INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

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MICROECONOMICS

advanced course

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1 Consumer Behaviour Theory

The basic principle for the functioning of markets is a tendency towards reaching market equilibrium, i.e. finding the optimal price and quantity. The general equilibrium on the market of goods and services anticipates the achievement of current balance on the demand (consumer's) side as well as on the supply (manufacturer's) side. The following chapter shall be focused on analysing consumer's equilibrium based on cardinal and ordinal utility theories. The principles of consumer behaviour clarified in the basic course of Microeconomics I. shall be used in the analysis.

1.1 Consumer's equilibrium

Consumer's equilibrium can be characterized as a condition when a consumer reaches maximum utility under the budget constraints given. The size of utility and consumer's surplus can be expressed by means of the theory of consumer's preferences.

1.2 Consumer's preferences

A complete list of goods and services which are subject to the consumer's choice is called a "consumer basket". For the purposes of clarifying consumer's preferences, we shall use the consumer basket which includes two commodities *P* (apples) and *B* (bananas). If it is necessary to consider a larger number of various goods, then, *B* shall mean "all other goods". Consumer baskets are described using the letters of the Greek alphabet α, β, γ, δ…

1.2.1 Presumptions for applying consumer's preferences (axioms)

1.2.1.1 Comparison completeness

One of the following statements shall apply to each pair of consumer baskets α and β.

- ⇒ The consumer basket α is preferred to the consumer basket β.
- ⇒ The consumer basket β is preferred to the consumer basket α.
- ⇒ The consumer baskets α and β are equally desirable to consumers.

There is a presumption that the consumer is always able to compare two baskets of goods.

1.2.1.2 Transitivity

In case of three consumer baskets α, β and γ, it must apply that if the consumer prefers γ to β and β to α, then he must also prefer γ to α.

1.2.1.3 Reflexivity, axiom of choice

The consumer makes every effort to gain the most preferred good, i.e. for any consumer basket (e.g. for the basket α), it shall apply: $\alpha \geq \alpha$, i.e. the basket α has higher or equal utility as it has in itself. It is a mathematical condition for the existence of a function.

1.2.1.4 Continuity

The condition of continuity anticipates that the consumer requires increasing the consumption of bananas *B* under any slight decrease in the consumption of apples *A*. This assumption shall ensure the function continuity.

1.2.1.5 Axiom of non-saturation (dominance)

Two consumer baskets α and β consist of a certain amount of two goods: $\alpha = [A_0;B_0]$ and $\beta = [A_1;B_1]$. The consumer shall prefer the basket α to the consumer basket β if:

- ⇒ either: $A_0 > A_1$ and at the same time $B_0 \geq B_1$
- ⇒ or: $B_0 > B_1$ and at the same time $A_0 \geq A_1$

1.2.1.6 Preference of average to extremes, axiom of diversity

The axiom assumes that the rational consumer prefers combinations of various commodities to the consumption in which only one commodity is represented extensively (or even exclusively).

1.2.2 Utility function

The above-mentioned assumptions are a sufficient condition for expressing arranged consumer's preferences using the **utility function *U***. The utility represents a level of satisfaction arising out of consumption of combinations of goods and services. It is the quantity which shows the tendency of consumer's preferences. However, the utility **is not** an objective quantity – it is a subjective feeling of a level of satisfaction of the given need of each consumer.

To derive the utility function, we shall use basic rules:

- ⇒ We shall ascribe the same real number to all baskets of goods which are equally desirable to consumers.
- ⇒ If the consumer prefers one basket to the other basket, we shall ascribe a higher real number to the more preferred basket.

The same fact can be formally expressed as:

⇒ If α is preferred to β, then $U(\alpha) > U(\beta)$.

⇒ If α is equally desirable as β, then $U(\alpha) = U(\beta)$.

Since any numerical values which comply with the above-mentioned rule can be essentially ascribed, we can create an infinite number of utility functions for arranged preferences. Overall satisfaction of needs under the consumption of the goods given is expressed by **total utility** (*TU*).

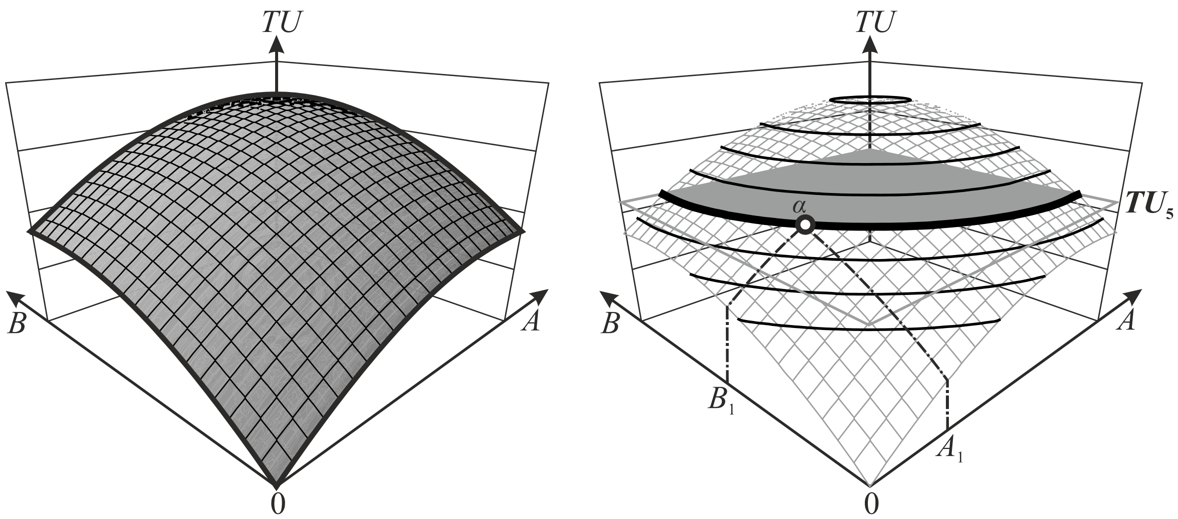
$$TU = f(A_1, A_2, A_3, ...), \tag{1.1}$$

where A_1, A_2, A_3 are amounts of consumed goods and *TU* measures the size of total utility.

Marginal utility (*MU*) represents a change in the total utility as a result of changing the goods consumed by one unit.

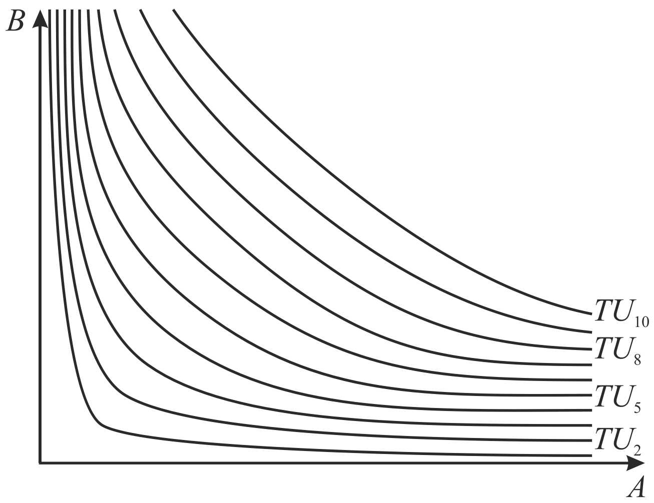
$$MU_A = \frac{\partial TU}{\partial A} \tag{1.2}$$

where MU_A represents the marginal utility of the goods A calculated as a partial derivative of the consumer's total utility (*TU*) function based on the amount of the goods *A*.



1-1 Utility hill

On the utility hill (graph 1-1), its "contours", which represent so called indifference curves and express consumer's preferences, can be defined. An indifference curve (**IC**) is a set of all combinations of goods under various consumed amounts of apples A and bananas *B* which bring the same size of total utility *TU* to the consumer. By projecting indifference curves ("contours") onto the horizontal plane, an indifference map is created.



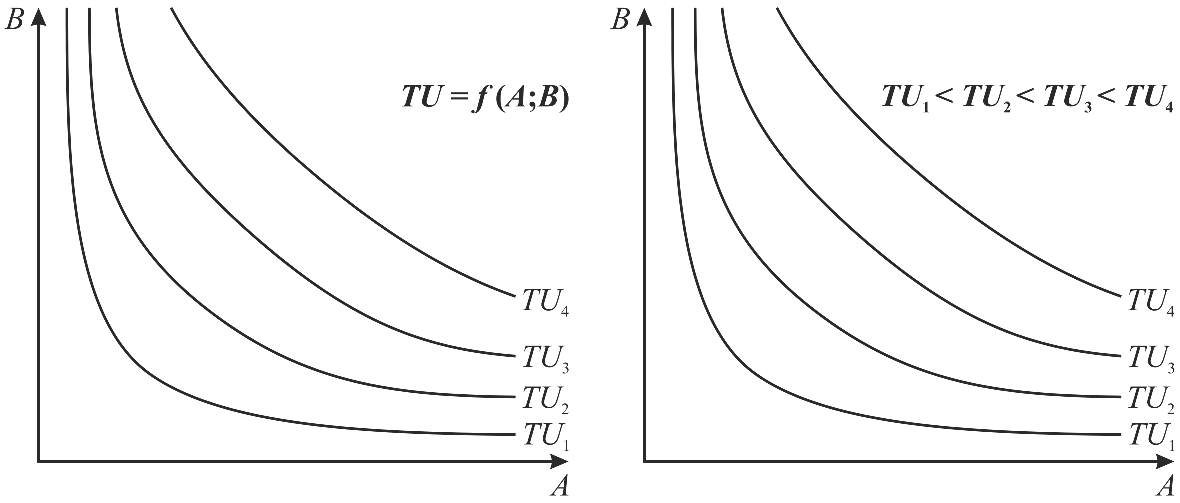
1-2 Indifference map

The issue of utility measurability led economists to elaborate two basic directions depending on the access to utility measurement.

1.2.3 Cardinal utility theory

The cardinal utility theory is based on the principle of directly measurable utility, the size of utility takes on a specific value.

Hermann Heinrich Gossen (1810-1858) was the first author to develop the marginal utility theory and theory of consumer behaviour. Other propagators of the cardinal utility theory were economists Carl Menger 1840-1921, William Jevons (1835-1882) and Léon Walras (1834-1910), who independently formulated a theory of marginal utility (in 1871) with which the beginning of the so called **marginal revolution** is associated and that became the onset of modern microeconomics. The cardinal theory of marginal utility was elaborated by Alfred Marshall (1842-1924) who recognised that the consumer measures his utility indirectly, i.e. by means of money.



1-3 Ordinal and cardinal utility concepts

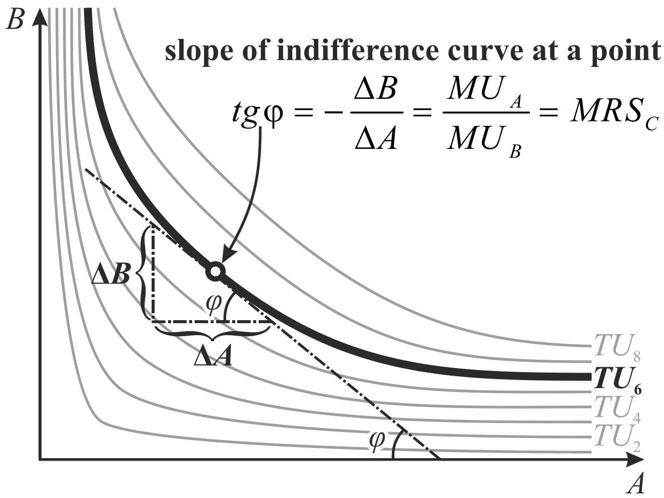
1.2.4 Ordinal utility theory

The ordinal utility theory is based on the presumption that utility is not directly measurable but the consumer is able to assess what combination of goods brings him higher utility compared to the other one. When constructing indifference curves, the ordinalist Vilfredo Pareto (1848-1923) utilized the fact that the consumer is able to arrange combinations of goods according to the size of utility (while specifying their order) which they bring him but he is not able to assign them cardinal numbers which would express the level of satisfaction. Pareto defined a theory of welfare and social optimum. Pareto optimality is such a situation (such distribution of goods) when it is not possible to increase (by redistributing the goods) the level of satisfaction of any individual (to move him to a higher indifference curve) without decreasing someone else's level of satisfaction (to move someone else to a lower indifference curve). Pareto suboptimality occurs when it was possible to increase someone's level of satisfaction by redistributing the goods without decreasing someone else's level of satisfaction.

The theory of marginal utility was extended by Eugen E. Slutsky (1880-1948), a Russian economist and statistician, who added an analysis of the consumer decision making process which is influenced by price changes and changes in real income on the market. John Richard Hicks (1904-1989) drew up a line of consumer's possibilities and derived conditions of consumer's equilibrium. A significant movement in microeconomic theory consisted in applying the ordinal theory of value to prices of production factors.

The slope (gradient) of an indifference curve represents the rate at which the consumer is willing to substitute (replace) the good A with the good *B* and vice versa whereas after this substitution, the consumer's position shall be as good as before the substitution, i.e. it shall reach the same size of total utility. This rate represents the **marginal rate of substitution in consumption** (MRS_C). In case of convex indifference curves, MRS_C (slope of indifference curves) is descending which means that the consumer's willingness to exchange applesA*A* for bananas *B* increases with the increasing consumption of apples *A*.

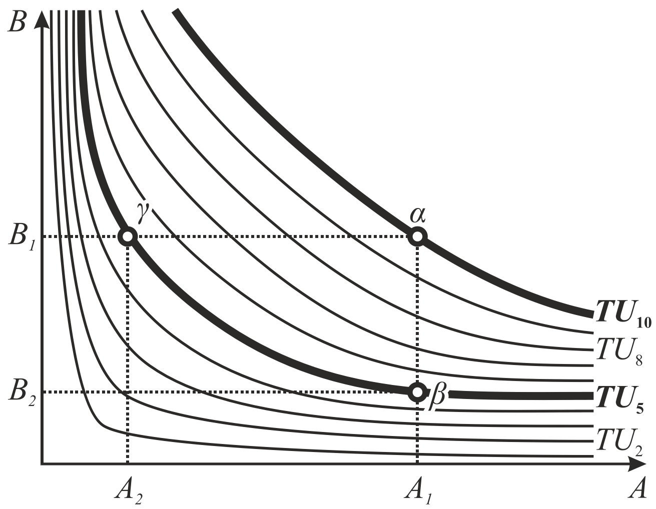
$$MRS_C = \frac{\Delta B}{\Delta A} = \frac{MU_A}{MU_B} = \frac{\frac{\partial TU}{\partial A}}{\frac{\partial TU}{\partial B}} \quad (1.3)$$



1-4 Slope (gradient) of an indifference curve at a point

The axiom of comparison completeness in the graphical representation of indifference curves means that if the consumer basket *α* is preferred to the consumer basket *β*, then, *α* lies on a higher indifference curve than *β*. It analogously applies that if the consumer basket *β* is preferred to the consumer basket *α*, then, *β* lies on a higher indifference curve than *α*. If the consumer baskets *α* and *β* are equally desirable to the consumer, then, α and β lie on the same indifference curve. The axiom of comparison completeness leads to the fact that exactly one indifference curve is at each point of the consumer's situation. **The axiom of transitivity** means that *α* lies on a lower indifference curve than *β* which lies on a lower indifference curve than *γ*. The axiom of transitivity ensures that the indifference curves of one rational consumer never intersect. **The axiom of non-saturation** eliminates the existence of goods with negative preferences and ensures that indifference curves have a negative gradient. Due to this axiom, the width of the indifference curve also corresponds exactly to one point in the graphical representation. **The axiom of diversity** or preferring the average to extremes ensures a strictly convex shape of indifference curves.

The above-mentioned assumptions allow for arranging consumer's preferences. If we suppose the existence of three baskets, the consumer can arrange them e. g. in the order $\alpha > \beta = \delta$ shown in the following figure.

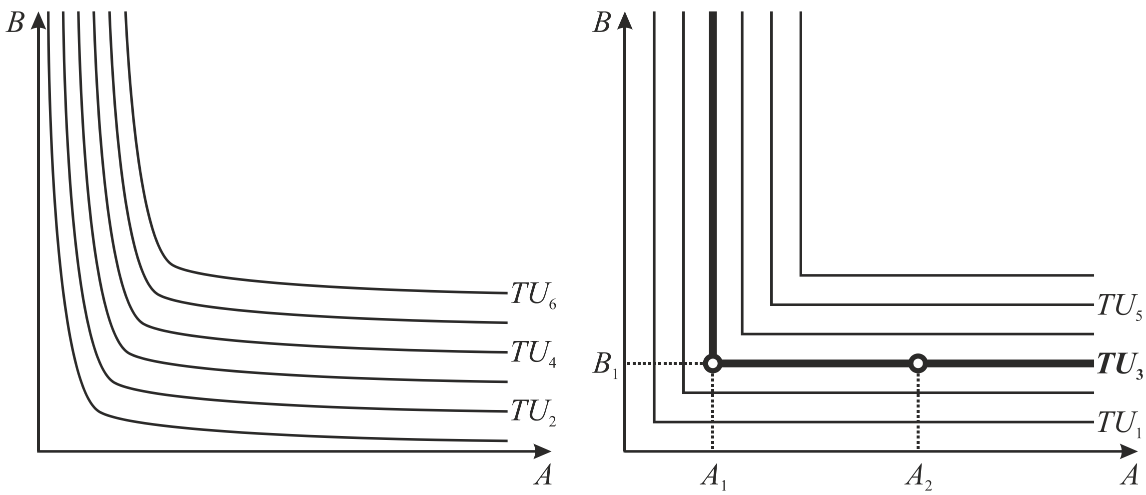


1-5 Consumer's preferences in the indifference map

1.2.5 Specific examples of preferences

In the case of specific products, the relation of preferences to indifference curves can be expressed as follows:

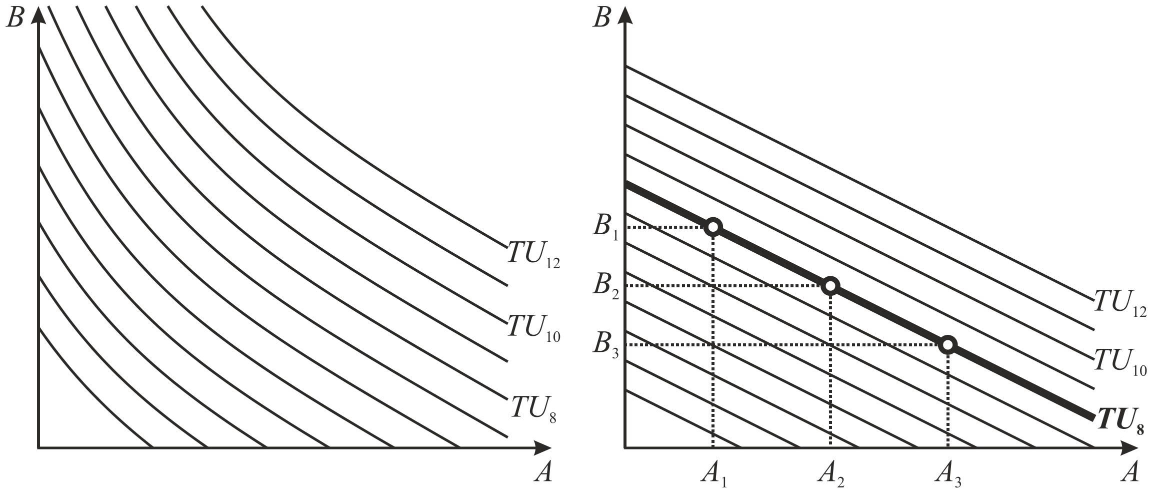
⇒ Complementary goods and perfect complements (MRS_C = 0 or MRS_C = ∞)



1-6 Indifference map for complements and perfect complements

⇒ Substitute goods and perfect substitutes (MRS_C = –1)

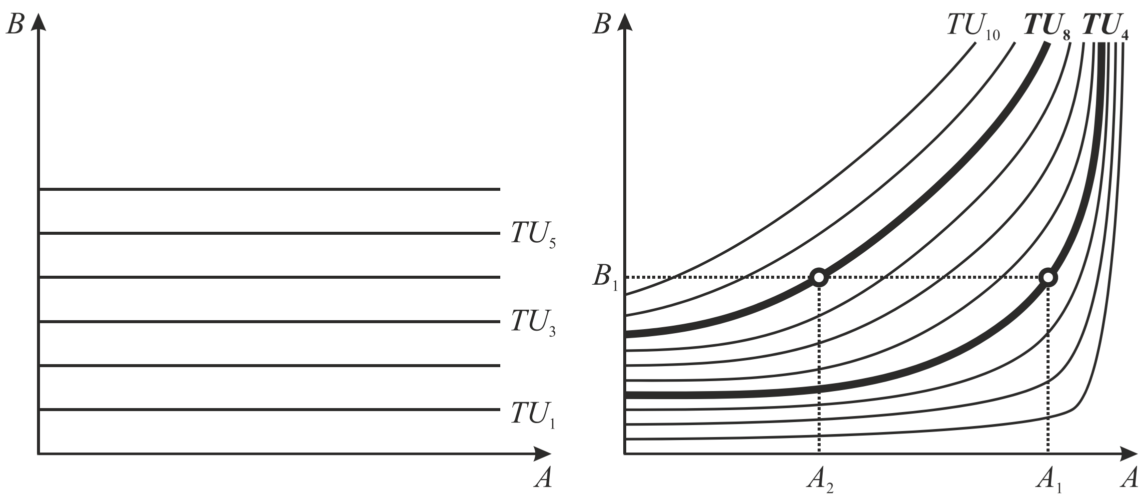
Let's compare the shape of indifference curves in the graph 1-5 with the left parts of the graphs 1-6 and 1-7. In the graphs 1-5 and 1-6, the consumer is not willing to **entirely** give up apples A or bananas *B*. Therefore, he considers both the commodities more as complements. However, the indifference curves intersect both the vertical axis and horizontal axis in the graph 1-7. The consumer is then willing to fully exchange the consumption of apples A for the consumption of bananas *B* and vice versa. Both the commodities are substitutes.



1-7 Indifference map for substitutes and perfect substitutes

We do not often find these crystal clear options of substitutes and complements in practice. Under low purchased amounts of both the goods A and B, the consumer often prefers the consumption of both the commodities to the option when he gives up one of them in favour of the other one (see the axiom of diversity). The goods A and B are more likely to have characteristics of complements. At the moment when the consumer buys a relatively high amount of both the commodities and reaches higher levels of total utility, he shall show greater willingness to mutually substitute the commodities (usually with regard to their individual prices). Thus, the deflection of indifference curves can be changed (reduced) with the growing utility reached, as shown e. g. in the graph 1-5.

⇒ The neuter A ($MRS_C = 0$) and the bad A ($MRS_C > 0$)



1-8 Indifference map for the neuter and the bad *A*

For the neuter, it applies that an increase in its consumption does not increase or decrease the total utility of the consumer. Since the commodity B is a desirable good, the total utility of the consumer is increasing only if the consumed amount of the good B is increasing. Thus, the total utility of the consumer is positively dependent on the consumed amount of the desirable good B but is not dependent on the consumed amount of the neuter A, and therefore, their indifference curves are of a horizontal shape and the marginal rate of substitution in consumption is neutral.

It analogously applies to the bad A that an increase in its consumption leads to a decrease in the total utility. The total utility of the consumer is then positively dependent on the consumed amount of the desirable good B and negatively dependent on the consumed amount of the bad A, therefore, the indifference curves have an ascending tendency and the marginal rate of substitution in consumption is positive.

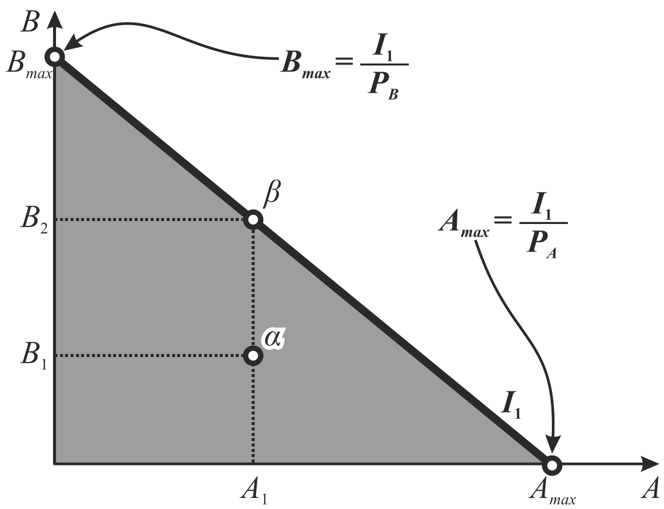
1.3 Budgetary possibilities of the consumer

We define a set of combinations of goods and services available to the consumer by means of **non-negativity** conditions (consumption of commodities is non-negative, i.e. $A \geq 0$ and $B \geq 0$) and **budget constraints**. It shall apply to budget constraints of the consumer:

$$I \geq P_A \cdot A + P_B \cdot B, \qquad (1.4)$$

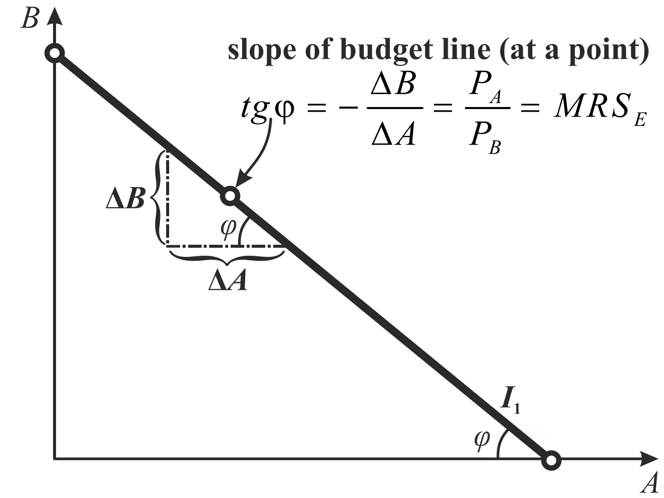
where A is the quantity of the goods A (apples), B is the quantity of the goods B (bananas), P_A is the price of the goods A (apples) and P_B is the price of the goods B (bananas).

Let us presume that the consumer buys only the above-mentioned goods A and B for the prices P_A and P_B, which are higher than zero, and he is limited by a certain amount of income I_1 in his decision. **The budget line** (*BL*) then represents the upper limit of budgetary possibilities of the consumer. All the combinations of the goods A and B for which the consumer shall spend all his income lie on this line segment. Together with the area under this line segment, the budget line forms a set of so called budgetary possibilities, i.e. such combinations of the goods A and B which the consumer can afford to buy for his income I_1.



1-9 Budgetary possibilities of the consumer

$$I = P_A \cdot A + P_B \cdot B \text{, or } B = \frac{I}{P_B} - \frac{P_A}{P_B} \cdot A \qquad (1.5)$$



1-10 Slope (gradient) of the budget line at a point

The slope (or gradient) of the budget line expresses the rate by which the consumer is ready to substitute the goods A and *B* on the market while spending all his disposable income. This rate represents the **marginal rate of substitution in exchange** (MRS_E).

$$MRS_E = \frac{\Delta B}{\Delta A} = -\frac{P_A}{P_B} \qquad (1.6)$$

With increasing income, budgetary possibilities of the consumer are increasing, with decreasing income, they are decreasing. If the prices of the goods P_A, P_B remain constant, the slope of the budget line remains the same, the marginal rate of substitution in exchange remains unchanged. When the prices of the goods (*A* or *B*) are changed, there shall be a change in the marginal rates of substitution in exchange as well as in the slope of the budget line (*BL*). If there is an increase e. g. in the market price of the goods A, the budgetary possibilities of the consumer shall be decreased and the budget line shall be lowered due to the movement of the end point on the axis of the goods A to the left (towards the origin). The slope of the budget line shall change since the rate of prices shall change as a result of increasing prices P_A.

1.4 Consumer's equilibrium (optimum)

We speak of the consumer's optimum when the utility is maximized depending on consumer's preferences and market possibilities. The market possibilities are influenced by disposable income and prices of consumed goods.

The situation can be described as follows:

$$\max U = f(A, B) \text{ under the restriction } I = P_A \cdot A + P_B \cdot B, \qquad (1.7)$$

where A ≥ 0 and $B \geq 0$.

In case of cardinal theories, it applies to the consumer's optimum

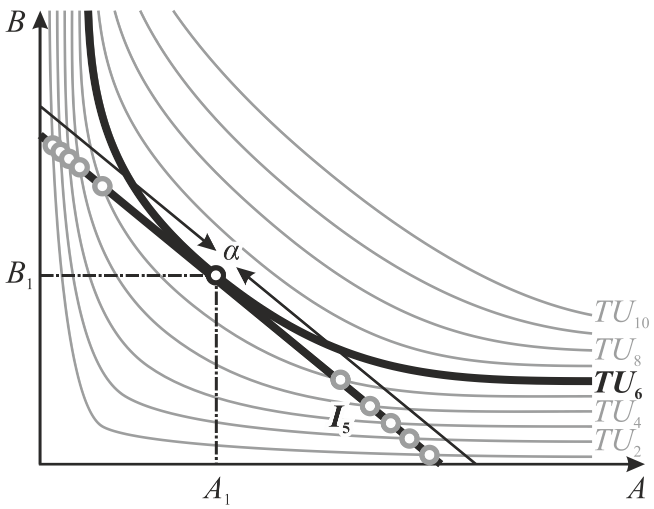
- ⇒ When consuming one commodity, $MU_A = P_A$ (for the last unit of the commodity consumed)
- ⇒ For the combination of two consumed commodities, it applies: $MU_A/P_A = MU_B/P_B$ (for the last consumed units of the goods A and B)

The ordinal theory defines the consumer's optimum as equality of the marginal rate of substitution in consumption MRS_C and the marginal rate of substitution in exchange MRS_E, i.e.:

$$MRS_C = MRS_E \text{ or } \frac{MU_A}{MU_B} = \frac{P_A}{P_B} \text{ or otherwise } \frac{MU_A}{P_A} = \frac{MU_B}{P_B} \qquad (1.8)$$

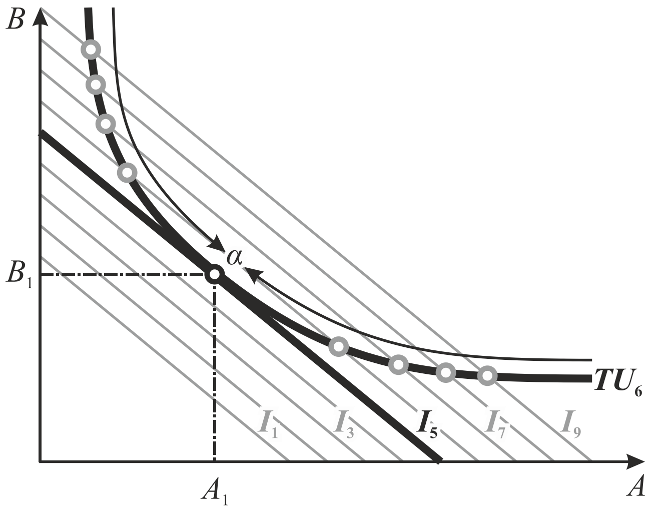
The consumer's optimum is graphically at the point where the budget line touches the indifference curve – an internal solution. The consumer's optimum can be specified based on model behaviour when the consumer's objective is:

- ⇒ Utility maximization under budget constraints, i.e. the consumer tries to reach the farthest indifference curve by moving over the budget constraint (graph 1-11).



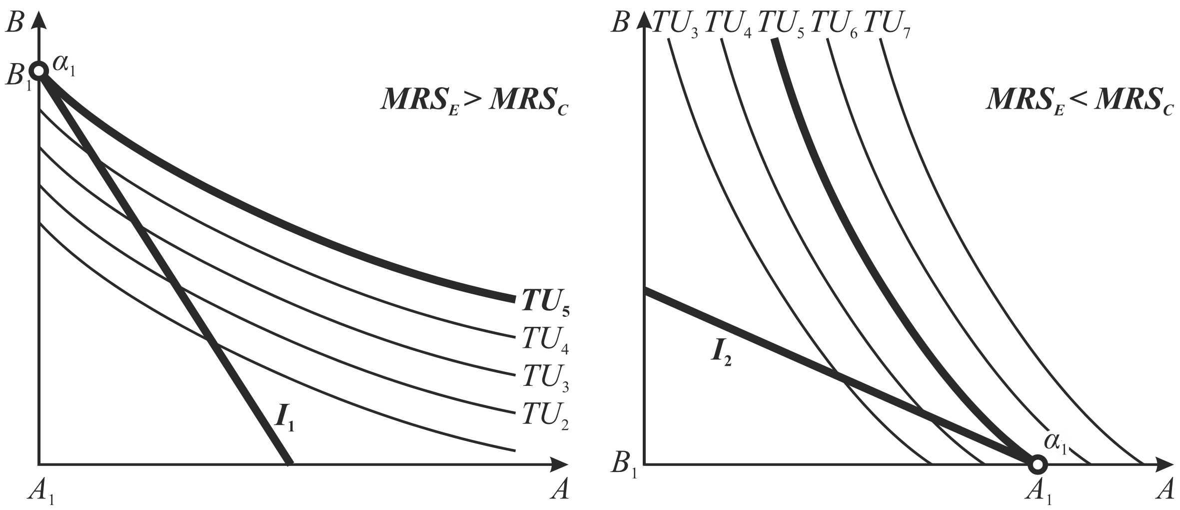
1-11 Reaching the consumer's optimum under utility maximization

⇒ Minimization of expenses towards the utility required, i.e. the consumer looks for the lowest budget line by moving on the indifference curve which represents the required level of utility (graph 1-12).



1-12 Reaching the consumer's optimum under minimization of expenses

Consumer's optimum – a corner solution. In this case, MRS_C is not usually equal to MRS_E. Both the commodities are logically substitutes, the rate of the prices of both the commodities shall be decisive for the consumer. If the apples A are substantially more expensive (graph 1-13 on the left) than the bananas *B*, the consumer shall naturally buy only bananas and vice versa (graph 1-13 on the right).

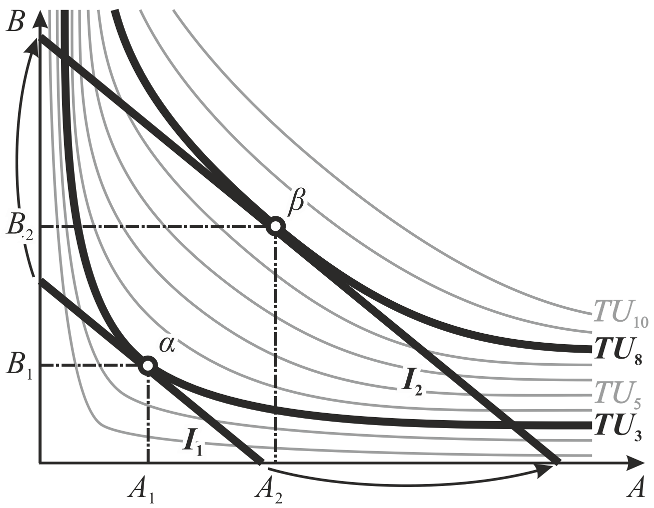


1-13 Corner solution of the consumer's optimum

The consumer's optimum, representing the optimal combination of consumed goods, is not invariable. Any changes in disposable income, changes in the goods A or *B*, changes in preferences are incentives for reaching a new state of equilibrium.

1.4.1 Change in disposable income

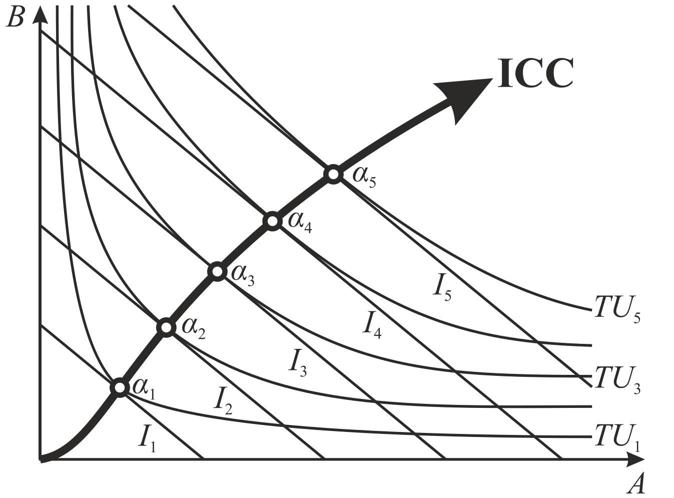
An increase in disposable income results in a parallel shift in the budget line from I_1 to I_2 in the graph 1-14. The gradient of all budget lines MRS_E remains the same (prices of the goods P_A a P_B remain unchanged). The consumer reaches higher total utility (consumer basket β) when consuming a higher amount of the goods A (increase from A_1 to A_2) and B (increase from B_1 to B_2). The size of MRS_C is still the same at both points of optimum since MRS_E also remains unchanged.



1-14Increasing the consumer's budget from I_1 to I_2

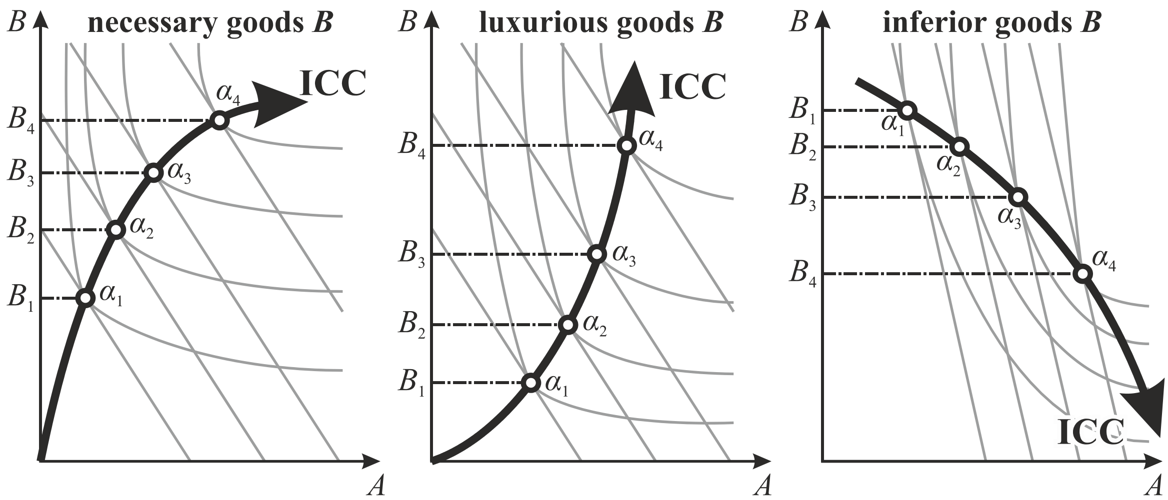
1.4.1.1 Income consumption curve ICC

If we connect the points of optimum (α_1, α_2, α_3,…) corresponding to individual levels of income (I_1, I_2, I_3,…), we get the income consumption curve (**ICC**).



1-1 Income consumption curve

ICC is a set of combinations of consumption of two commodities A and B at which the consumer maximizes the utility at various levels of income. For normal goods, ICC is an ascending curve since the purchased amount of the goods increases with growing income (for more detailed characteristics and classification see the chapter 1.6.2 on the page 28).



1-15 Shape of the income consumption curve depending on the type of the goods *B*

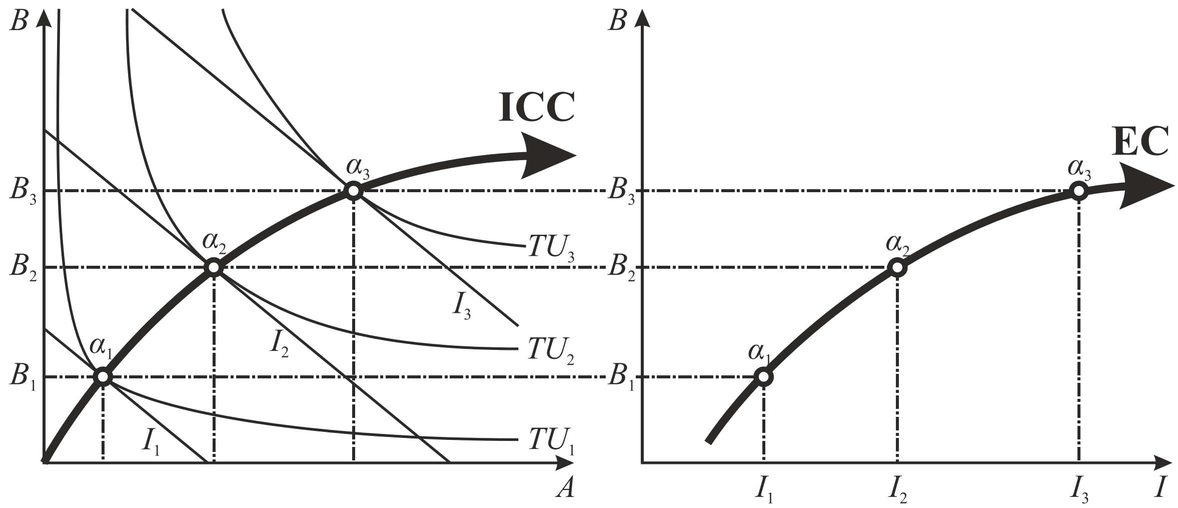
⇒ For necessary goods (with increasing income, the consumed amount grows slower than income), ICC is ascending, positively sloped and concave.

⇒ For luxurious goods (with increasing income, the consumed amount grows faster than income), ICC is ascending, positively sloped and convex.

⇒ For inferior goods (the consumed amount falls with increasing income), ICC is descending and negatively sloped.

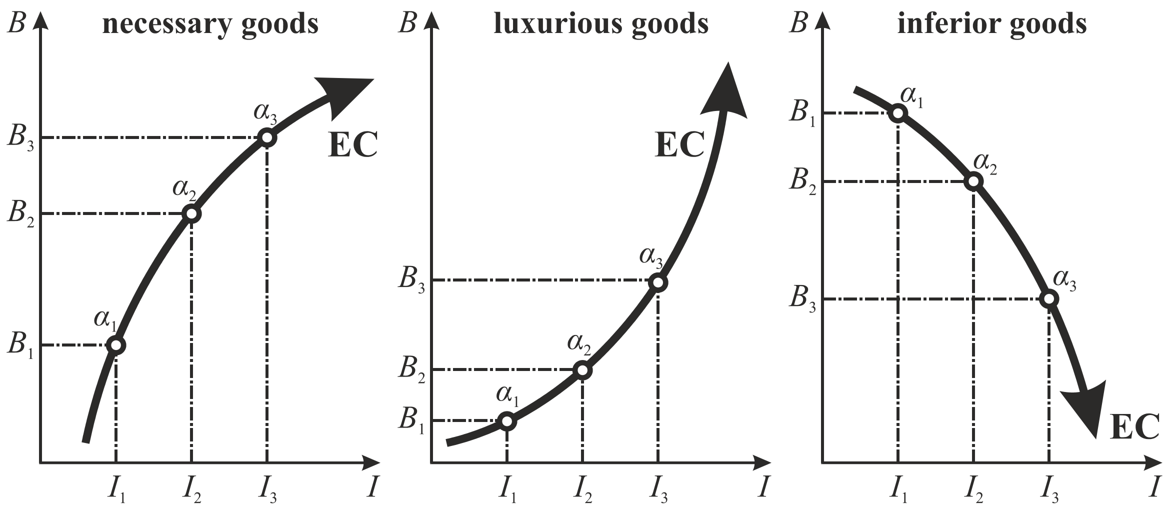
1.4.1.2 Engel curve

The dependence between the consumed amount of a certain commodity and the amount of total income is expressed by an Engel curve (**EC**).



1-16 Deriving an Engel curve from an income consumption curve

We can derive the Engel curve from the income consumption curve when the amount of consumed goods can be expressed for each level of income from the point of optimum. For normal goods, EC is ascending and positively sloped (consumed amount is growing with increasing income).



1-17Shape of the Engel curve depending on the type of the goods *B*

- ⇒ For the necessary goods *B*, EC is ascending, positively sloped and concave (with increasing income, consumed amount grows slower than income).
- ⇒ For the luxurious goods *B*, EC is ascending, positively sloped and convex (with increasing income, consumed amount grows faster than income).
- ⇒ For the inferior goods *B*, EC is descending and negatively sloped (with increasing income, consumed amount falls).

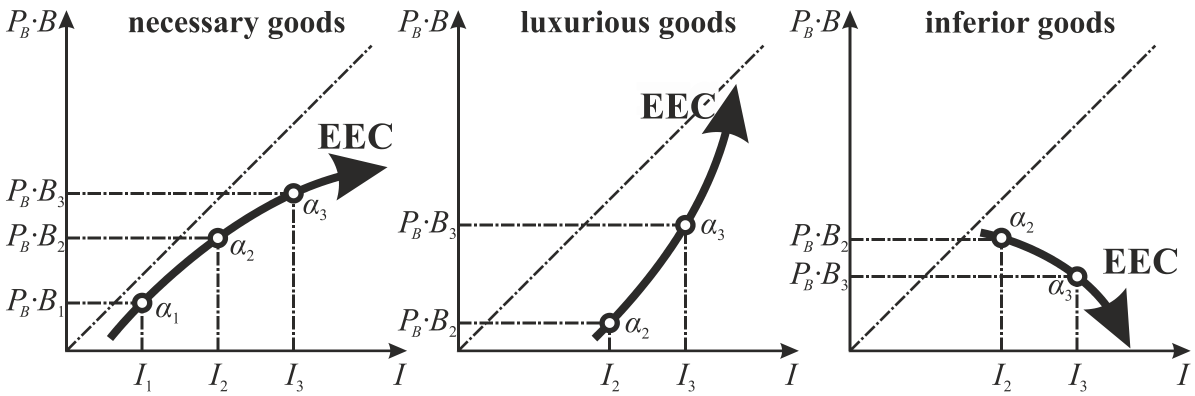
1.4.1.3 Engel's expenditure curve

The dependence between the amount of total expenditure for purchasing a certain commodity (e. g. $P_B \cdot B$ in case of the goods B) and the amount of consumer's income I is expressed by the Engel expenditure curve (**EEC**). For normal goods, EEC is ascending (with increasing income, the consumed amount grows). The axis 45° captures all points when expenditure for purchasing the goods B exactly corresponds to the amount of total income, therefore, it is marginal for EEC.

⇒ For the necessary goods, it applies that the share in total expenditure falls with increasing income and that EEC is ascending, it moves away from the axis 45° (with increasing income, the consumed amount grows slower than income).

⇒ For the luxurious goods, it applies that the share in total expenditure grows with increasing income and that EEC is ascending, it moves towards the axis 45° (with increasing income, the consumed amount grows faster than income).

⇒ For the inferior goods, it applies that expenditure falls with increasing income and that EEC is descending, it moves away from the axis 45° (with increasing income, the consumed amount falls).



1-18 Shape of the Engel expenditure curve depending on the type of the goods *B*

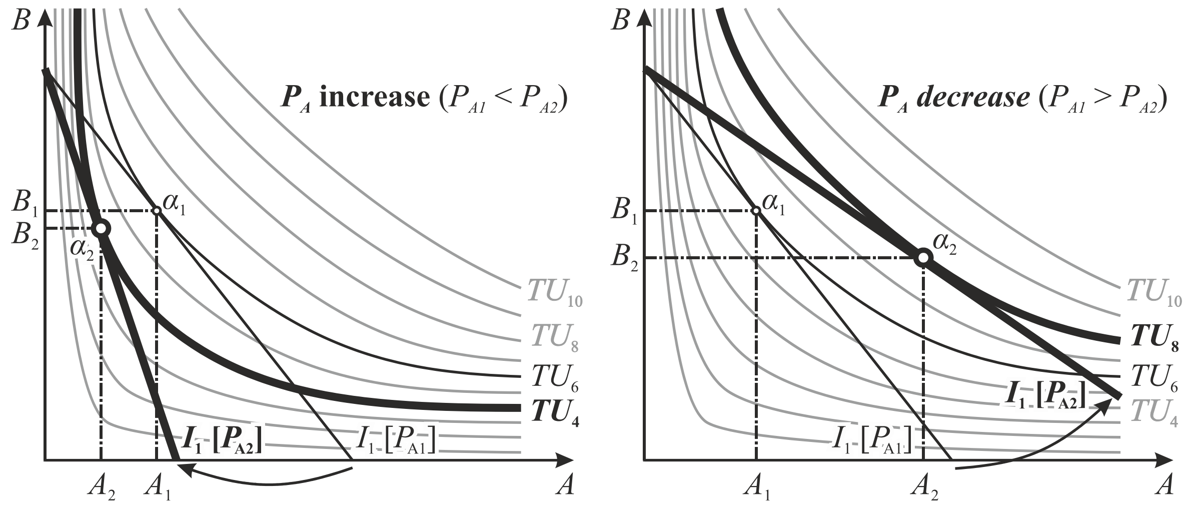
1.4.2 A change in the price of the goods A

The change in the price of the goods A results in changing the slope of the budget line, the gradient of the budget line MRS_E is changing at the constant price of the goods B. For a new point of consumer's equilibrium, the budget line becomes a tangent of another indifference curve, i.e. the total utility achieved is changed and so is MRS_C.

The change in the price P_A results in changing the **real income** and **relative prices**. The total change in the demanded amount is given by the sum of:

⇒ a change in the demanded amount caused only by changing the real income – so called an **income effect** when a decrease in the price P_A results in real valuation of consumer's income (he can buy larger amounts of apples A and bananas B for the same income I_1 – naturally provided that both the commodities considered are normal and desired commodities for the consumer),

⇒ a change in the demanded amount caused only by changing relative prices ˜– so called a **substitution effect** when a decrease in the price P_A makes the customer to substitute a relatively more expensive commodity (in this case, bananas *B*) with a relatively cheaper commodity (in our situation, apples *A*).

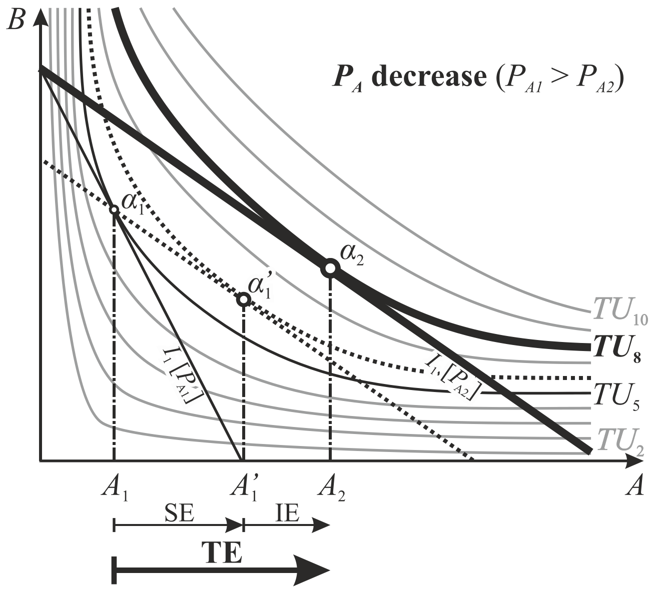


1-19 Impact of changing the price P_A on the consumer's optimum

However, the approach to defining a substitution effect admits two possible variants.

1.4.2.1 E. Slutsky's decomposition

In this case, a (Slutsky's) substitution effect is defined on the basis of preserving **constant and real income** which is such income at which the consumer is able to buy the original consumer basket (purchase power is preserved at the original level, the size of the utility reached changes when P_A decreases, it is higher). The budget line moves around the original consumer's choice α_1. We get the income effect by adjusting purchase power while preserving price ranges. The budget line moves parallel up to the right to the final $I_1[P_{A2}]$.

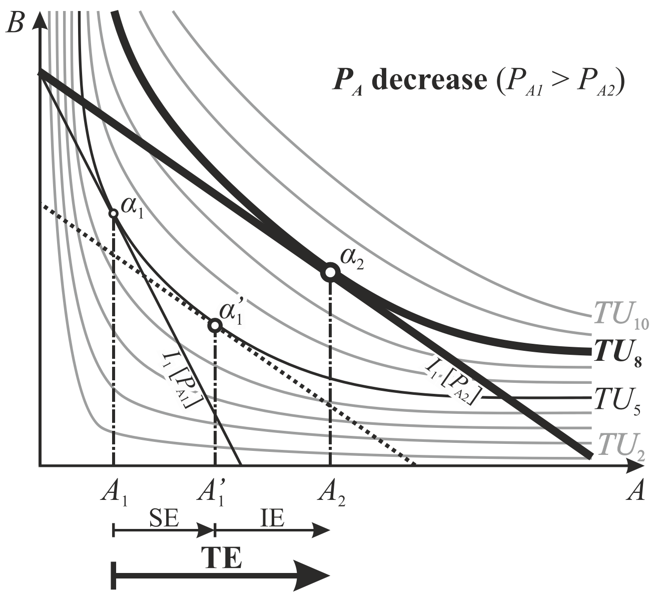


1-20 Slutsky's decomposition of a decrease in price to substitution and income effects

The substitution effect usually acts against changing the price (with increasing P_A, the demanded amount A is falling as a result of the substitution effect), i.e. it has a **negative direction**. The income effect acts against changing the price (with increasing price P_A, the real income of the consumer falls and so falls the amount A) and it has a **negative direction** if it concerns a normal commodity (i.e. necessary or luxurious). In case of inferior commodities, the income effect works in the same direction in which the price is changed (with increasing P_A, the real income of the consumer falls and the amount A increases), i.e. it has a **positive direction**. In the figure shown, the goods A is a normal commodity, the substitution effect and income effect shall be added together into the total effect.

1.4.2.2 J. R. Hicks' decomposition

(Hicks') substitution effect preserves **constant utility** which means that the budget line is moving on the indifference line going through the original consumer's choice (size of achieved utility is equal, purchasing power of the consumer is not sufficient for purchasing the original consumer basket but it enables purchasing the consumer basket α_1', which is indifferent to the original consumer basket α_1). We get the income effect by adjusting purchase power while preserving price ranges for the new consumer basket required. The budget line moves parallel up to the right to the final $I_1[P_{A2}]$.

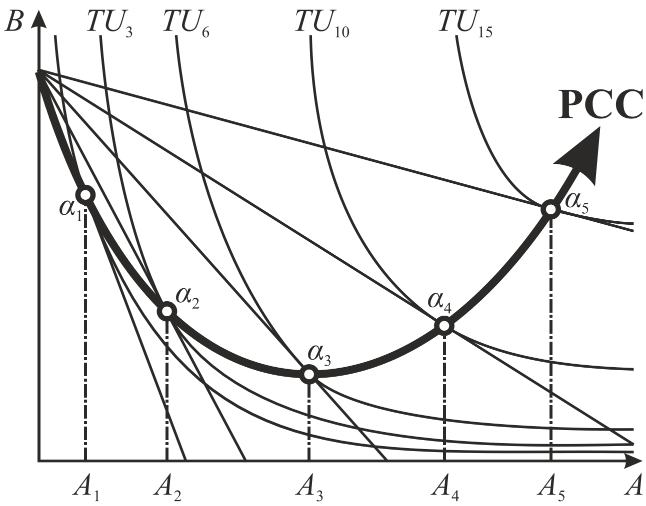


1-21 Hicks' decomposition of a decrease in price to substitution and income effects

The substitution effect usually acts against changing the price (with increasing P_A, the demanded amount A falls as a result of the substitution effect), i.e. it has a **negative direction**. The income effect acts against changing the price (with increasing price P_A, the real income of the consumer falls and so falls the amount A) and it has a **negative direction** if it concerns a normal commodity (i.e. necessary or luxurious). In case of inferior commodities, the income effect works in the same direction in which the price is changed (with increasing P_A, the real income of the consumer falls and the amount A increases), i.e. it has a **positive direction**. In the figure shown, the good A is a normal commodity, the substitution effect and income effect shall be added together into the total effect.

1.4.2.3 Price consumption curve PCC

The price consumption curve PCC is a set of combinations of consumption of the goods A and B maximizing the utility at various prices of the goods A (or at various prices of the goods B if the price of the goods A is constant). We get PCC by connecting points of optimum at various prices of the goods A. If PCC is descending in the interval (0-A_3), then, the decrease in the price of the goods A leads to increasing the consumption of the goods A and to decreasing the consumption of the goods B. In the interval from the point A_3, when PCC is ascending, there is an increase in the consumption of the goods A as well as an increase in the consumption of the goods B with the decrease in the price of the goods A. If there is a corner solution of optimum when A = 0 (prices of B and income amount are constant), PCC can only reach the threshold for maximum purchase of the goods B.



1-22 Price consumption curve PCC

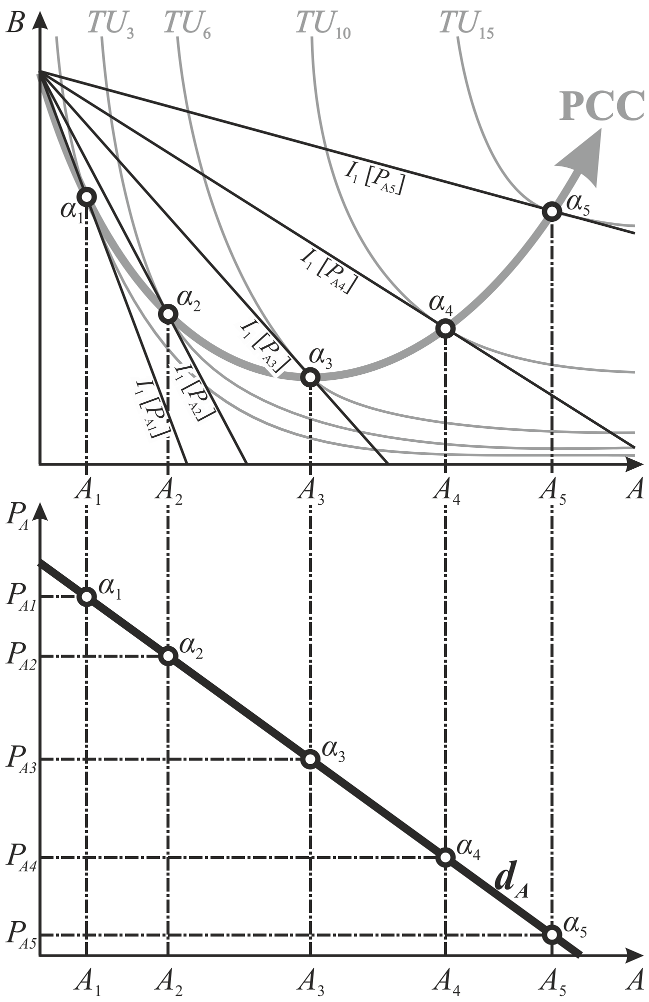
1.5 Individual demand

Points on the price consumption curve are a basis for deriving a curve of demand for the goods *A*. The individual demand curve d_A expresses the dependence of the demanded amount of the goods A on the price P_A at the assumption of rational optimization of consumption, i.e. maximization of utility given the budget constraints. The individual demand for the goods A is then the function of price P_A, prices of other goods (P_B) and consumer's income *I*.

1.5.1 Deriving demand based on utility maximization

Functions when the purchased amount of the goods depends on the consumer's income and prices of commodities (under the preferences given) are referred to as **Marshallian demand functions:**

$$\begin{matrix} MRS_C = MRS_E \\ I = P_A \cdot A + P_B \cdot B \end{matrix} \Rightarrow \begin{matrix} q_A = f(I, P_A, P_B) \\ q_B = f(I, P_A, P_B) \end{matrix} \tag{1.9}$$



1-23 Deriving individual demand from the price consumption curve

1.5.2 Deriving demand based on expenditure minimization

When deriving, we come out of the consumer's optimum minimizing his expenditure under the given prices of the goods and size of required utility. The demand function when the purchased amount of the goods depends on the prices of the goods under the given amount of utility is referred to as the **Hicksian demand function**.

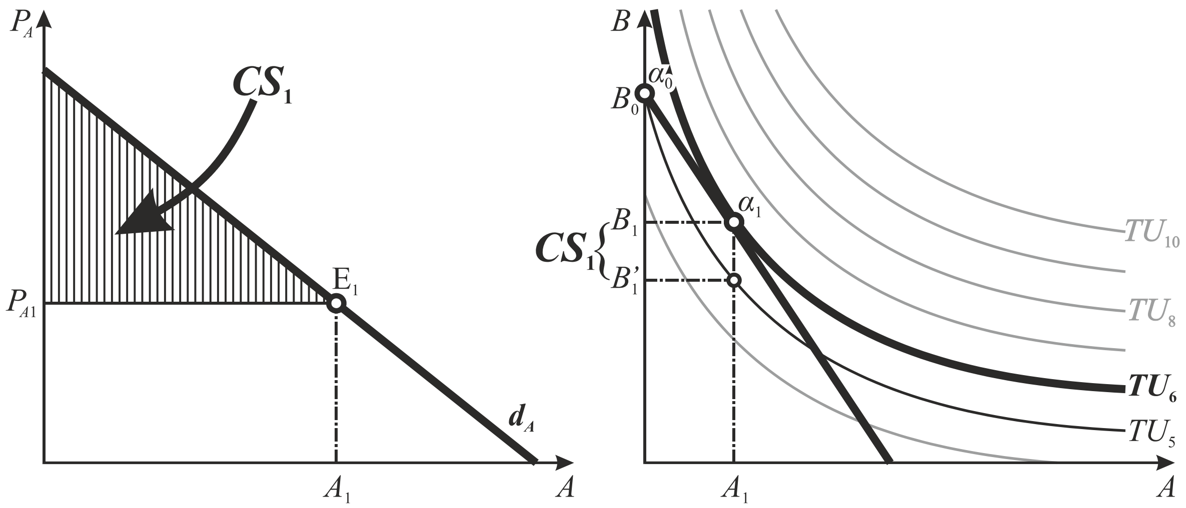
$$\begin{matrix} MRS_C = MRS_E \\ U_o = f(A,B) \end{matrix} \Rightarrow \begin{matrix} q_A = f(U_o, P_A, P_B) \\ q_B = f(U_o, P_A, P_B) \end{matrix} \tag{1.10}$$

1.5.3 Deriving demand based on maximization of consumer surplus

The term "consumer surplus" (*CS*) means the difference between the total utility from the consumption of the goods TU_A and total expenditure of the consumer spent on purchasing the given amount of the commodity $P_A \cdot A$, or it is the difference between the willingness and necessity to sacrifice a part of income for acquiring the required amount of the goods. The consumer surplus from purchasing and consuming the goods A can be calculated as:

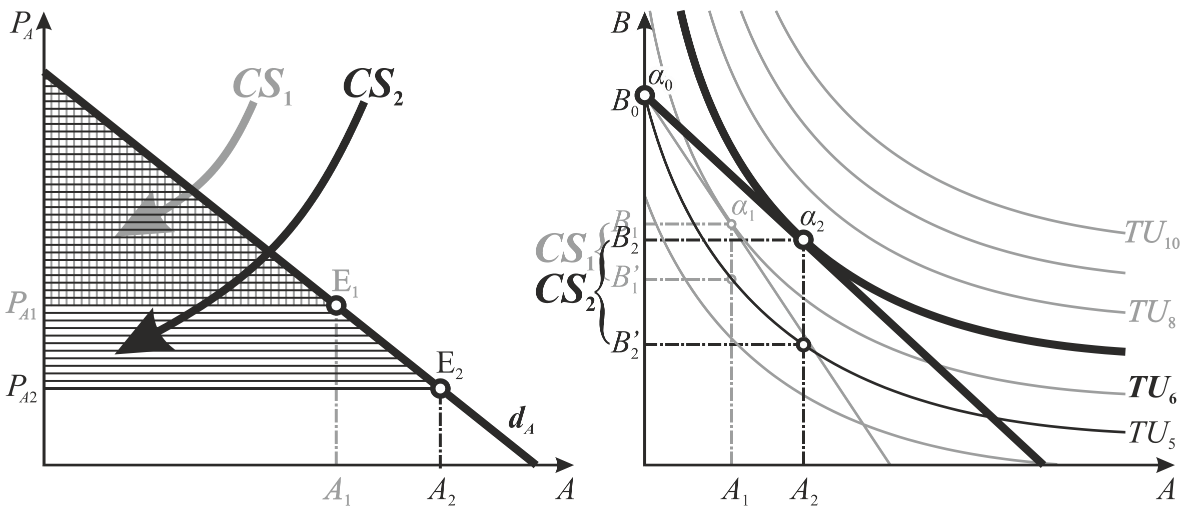
$$CS_A = TU_A - P_A \cdot A \quad (1.11)$$

The consumer surplus can be expressed by using the function of marginal utility as well as by means of indifference curves. Let's bear in mind that in this situation, the goods B represent all other consumed commodities or expenditure spent on them in the indifference analysis.



1-24 Consumer surplus in the graph of marginal utility and in the indifference analysis

The change in the consumption of the goods and amount of consumer surplus caused by the decrease in price of the goods is expressed in the following graphs.



1-25 Change in the amount of consumer surplus at the decrease in price P_A

1.6 Demand elasticity

The demand elasticity quantifies the ability or willingness of the consumer to react to various changes on the market (especially to changes in prices of purchased goods and income changes).

1.6.1 Price elasticity of demand

Price elasticity of demand $\boldsymbol{e_{PD}}$ expresses a percentage change in the demanded amount which is caused by a one-percent change in the price. The price elasticity of demand can be determined according to the following formulas:

⇒ Generally:

$$e_{PD} = \frac{\%\Delta A}{\%\Delta P_A} = \frac{A_2 - A_1}{A_1} : \frac{P_{A2} - P_{A1}}{P_{A1}} \tag{1.12}$$

⇒ More exactly:

$$e_{PD} = \frac{A_2 - A_1}{\frac{A_2 + A_1}{2}} : \frac{P_{A2} - P_{A1}}{\frac{P_{A2} + P_{A1}}{2}} = \frac{A_2 - A_1}{A_2 + A_1} : \frac{P_{A2} - P_{A1}}{P_{A2} + P_{A1}} \tag{1.13}$$

⇒ At the point $[A_1; P_{A1}]$:

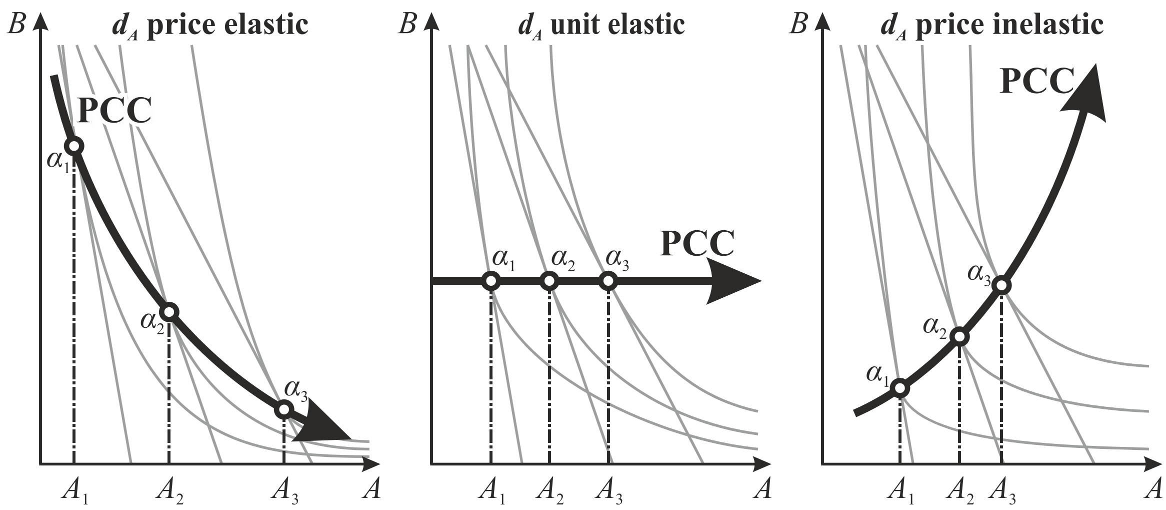
$$e_{PD} = \frac{\partial A}{\partial P_A} : \frac{A_1}{P_{A1}} \tag{1.14}$$

In the formulas (1.12) and (1.13), we measure the elasticity between two points the coordinates of which on the demand curve are $[A_1;P_{A1}]$ and $[A_2;P_{A2}]$. The demand elasticity directly at the only specific point of the curve with the coordinates $[A_1;P_{A1}]$ is calculated by means of a partial derivative of the demand curve function according to the price P_A.

The price elasticity of demand usually reaches negative values (with increasing price P_A, the demanded amount A is decreased).

- ⇒ At the value $e_{PD} = 0$, we speak of perfectly price inelastic demand.
- ⇒ If the value $-1 < e_{PD} < 0$, it concerns price inelastic demand.
- ⇒ If the value $e_{PD} = -1$, it concerns unit elastic demand.
- ⇒ If the value $e_{PD} < -1$, it concerns price elastic demand.
- ⇒ At the value $e_{PD} = -\infty$, we speak of perfectly price elastic demand.

The shape of PCC curve depends on the price elasticity of demand, i.e. as follows:



1-26 Shape of the price consumption curve depending on the price elasticity of demand

Let's pay attention to the relation between the graph 1-27 and the graph 1-24 on the page 25. It is clear that the linear demand curve d_A shall be in compliance with the descending course of PCC price elastic in its upper part while in its lower part, d_A shall be price inelastic (ascending part of PCC). The point (α_3 in the graph 1-24 on the page 25) at which the price elasticity of demand shall be equal to – 1 (minimum PCC) shall then lie just in the middle of the demand curve.

1.6.2 Income elasticity of demand

Income elasticity of demand e_{ID} measures the relationship between a percentage change in the purchased amount of the given goods which is caused by a one-percent income change. The income elasticity of demand can be calculated by means of formulas:

⇒ Generally:

$$e_{ID} = \frac{\%\Delta A}{\%\Delta I} = \frac{A_2 - A_1}{A_1} : \frac{I_2 - I_1}{I_1} \tag{1.15}$$

⇒ More exactly:

$$e_{ID} = \frac{A_2 - A_1}{\frac{A_2 + A_1}{2}} : \frac{I_2 - I_1}{\frac{I_2 + I_1}{2}} = \frac{A_2 - A_1}{A_2 + A_1} : \frac{I_2 - I_1}{I_2 + I_1} \tag{1.16}$$

⇒ At the point [A_1; P_{A1}]:

$$e_{ID} = \frac{\partial A}{\partial I} : \frac{A_1}{I_1} \tag{1.17}$$

The income elasticity reaches positive values or negative values depending on a specific type of goods. For **normal** goods, the income elasticity reaches positive values. As normal goods, we consider necessary and luxurious goods.

⇒ For inferior goods, the value shall be $e_{ID} < 0$.

⇒ For necessary goods, the income elasticity of demand shall reach the values $0 < e_{ID} < 1$.

⇒ For luxurious goods, it shall apply that the value shall be $e_{ID} > 1$.

Let's pay attention to the relationship between the income elasticity of demand and the shape of the Engel curve (EC) in the graph 1-18 on the page 19 and the shape of the Engel curve for expenditure (EEC) in the graph 1-19 on the page 20. These relationships can be simply summarized by means of the following table:

1-1 Relationship between the shape of the Engel curve and the Engel curve for expenditure and income elasticity of demand

	income elasticity	EC course	EEC course
inferior goods	$e_{ID} < 0$	descending	descending
necessary goods	$0 < e_{ID} < 1$	ascending, concave	ascending, concave
luxurious goods	$e_{ID} > 1$	ascending, convex	ascending, convex

1.6.3 Cross elasticity of demand

In relation to income changes and changes in prices of individual commodities, the so called **cross elasticity of demand e$_{CD}$** is also enforced. The cross elasticity expresses the percentage by which the demanded amount of the goods A changes when the price P_B changes by one percentage. For calculation of income elasticity, the following formulas shall be used:

⇒ Generally:

$$e_{CD} = \frac{\%\Delta A}{\%\Delta P_B} = \frac{A_2 - A_1}{A_1} : \frac{P_{B2} - P_{B1}}{P_{B1}} \tag{1.18}$$

⇒ More exactly:

$$e_{CD} = \frac{A_2 - A_1}{\frac{A_2 + A_1}{2}} : \frac{P_{B2} - P_{B1}}{\frac{P_{B2} + P_{B1}}{2}} = \frac{A_2 - A_1}{A_2 + A_1} : \frac{P_{B2} - P_{B1}}{P_{B2} + P_{B1}} \tag{1.19}$$

⇒ At the point [A_1; P_{A1}]:

$$e_{CD} = \frac{\partial A}{\partial P_B} : \frac{A_1}{P_{B1}} \tag{1.20}$$

The cross elasticity of demand depends on the mutual relation of the goods A and B:

⇒ For perfect complements, the value shall be $e_{CD} = -\infty$.

⇒ For complements, the value shall be $e_{CD} < 0$.

⇒ For substitutes, the value shall be $e_{CD} > 0$.

⇒ For perfect substitutes, the value shall be $e_{CD} = +\infty$.

The sum of price, income and cross elasticity of demand equals to 0.

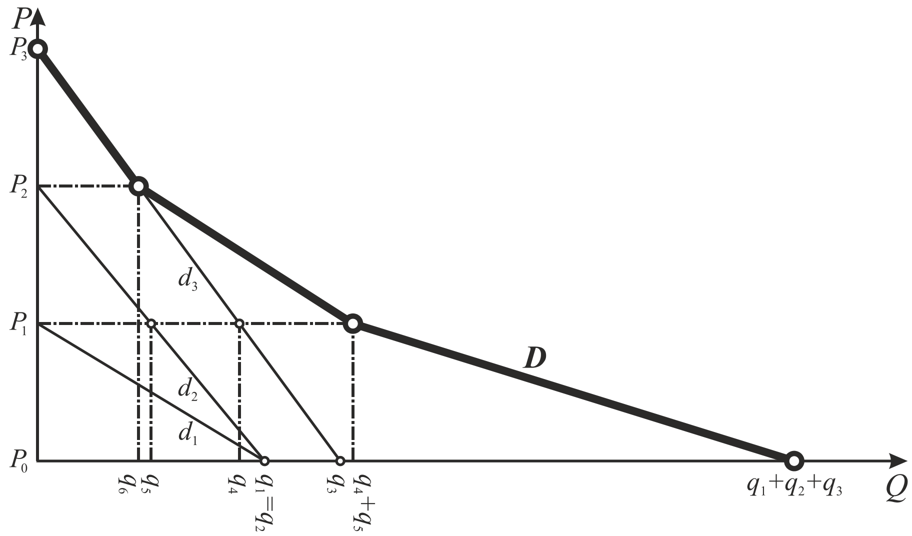
$$e_{PD} + e_{ID} + e_{CD} = 0 \qquad (1.21)$$

From the above-mentioned formula, one key characteristic of the demand curve can be derived. If prices of all products, which are bought by the consumer, rise in the same proportion (e. g. by 10%) and the consumer's income shall be increased in due proportion as a whole (i.e. it shall also increase by 10%), then, the consumer's demand function shall remain unchanged.

1.7 Deriving a market demand curve

By summing individual demands d_A of individual consumers, we get **market demand D_A** for the goods *A*. Graphically, it is the horizontal sum of individual demand curves (see the graph 1-28). The consumers' market demand *m* for the goods *A* shall have a general formula:

$$Q_A = \sum_{i=1}^{m} q_{Ai} = f(P_A, P_B, I_1, ...I_m) = D_A \qquad (1.22)$$



1-27 Market demand curve

1.8 Consumer's preferences under conditions of risk

In contrast to the previous (as well as the following) parts of this textbook, the consumer's decision-making under conditions of risk is characterized by considering various variants which can occur. We shall call these variants **states of the world**. According to information about these various states of the world which is available to the consumer, it is then necessary to distinguish four basic situations:

- **certainty:** The consumer knows all possible states of the world (S_1, S_2 … S_n), he knows consequences which arise for him from these states (X_1, X_2 … X_n), and at the same time, he knows that one of the states of the world occurs with the probability of 100 %, i.e. that other states of the world can never occur.

- **risk:** The consumer knows all possible states of the world (S_1, S_2 … S_n), he knows consequences which arise for him from these states (X_1, X_2 … X_n), and he knows probabilities (π_1, π_2… π_n) with which individual states of the world can occur. The sum of these probabilities is 100%.
- **uncertainty:** The consumer knows all possible states of the world (S_1, S_2 … S_n), he knows consequences (X_1, X_2 … X_n) which arise for him from these states but he is not able to specify (or find out) probabilities with which individual states of the world can occur.
- **indefiniteness:** The consumer knows all possible states of the world, he does not know consequences which arise for him from these states and he is not able to specify (or find out) probabilities with which individual states of the world can occur.

A conditional consumption plan then specifies what shall be consumed under individual resulting states of the world. **An expected result** *EX* is an average value of all *n* possible results whereas weights are probabilities with which these results or states of the world occur:

$$EX = \sum_{i=1}^{n} X_i \cdot \pi_i \ , \qquad (1.23)$$

where π_i is the probability that the resulting state X_i shall occur and *n* is the number of various possible resulting states (in our simplified conception, we shall first suppose only one risky variation, subsequently, we extend interpretation to two risky variants).

In this case, **the utility function U_c** is dependent on the probability and consumer level.

$$U_c = f(C_1, C_2 ... C_n, \pi_1, \pi_2 ... \pi_n), \qquad (1.1)$$

where C_1, C_2 … C_n represent the levels of consumption if the states S_1, S_2… S_n, that occur with the probability π_1, π_2 … π_n, arises.

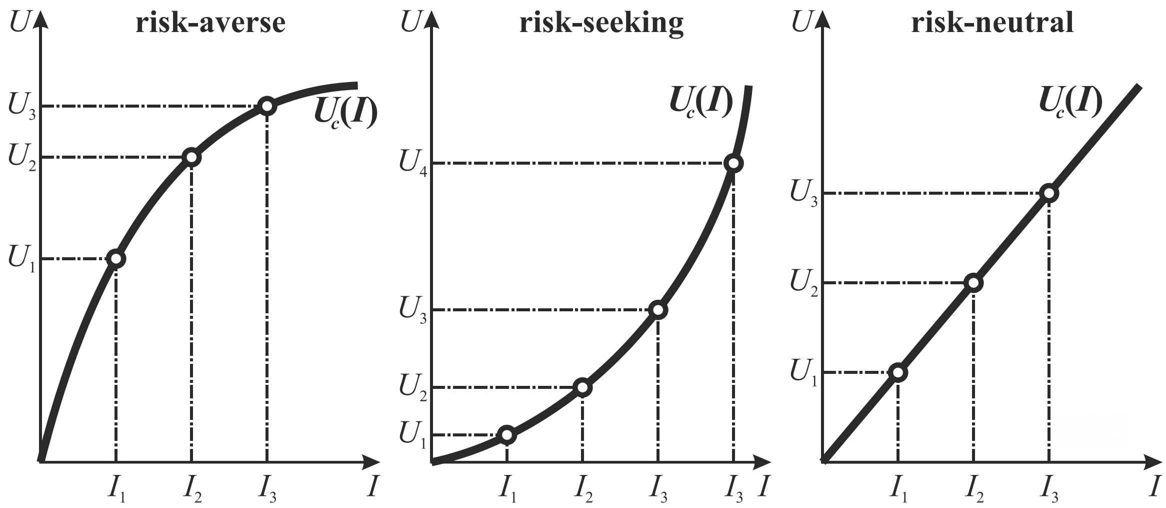
Analogously to the expected result, **the expected utility** represents the average value of utility of individual results weighted by their probability.

$$U_c = \sum_{i=1}^{n} C_i \cdot \pi_i \ , \qquad (1.24)$$

where C_i represents the level of consumption if the state S_i occurs and π_i is the probability of this variant S_i.

1.8.1 The only risky situation

Consumers prefer various consumption plans as they have various preferences for real consumption. If the consumer prefers certainty of expected value of his assets U_c to the risk with the same expected result U_r, then, it is a **risk-averse** consumer. The progress of utility depending on increasing income shall represent concave functions. With increasing income, the total utility U_c grows slower than the consumer's income I since with another risk taken which is necessary for gaining higher income, the utility grows with a decreasing rate.



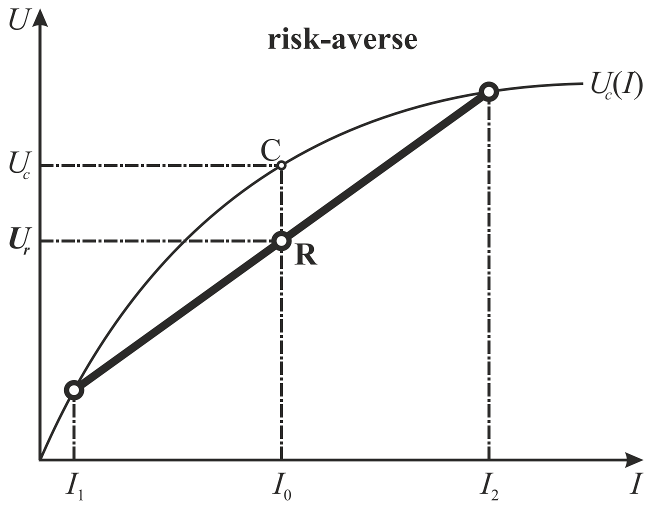
1-28 Shape of the utility function according to the consumer's relation to risk

If the consumer prefers random distribution of assets U_r to their certain expected value U_c, we speak of a **risk-seeking** consumer. His utility function is convex, the utility U_c grows faster than the consumer's income I since with another risk taken which is necessary for gaining higher income, the utility grows with an increasing rate. If the consumer is **risk-neutral**, then, the certain utility from the assets U_c is equal to the utility from their expected value U_r. In this case, the utility function is linear since with another risk taken, which is necessary for gaining higher income, the utility keeps on growing with the same rate.

We specify consumer's preferences if we apply the theory of expected utility to a simple **example with 50% probability of choosing** a resultant state. The consumer whose assets are I_0 (see the graph 1-30) can undertake a risk where his income grows up to I_2 with 50% probability and it can fall to the level I_1 with the same probability. The line segment in the graph 1-30 expresses the amount of expected utility under various probabilities of a risky variant, the point R is in the middle since we have chosen the example with 50% probability of a risky variant (if the probability of the risky variant I_2 was higher, the point R would lie higher on the line segment, if the probability of the variant I_1 was higher, the point R would lie lower on the line segment). The expected utility U_r is equal to the sum of the probability and result $U_r = 0.5 \cdot U(I_1) + 0.5 \cdot U(I_2)$. The risk-averse consumer shall prefer the alternative of certainty C to the risky alternative R since $U_r < U_c$.

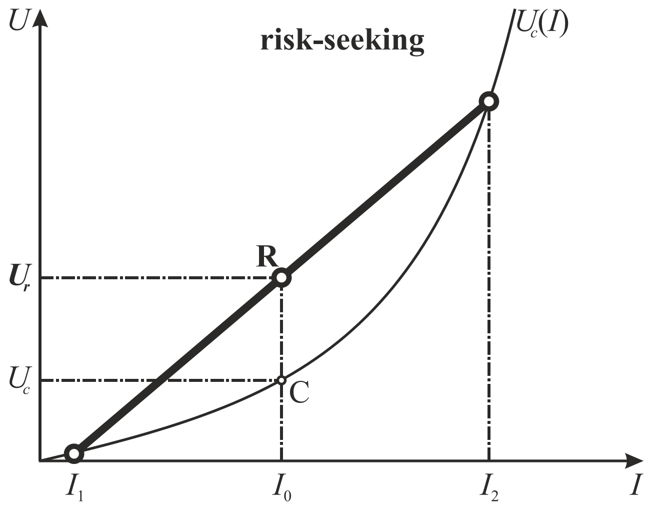
The case when the probability of choosing the risky variant amounts to 50% can be considered as one of examples of a so called **fair bet**. The fair bet is such a bet when the expected result size is identical to the initial value of the consumer's income. E. g.: The consumer disposes of CZK 100 and is guessing one number from 0 to 9. The probability that he manages to guess the number is 10% and he wins CZK 1,000. If he does not manage to guess the number, he wins nothing. It applies that the expected result value is: EX = CZK 1,000·0.1 + CZK 0·0.9 = CZK 100. If the win did not reach CZK 1,000, it would not be a fair bet.

Now, we assume the situation when the consumer tries to guess if the number, which shall fall e. g. in the roulette, shall be an even or odd number. The probability of his win is then 50%, he shall lose his deposit with the same probability.



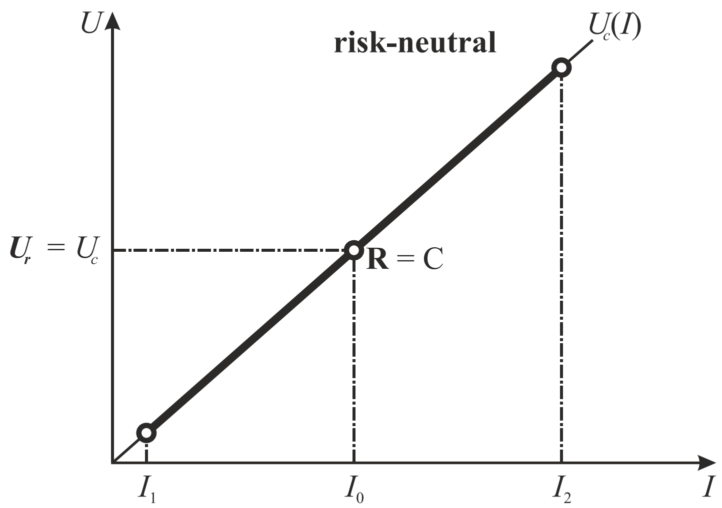
1-29 Example with 50% probability of a risky situation by the risk-averse consumer

The risk-seeking consumer shall prefer the risky variation to the certainty, his expected utility from the risky variation is higher than the utility from the variation of certainty ($U_r > U_c$).



1-30 Example with 50% probability of a risky situation by the risk-seeking consumer

For the risk-neutral consumer, the expected utility of the risky variation shall be as big as the utility from the variation of certainty, $U_r = U_c$. This consumer is indifferent between the certainty and risky variation of the situation.



1-31 Example with 50% probability of a risky situation by the risk-indifferent consumer

1.8.2 Two risky situations

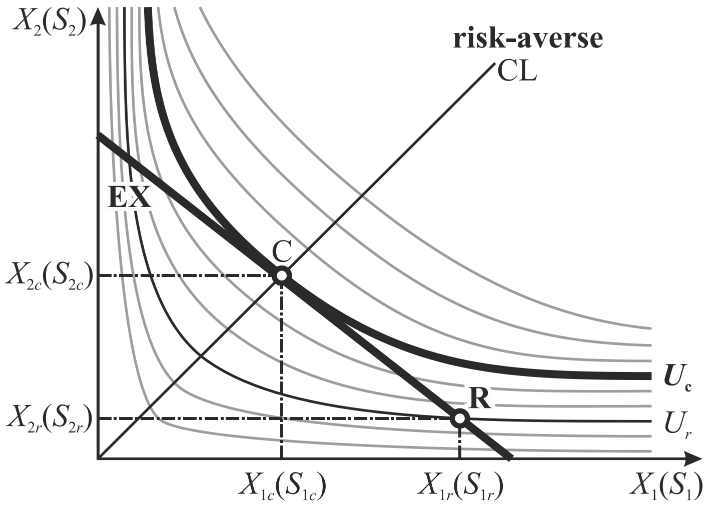
If the consumer considers two risky situations (e.g. a bet to odd and even numbers or red and black in the roulette, home insurance covering against flood and fire or accident insurance and damage liability and others), then, for the consumer's optimum decision-making under conditions of risk, we use – analogously to the consumer's decision-making regarding the combination of consumption of two commodities – an indifference analysis. The construction of indifference curves is again dependent on the shape of utility functions which now express the relation of the consumer to the risk.

For reaching the consumer's optimum under conditions of risk (we suppose two situations S_1 and S_2 with two resultant states X1 and X2), we must define:

- ⇒ **certainty line** (**CL**) which represents a set of points with the same revenue from both the situations considered. If the probability of both states S_1 and S_2 is the same, i.e. 50%, the slope of the certainty line shall then be 45°.
- ⇒ **line of the same expected result** (**EX**) which represents the same expected result of both situations under different distribution of probabilities between the situation S_1 and S_2. The slope of the EX line segment is then given by the ratio of probabilities $\pi_1:\pi_2$. Graphically, we express the results X_1 for the situation S_1 on the horizontal axis and results X_2 for the situation S_2 on the vertical axis.

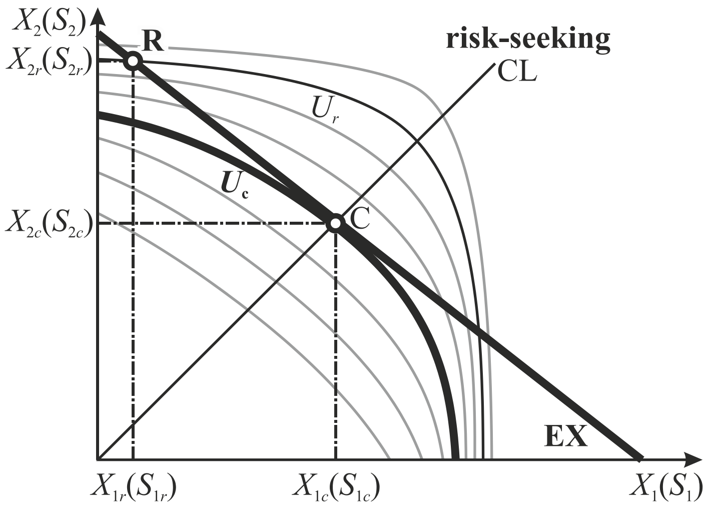
Naturally, the resultant optimum again depends on the relation of the consumer to the risk:

- ⇒ In case of risk aversion, the point C representing a certain variation lies on the same line of the expected result as the point R which represents a risky variation. At the same time, C lies on the higher indifference curve U_c – a risk-averse consumer shall prefer it.



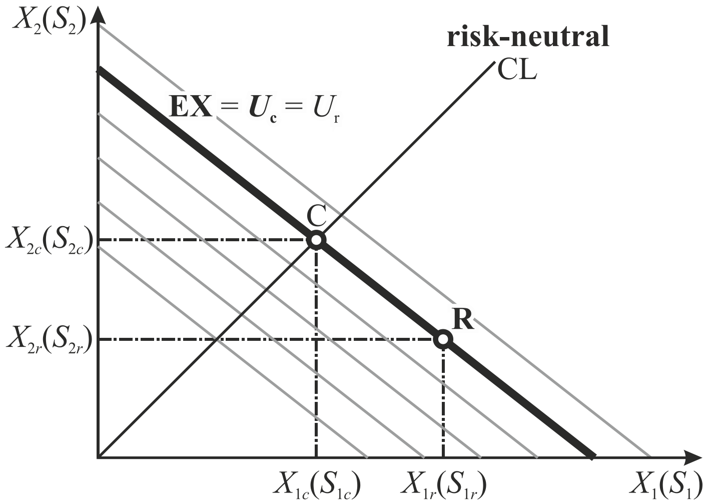
1-32 Decision-making of the risk-averse consumer in the conditions of uncertainty

⇒ In case of seeking a risk, the point R lies on the higher indifference curve and therefore it shall be more favourable for the consumer.



1-33 Decision-making of the risk-seeking consumer in the conditions of uncertainty

⇒ In case of risk neutrality, the points R and C bring the same level of utility, the line of the same expected result is identical to a certain indifference curve.



1-34 Decision-making of the risk-neutral consumer in the conditions of uncertainty

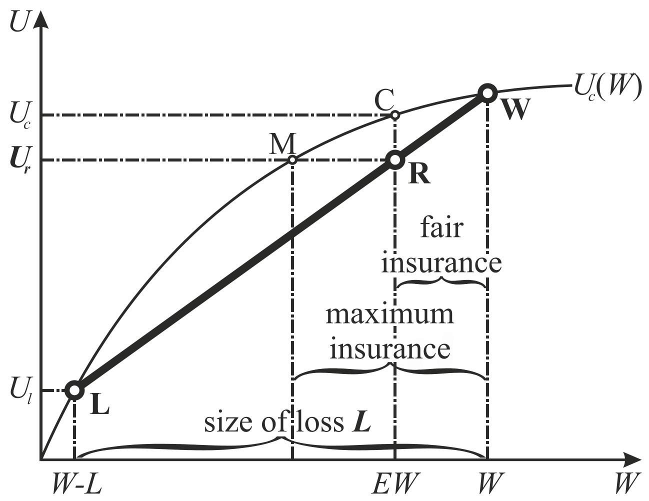
1.8.3 Insurance

Insurance is one of possibilities how to decrease the risk. Let's consider a man whose assets reach the value *W (welfare, **W**)*. If his assets are decreased by *L (loss, **L**)* as a result of a loss, deterioration, stealing, disasters and others, his resultant assets shall be $W - L$. The expected welfare (***EW***) for the case of non-insurance is identical to the value of assets achieved by means of insurance under the assumption of fair insurance. We speak of **fair insurance** if the value of assets in the situation of certainty achieved by insurance is the same as the expected value of assets in the situation of risk (without insurance).

$$EW = (W - L) \cdot \pi + W \cdot (1 - \pi) = W - L \cdot \pi \ , \qquad (1.25)$$

where $(W - L)$ is the value of assets in case of loss, π is the probability that this insurance event occurs, $(1 - \pi)$ is the probability that the assets shall not be damaged and W is the total value of assets.

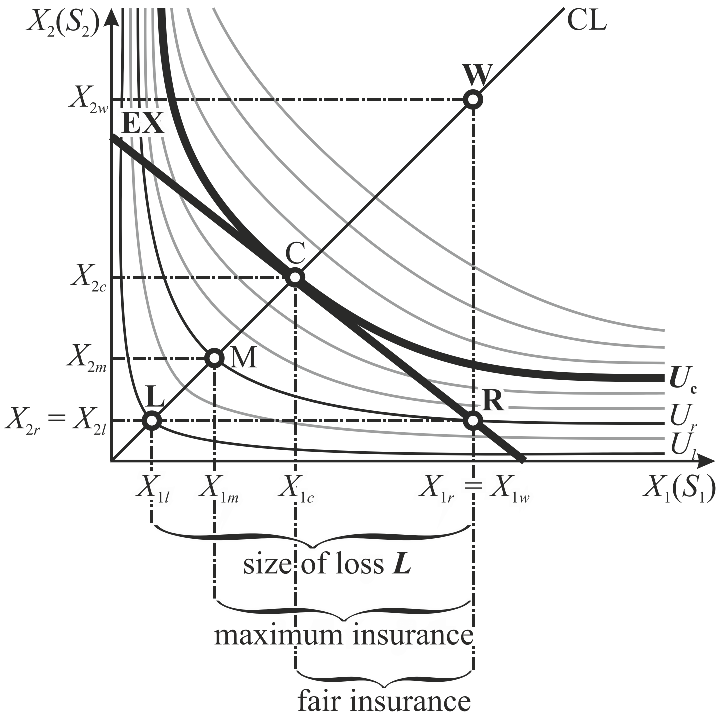
The situation is graphically expressed by the graph 1-36. The situation is illustrated for a risk-averse man since for a risk-seeking or risk-indifferent individual, the issue of insurance is understandably irrelevant.



1-35 Fair insurance by the risk-averse consumer

The maximum insurance is such an amount of insurance which leads to the fact that the utility associated with certainty achieved by insurance (point C) is identical to the expected utility associated with a risky variation without insurance (point R).

The situation of insurance can be also expressed by means of indifference curves. For logical reasons, we again consider a risk-averse consumer.



1-36 Fair insurance by the risk-averse consumer in the indifference analysis

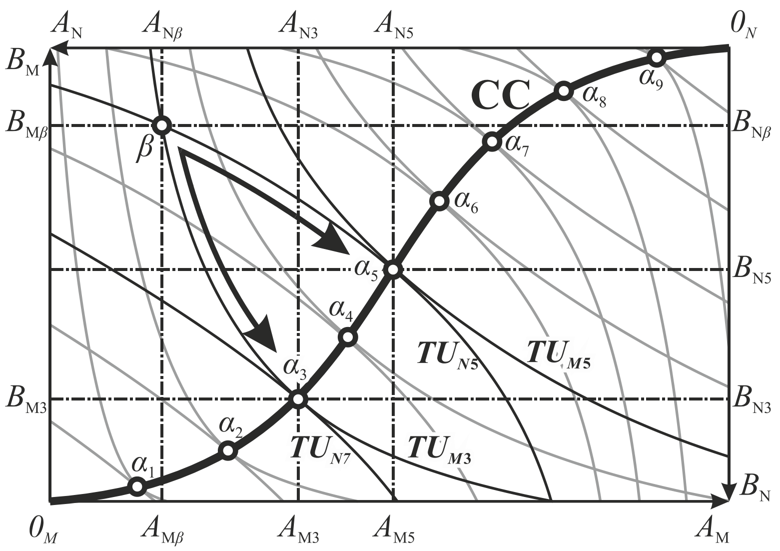
1.9 Efficiency in consumption

The efficiency in consumption means finding the Pareto-efficient distribution of fixed amount of the goods A and *B* (at the prices of the goods P_A and P_B) between the consumer *M* (Martin) and *N* (Nancy) limited by individual income $I_{M\ a}\ I_N$.

For the analysis, we shall use a graphical tool of the Edgeworth Box Diagram, named after **Francis Ysidro Edgeworth** (1845-1926). This diagram enables to express preferences of two consumers *M* (Martin) and *N* (Nancy) towards two commodities A (apples) and *B* (bananas) in one graph. In the graph, possible consumer baskets of both consumers and their preferences can be expressed by means of indifference curves. Choice of Martin's consumption level *M* is measured from the left lower corner up to the right, choice of Nancy *N* is determined from the right upper corner down to the left. The diagram width specifies the total amount of apples A in economics, the height is limited by the total amount of bananas *B*. **The contract curve** connects all points α where indifference curves of both the consumers connect or points where it applies that:

$$MRS_{C(M)} = MRS_{C(N)} \qquad (1.26)$$

On the contrary, intersection points of indifference curves (e. g. point β) are suboptimal points since by moving from the point β e. g. to the point α_5, the situation of Nancy does not worsen (indifference curve TU_{N5}) while the situation of Martin improves (from TU_{M3} to TU_{M5}).

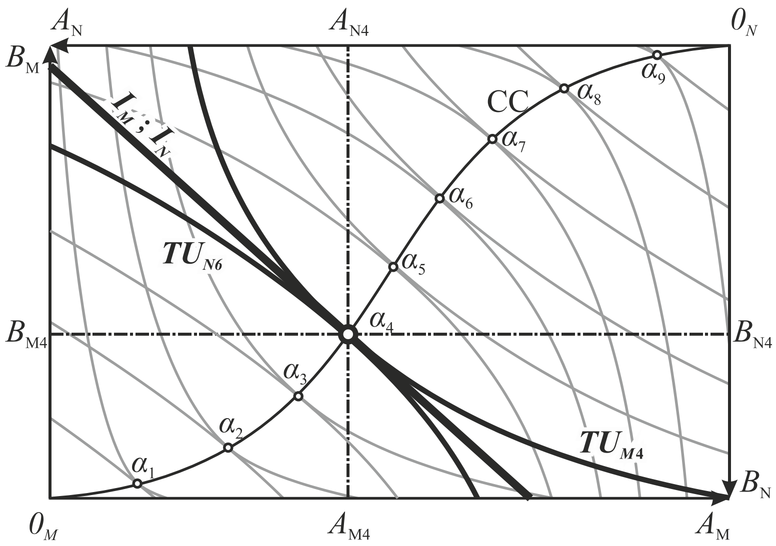


1-37 Contract curve in the Edgeworth box

On the contrary, by moving from the point β to the point α_3, the situation of Martin does not worsen (indifference curve TU_{M3}) while the situation of Nancy improves (from TU_{N5} to TU_{N7}). And finally, by moving from the point β to the point α_4, both the situation of Martin (from TU_{M3} to TU_{M4}) and the situation of Nancy (from TU_{N5} to TU_{N6}) shall improve. The point β does not fulfil the conditions of the Pareto optimality defined as a situation when the position of one subject (e. g. increasing the total utility of Martin) cannot be improved without worsening the position of the other subject (i.e. without decreasing the total utility of Nancy).

To search Pareto-efficient combinations except for preferences of consumers, we must also consider the prices of the commodities P_A and P_B and amount of income of consumers I_M and I_N. The consumer's optimum can be found at the point of touching the indifference curve and budget line the slope of which MRS_E is given by the rate of prices P_A/P_B. Both the consumers buy for the same prices and Pareto-efficient equilibrium is at the point where:

$$MRS_{C(M)} = MRS_E = MRS_{C(N)} \quad (1.27)$$



1-1 Pareto-efficient consumer's decision

Under the Pareto-efficient allocation (at the point α_4), both the consumers are, under the given indifference curves of the other consumer, on their highest possible indifference curves, any redistribution cannot bring any improvement to any consumer without damaging the other consumer.

Further Reading

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