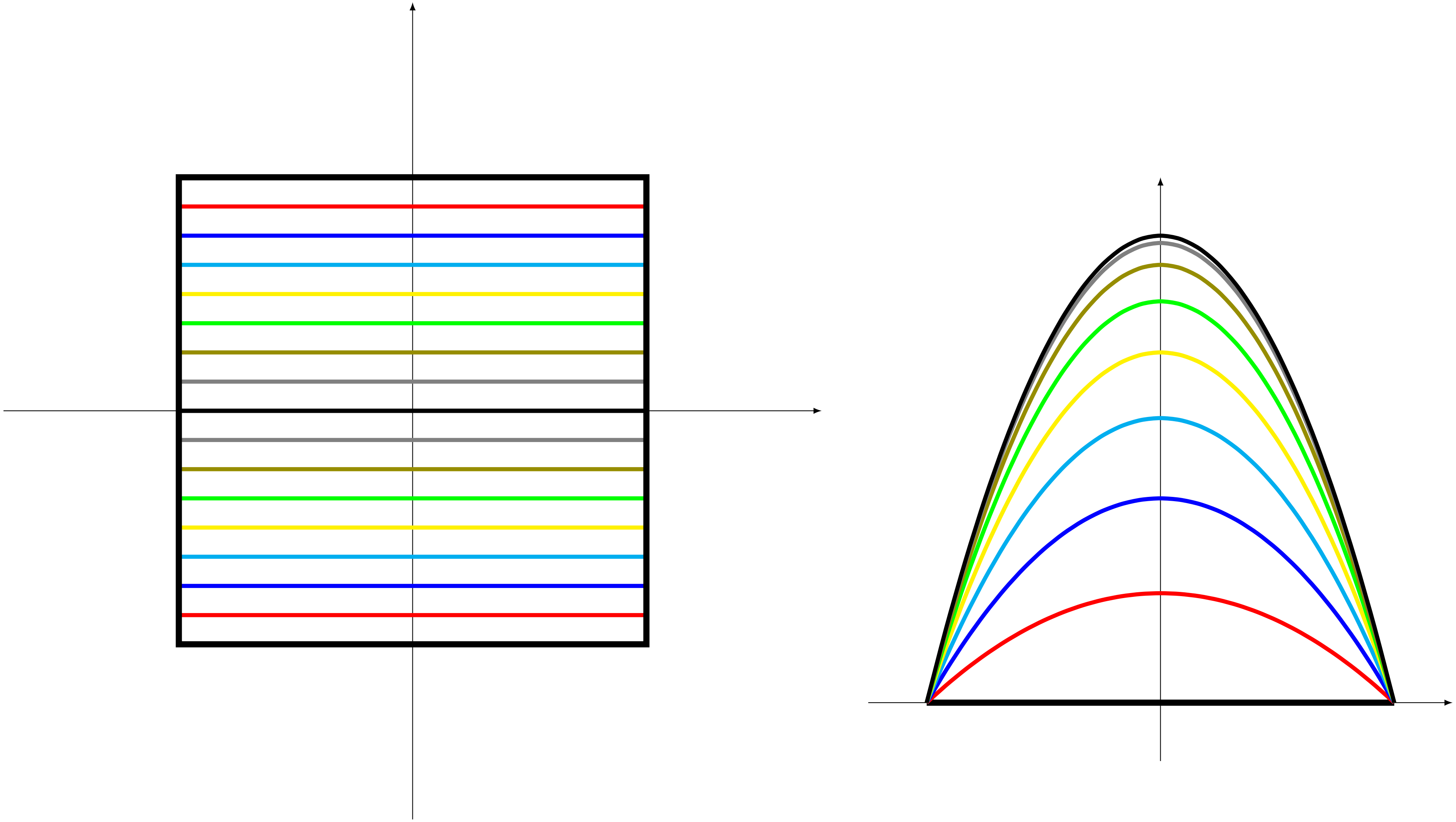
$$\xi \doteq \frac{5}{16 \cdot a^2} \cdot \frac{p}{F} \cdot \left(a^2 - y^2\right) \cdot \left(a^2 - z^2\right)$$

Řešení 1:





Řešení 1:



Řešení 1:

$$2\cdot\iint_S \xi\left(y,z\right)dy\cdot dz = M_k,\quad 2\cdot G\cdot\vartheta = \frac{p}{F}$$

$$M_k = 2\cdot\iint_S \frac{5}{16\cdot a^2}\cdot 2\cdot G\cdot\vartheta\cdot\left(a^2-y^2\right)\cdot\left(a^2-z^2\right)dy\cdot dz =$$

$$= \frac{10}{8}\cdot\frac{G\cdot\vartheta}{a^2}\cdot\int_{-a}^{a}\int_{-a}^{a}\left(a^2-y^2\right)\cdot\left(a^2-z^2\right)dy\cdot dz =$$

$$= \frac{10}{8}\cdot\frac{G\cdot\vartheta}{a^2}\cdot\int_{-a}^{a}\left(a^2-z^2\right)\cdot\left[a^2\cdot y-\frac{y^3}{3}\right]_{-a}^{a}\cdot dz =$$

$$= \frac{10}{8}\cdot\frac{G\cdot\vartheta}{a^2}\cdot\left(\frac{4\cdot a^3}{3}\right)^2$$

$$M_k = \frac{20}{9}\cdot G\cdot\vartheta\cdot a^4 = G\cdot J_k\cdot\vartheta \rightarrow J_k = \frac{20}{9}\cdot a^4$$

Řešení 1:

$$\Phi \doteq \frac{5}{16 \cdot a^2} \cdot 2 \cdot G \cdot \vartheta \cdot \left(a^2 - y^2\right) \cdot \left(a^2 - z^2\right)$$

$$\tau_{xy} = \frac{\partial \Phi}{\partial z}, \quad \tau_{zx} = -\frac{\partial \Phi}{\partial y}$$

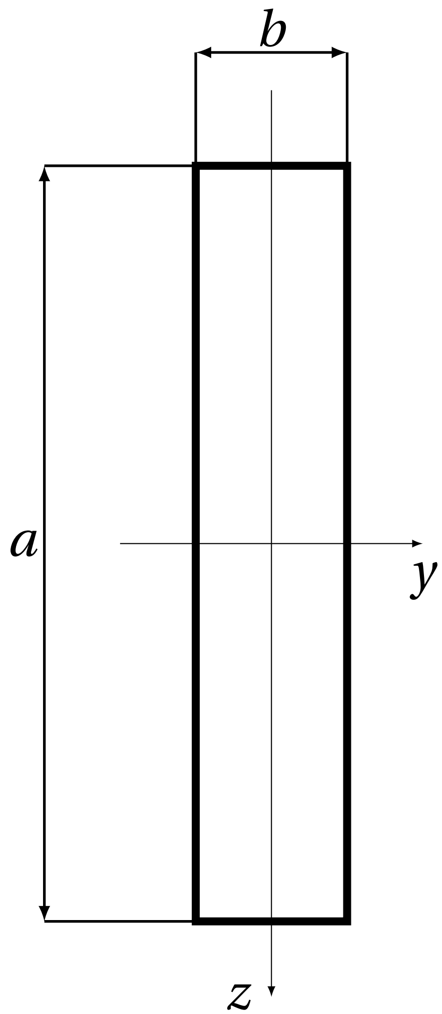
$$\tau_{xy} = \frac{5 \cdot G \cdot \vartheta}{8 \cdot a^2} \cdot \left(a^2 - y^2\right) \cdot \frac{\partial \left(a^2 - z^2\right)}{\partial z} = \frac{5 \cdot G \cdot \vartheta}{8 \cdot a^2} \cdot \left(a^2 - y^2\right) \cdot (-2) \cdot z$$

$$\tau_{xy_{max}} = \tau_{xy}\Big|_{\substack{y=0 \\ z=a}} = \left| -\frac{5 \cdot G \cdot \vartheta}{4 \cdot a^2} \cdot a^3 \right|$$

Protože $G \cdot \vartheta = \frac{9}{20} \cdot \frac{M_k}{a^4}$, dostaneme:

$$\tau_{xy_{max}} = \frac{M_k}{W_k} = \frac{5}{4} \cdot \frac{9}{20} \cdot \frac{M_k}{a^4} \cdot a = \frac{9}{16 \cdot a^3} \cdot M_k \rightarrow W_k = \frac{16}{9} \cdot a^3 \doteq 1,78 \cdot a^3$$

Příklad 2:



Dáno: $a \gg b$, b
Určete J_k a W_k.

Průhyb membrány nad obdélníkem $a \gg b$:

$$\xi \doteq \frac{p}{8 \cdot F} \cdot \left(b^2 - 4 \cdot y^2\right)$$

Řešení 2:

$$2\cdot\iint\limits_S \xi\left(y,z\right)dy\cdot dz = M_k,\quad 2\cdot G\cdot\vartheta=\frac{p}{F}$$

$$M_k = 2\cdot a\cdot\int\limits_{-\frac{b}{2}}^{\frac{b}{2}}\frac{1}{8}\cdot 2\cdot G\cdot\vartheta\cdot\left(b^2-4\cdot y^2\right)dy=$$

$$=\frac{G\cdot\vartheta\cdot a}{2}\cdot\left[b^2\cdot y-\frac{4\cdot y^3}{3}\right]_{-\frac{b}{2}}^{\frac{b}{2}}=G\cdot\vartheta\cdot\frac{a\cdot b^3}{3}$$

$$M_k = G\cdot J_k\cdot\vartheta\rightarrow J_k=\frac{1}{3}\cdot a\cdot b^3$$

$$G\cdot\vartheta=\frac{3\cdot M_k}{a\cdot b^3}$$

Řešení 2:

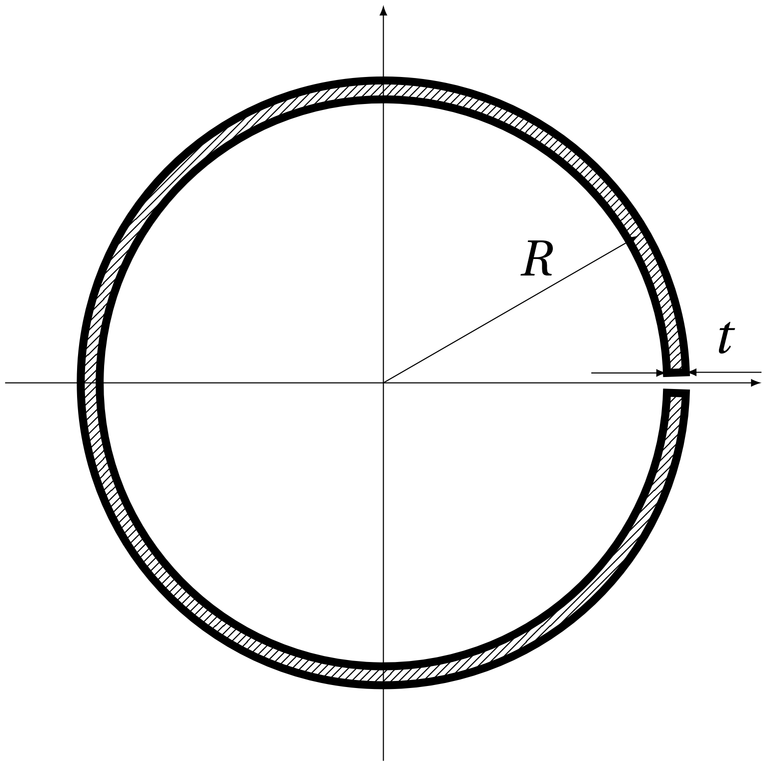
$$\tau_{max} = \tau_{zx}\Big|_{\substack{y=\frac{b}{2}\\ z=0}} = \left|\frac{\partial \Phi}{\partial y}\right|_{\substack{y=\frac{b}{2}\\ z=0}} = \frac{G\cdot\vartheta}{4}\cdot\left|\frac{\partial\left(b^2-4\cdot y^2\right)}{\partial y}\right| = \frac{3\cdot M_k}{4\cdot a\cdot b^3}\cdot 8\cdot\frac{b}{2}$$

$$\tau_{max} = \frac{M_k}{W_k} = \frac{3\cdot M_k}{a\cdot b^2} \longrightarrow W_k = \frac{1}{3}\cdot a\cdot b^2$$

Štíhlý obdélník: $J_k = \frac{1}{3}\cdot a\cdot b^3$ a $W_k = \frac{1}{3}\cdot a\cdot b^2$

Příklad 3:

Dáno: $R \gg t$, t
Určete J_k a W_k.
Porovnejte J_k a W_k otevřeného a uzavřeného profilu.



Řešení 3:

Otevřený profil:

$$J_{k_o} = \frac{2}{3} \cdot \pi \cdot R \cdot t^3 \doteq 2,1 \cdot R \cdot t^3, \quad W_{k_o} = \frac{2}{3} \cdot \pi \cdot R \cdot t^2$$

Uzavřený profil:

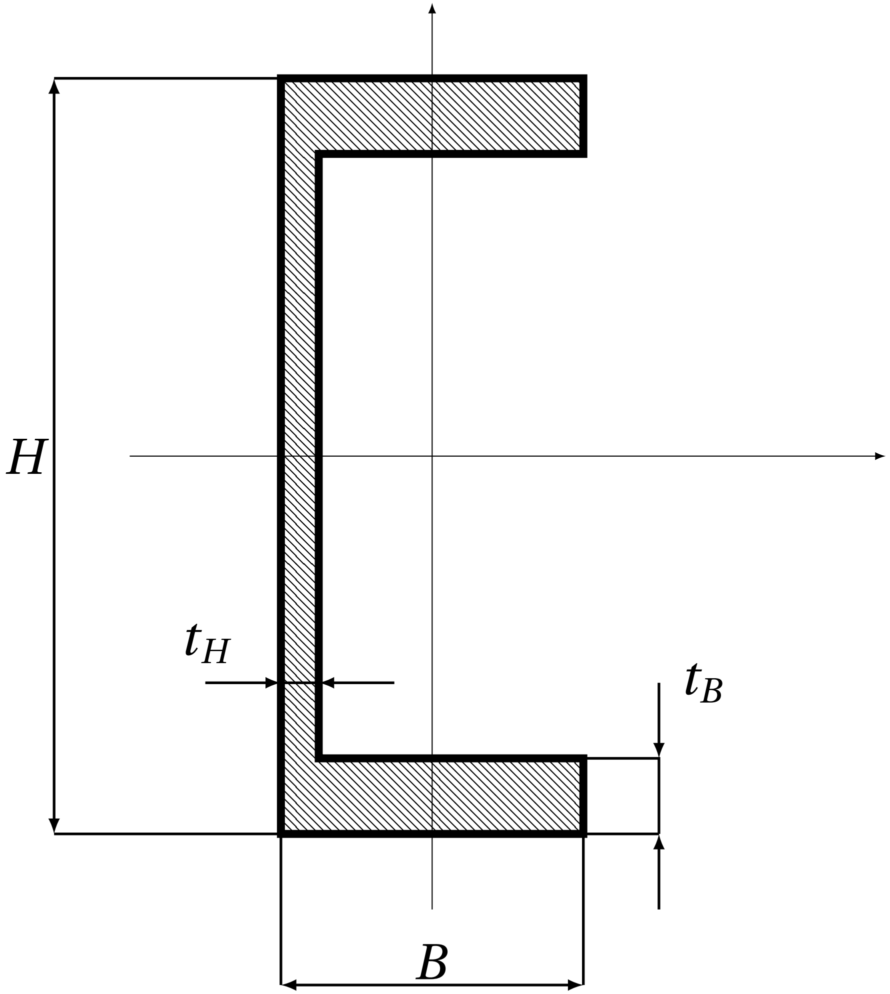
$$J_{k_u} = 2 \cdot \pi \cdot R^3 \cdot t + \frac{\pi}{2} \cdot R \cdot t^3, \quad W_{k_u} = \frac{4 \cdot \pi \cdot t \cdot R^3 + \pi \cdot t^3 \cdot R}{2 \cdot R + t}$$

$$\frac{J_{k_u}}{J_{k_o}} = 3 \cdot \frac{R^2}{t^2} + \frac{3}{4} \gg 1$$

$$\frac{W_{k_u}}{W_{k_o}} = \frac{12 \cdot R^2 + 3 \cdot t^2}{4 \cdot t \cdot R + 2 \cdot t^2}$$

Příklad 4:

Dáno: $H \gg t_H$, $B \gg t_B$, $t_B > t_H$
Určete J_k a W_k.



Řešení 4:

$$J_k = \frac{1}{3} \cdot \left(H \cdot t_H^3 + 2 \cdot B \cdot t_B^3 \right)$$

$$\vartheta = \frac{M_k}{G \cdot J_k} = konst. \rightarrow \tau_B = G \cdot \vartheta \cdot t_B, \quad \tau_H = G \cdot \vartheta \cdot t_H$$

$$W_k = \frac{M_k}{\tau_{max}} \rightarrow W_k = \frac{J_k}{t_B} = \frac{1}{3} \cdot H \cdot \frac{t_H^3}{t_B} + \frac{2}{3} \cdot B \cdot t_B^2$$