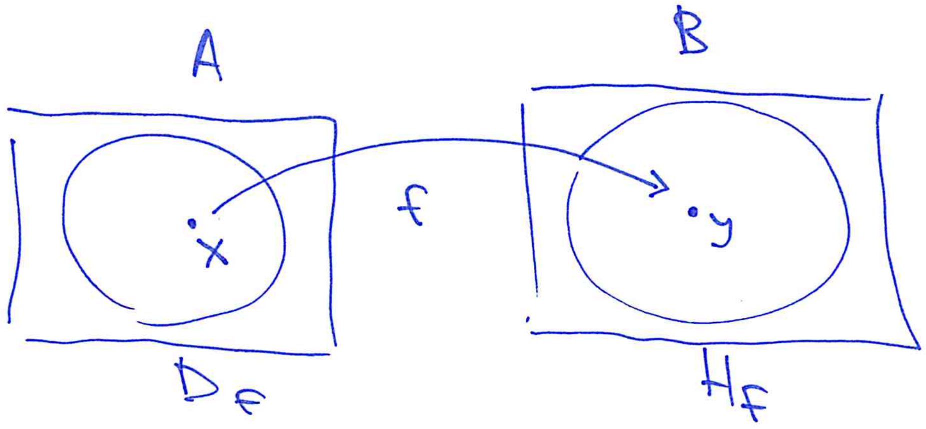
Funkce (reálné)

$f: y = f(x)$



cv2/1b

$$f(x) = \frac{x-2}{x+2}$$

$D_f: \; x + 2 \neq 0$

$x \neq -2$

$$D_f = \mathbb{R} \setminus \{-2\} = (-\infty, -2) \cup (-2; +\infty)$$

1c $f(x) = \dfrac{2x+3}{x^2+3x+2}$

$D_f: x^2+3x+\boxed{2} \neq 0$

$(x+1)\cdot(x+2) \neq 0$

$\swarrow$

$x+1 \neq 0 \wedge x+2 \neq 0$

$x \neq -1 \qquad x \neq -2$

$D_f = \mathbb{R} \setminus \{-2; -1\}$

CV3

1d $f(x) = \dfrac{1}{\sqrt{x-2}}$

D_F: $\underbrace{\sqrt{x-2} \neq 0 \wedge x-2 \geq 0}$

$x-2 \neq 0$

$x-2 > 0$

$x > 2$

$D_F = (2; +\infty)$

1F $f(x) = \dfrac{\sqrt[3]{4-x^2}}{x^2+16}$

D_F: $x^2+16 \neq 0$

$x^2 \neq -16$

... platí vždy

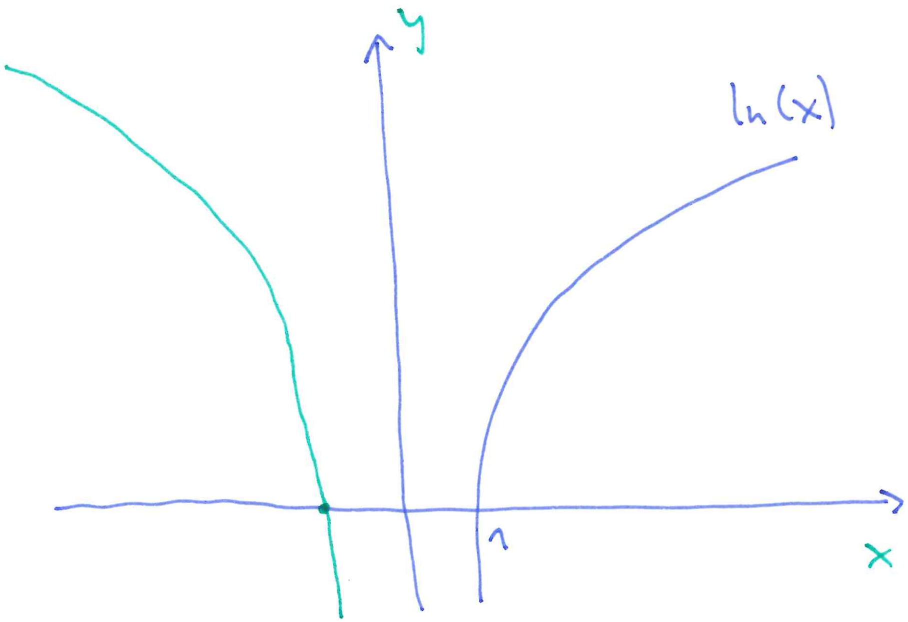
$D_F = \mathbb{R} = (-\infty; +\infty)$

Transformace grafu funkce

a) $g(x) = f(-x)$ [osová sym. podle y]

$f(x) = \ln(x)$

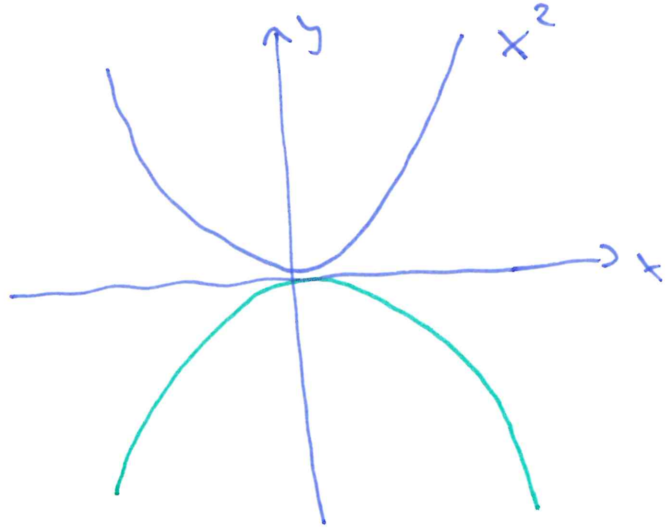
$g(x) = \ln(-x)$



b) $g(x) = -f(x)$ [os. sym podle x]

$f(x) = x^2$

$g(x) = -x^2$

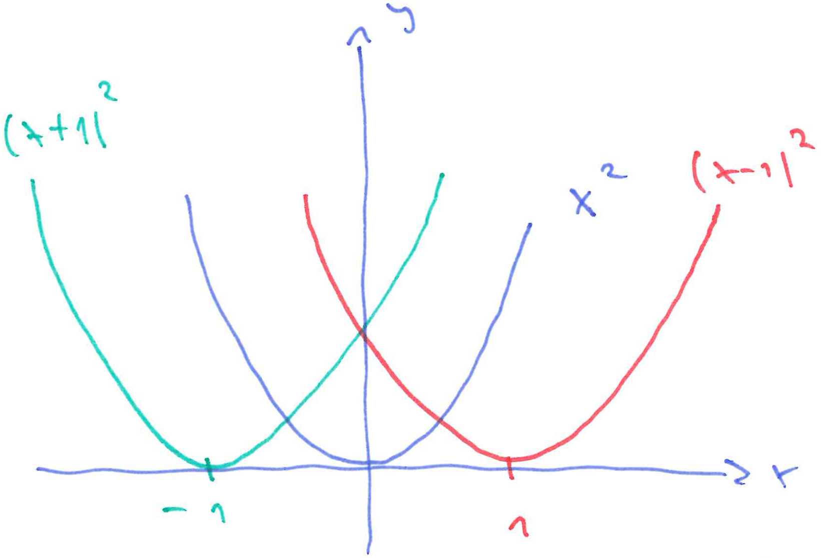


2c $g(x) = f(x+c)$, $c \in \mathbb{R}$

$f(x) = x^2$

$g(x) = (x+1)^2$

$h(x) = (x-1)^2$



… posun na ose x o $|c|$ dílků

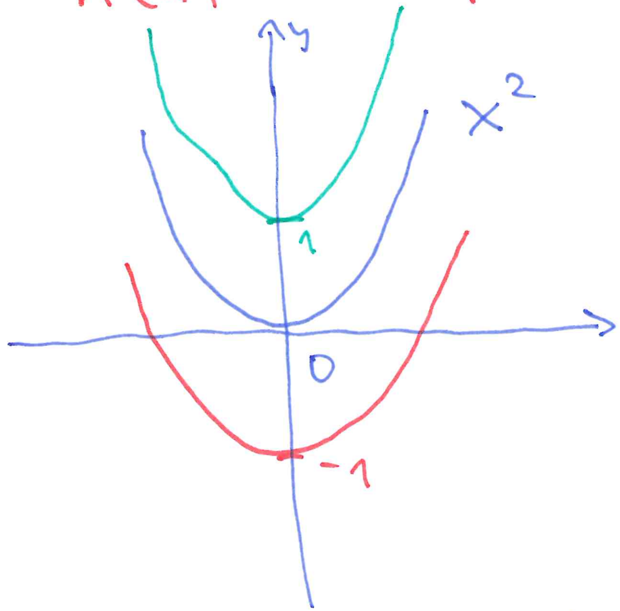
($c>0$ ←) ($c<0$ →)

d $g(x) = f(x) + c$, $c \in \mathbb{R}$

$f(x) = x^2$

$g(x) = x^2 + 1$

$h(x) = x^2 - 1$



… posun na ose y o $|c|$ dílků

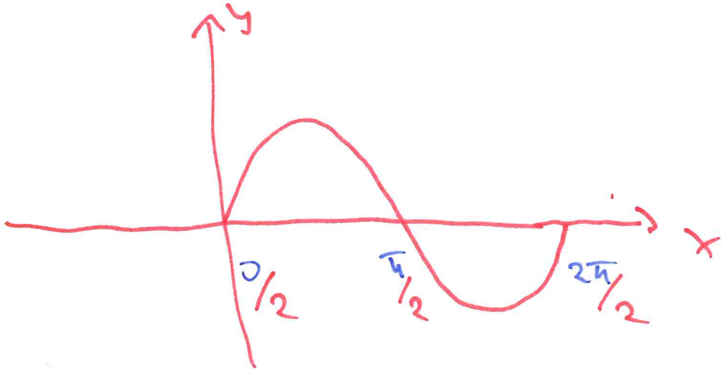
($c>0$ ↑) ($c<0$ ↓)

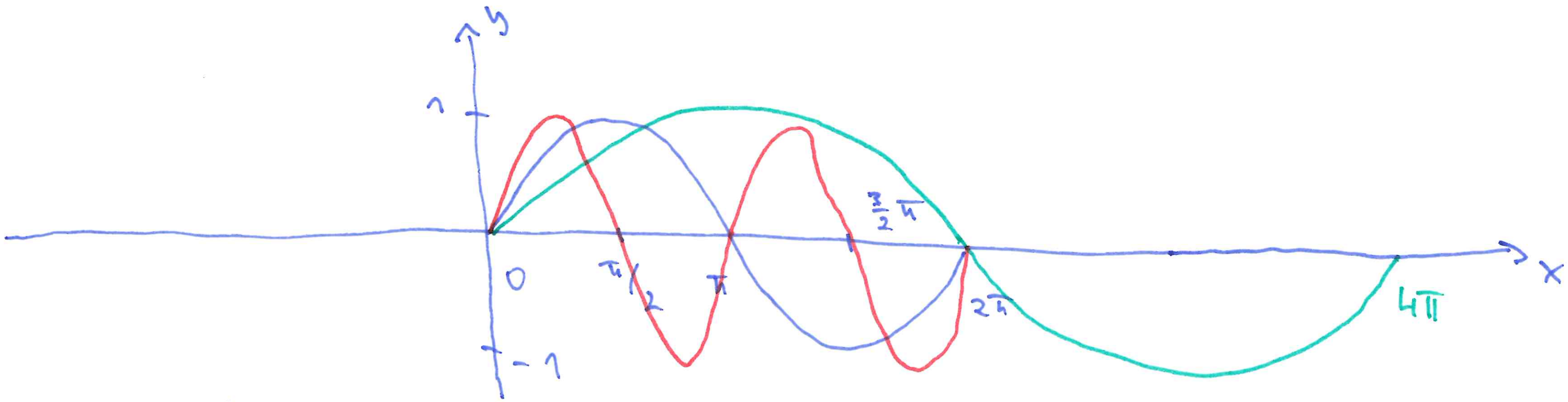
(e) $g(x) = F(c \cdot x)$, $c > 0$

$F(x) = \sin(x)$

$g(x) = \sin(2x)$

$h(x) = \sin\left(\frac{x}{2}\right)$





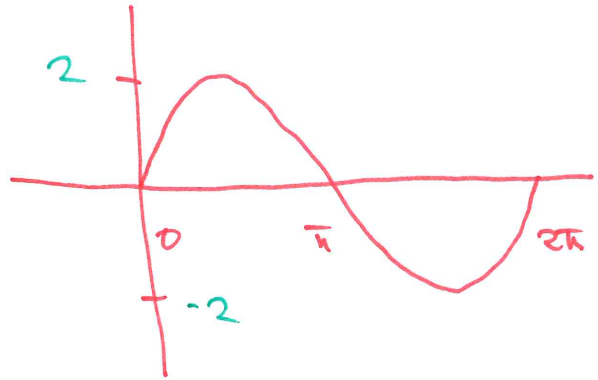
... přeškálování (změna měřítka) osy x

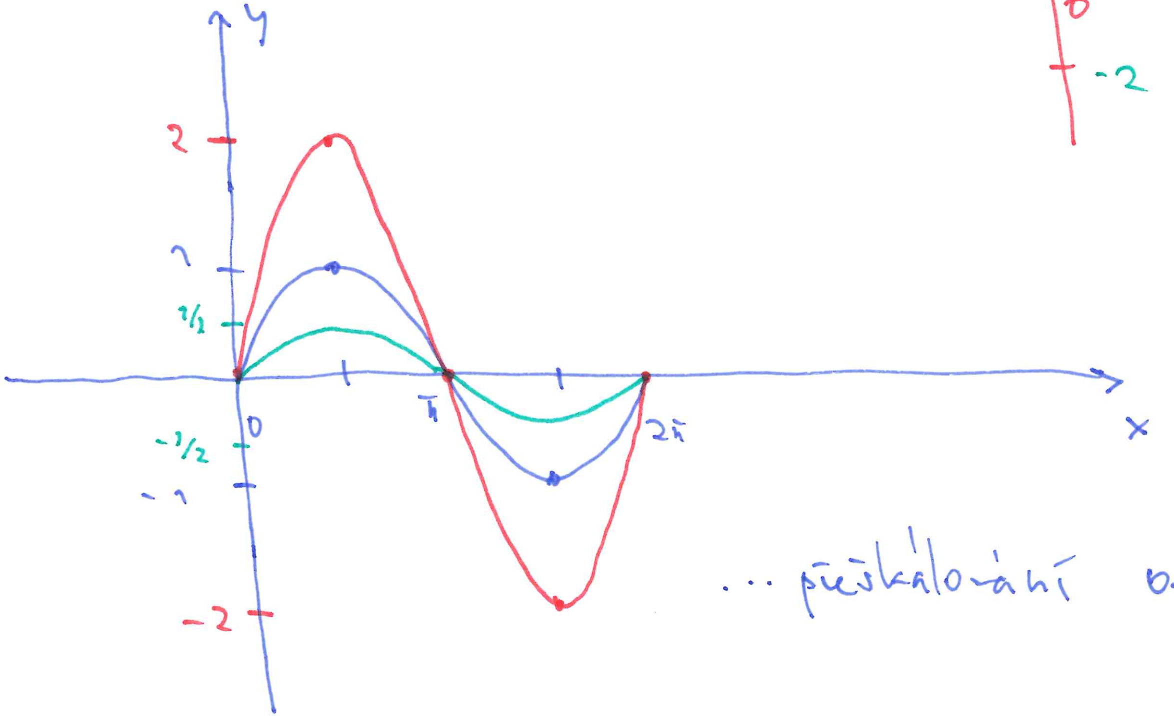
F $\quad g(x) = c \cdot f(x), \ c > 0$

$f(x) = \sin(x)$

$g(x) = 2 \cdot \sin(x)$

$h(x) = \frac{1}{2} \cdot \sin(x)$



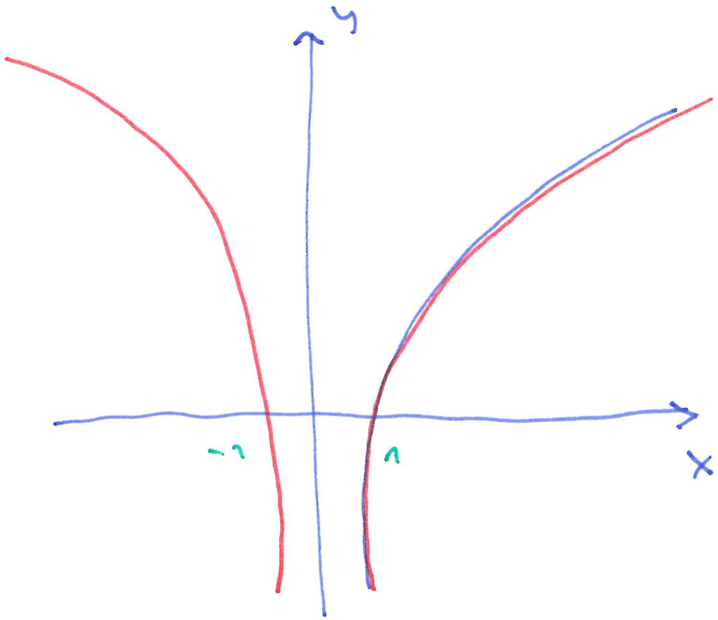


… přeškálování osy y

(g) $g(x) = F(|x|)$

$F(x) = \ln(x)$

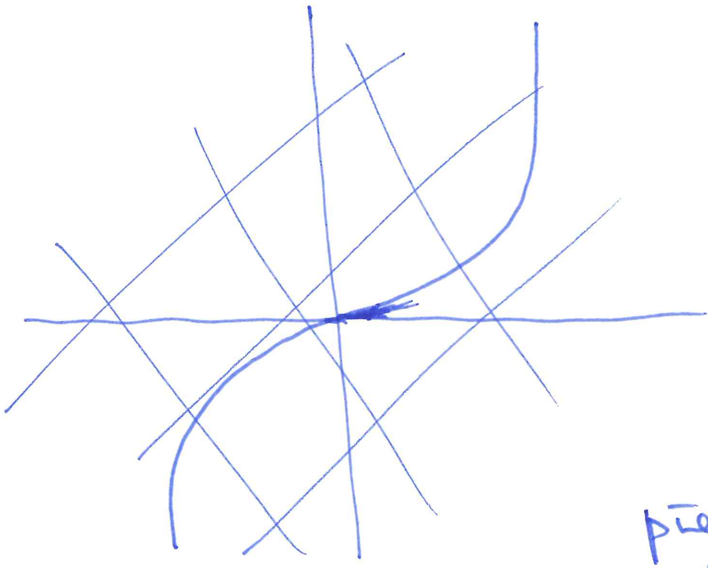
$g(x) = \ln(|x|)$ $\begin{cases} \ln(x), & x > 0 \\ \ln(-x), & x < 0 \end{cases}$

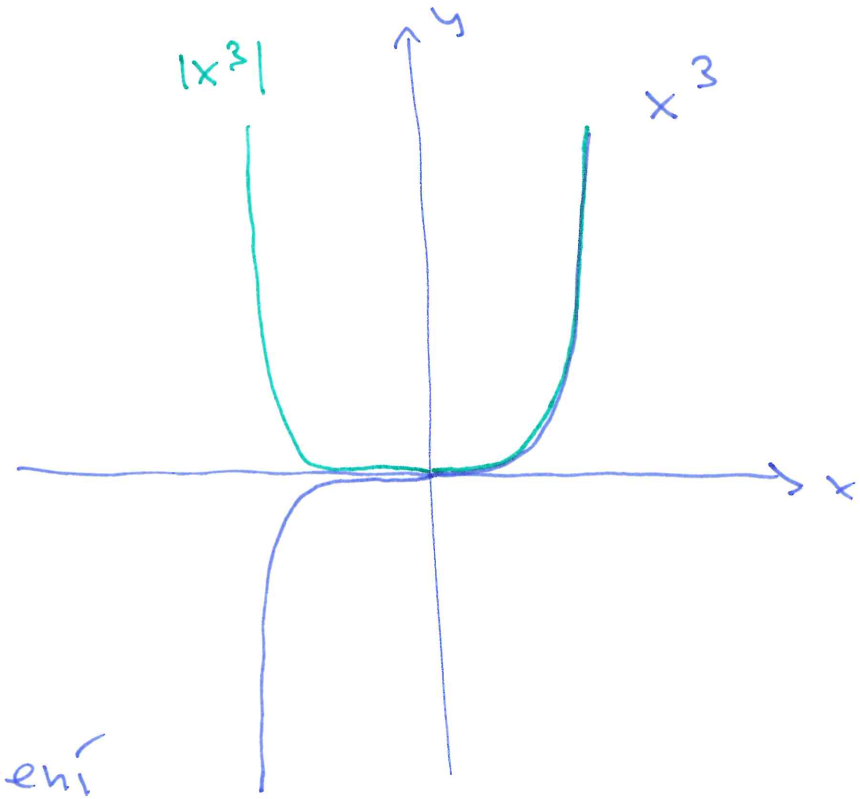


... na kopírování graf z I. a IV. do II. a III. podle osové symetrie y h $g(x) = |f(x)|$

$f(x) = x^3$

$g(x) = |x^3|$





překlopení

... ~~nakopírování~~ grafu z III. a IV. do I. a II. podle osové symetrie x

3j

$$f(x) = -x^2 + 4x + 1 = -\left[x^2 - \boxed{4}x\right] + 1 =$$

doplnění na $\square$

$(a \pm b)^2 = a^2 \pm 2ab \boxed{+b^2}$

$$b^2 = \left[\frac{\square}{2}\right]^2 = \left(\frac{4}{2}\right)^2 = 4$$

$$= -\left[x^2 - 4x + 4 - 4\right] + 1$$

$$= -\underbrace{\left[x^2 - 4x + 4\right]} + 5$$

$$f(x) = \boxed{-(x-2)^2 + 5}$$ ... vrcholový tvar paraboly: $V = [2; 5]$

$$x^2 \rightsquigarrow (x-2)^2 \rightsquigarrow -(x-2)^2 \rightsquigarrow -(x-2)^2 + 5$$

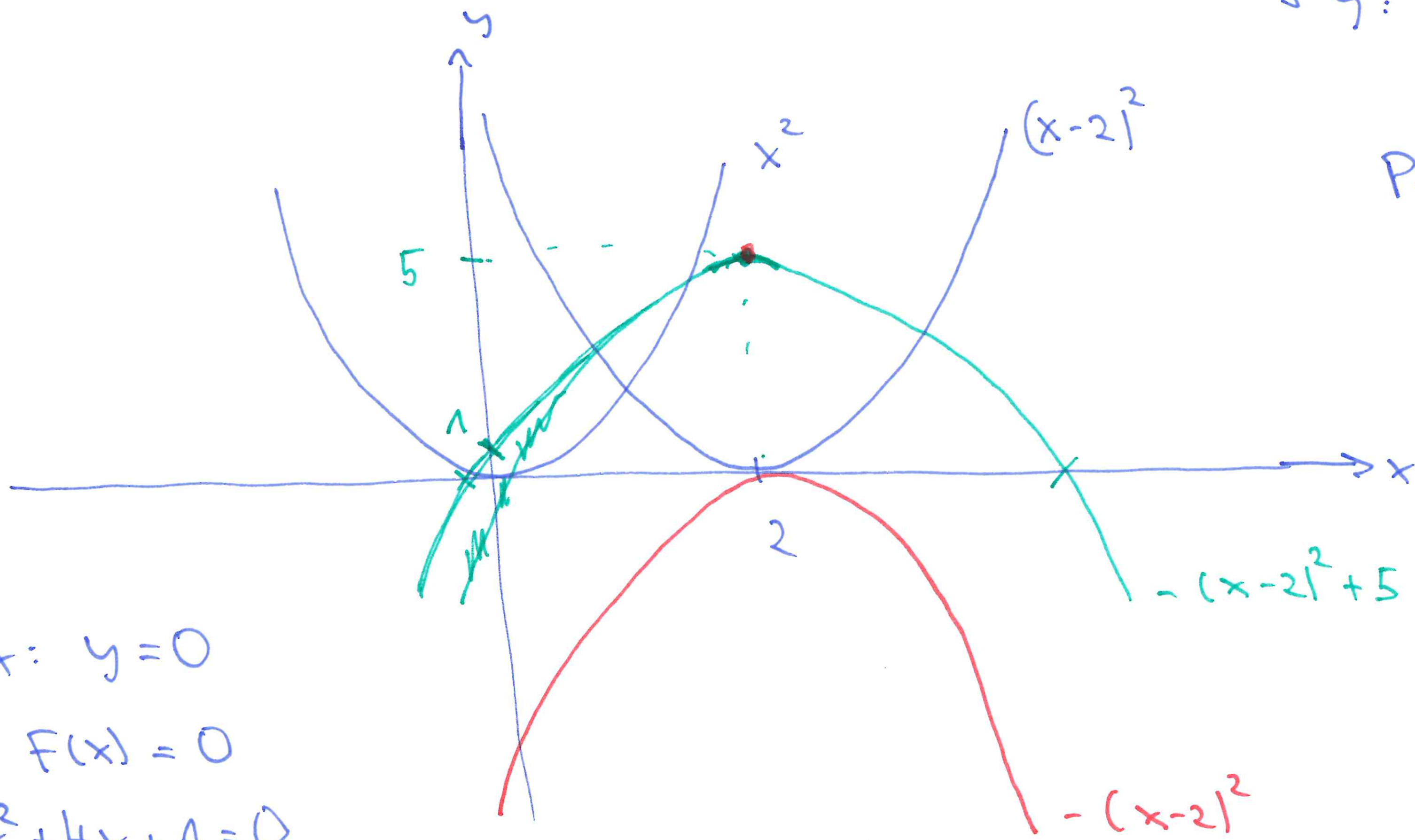
o $2 \rightarrow$ — překlopení podle x — o $5 \uparrow$

Průsečíky:

s y: $x = 0$

$f(0) = 1$

$P_y = [0;1]$



s x: $y = 0$

$f(x) = 0$

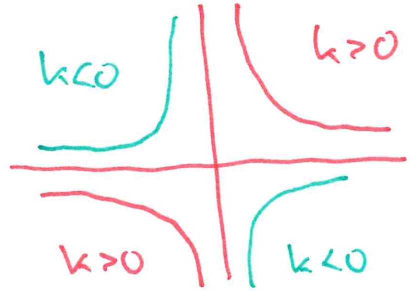
$-x^2+4x+1=0$

Vrchol: $x_v = \frac{-b}{2a} = \frac{-4}{2\cdot(-1)} = 2$ $\boxed{y = ax^2+bx+c}$

3l

$f(x) = \dfrac{x+1}{x-3}$ ... lineární lomená

grafem hyperbola $\boxed{y = \dfrac{k}{x}}$



$$(\boxed{x}+1) : (\boxed{x}-3) = 1$$
$$\underline{-(x-3)}$$
$$4$$ ... zbytek

výsledek, zbytek

$$\frac{5}{3} = \boxed{1} + \frac{\boxed{2}}{3}$$

$$f(x) = \frac{x+1}{x-3} = 1 + \frac{4}{x-3}$$ ... středový tvar: $S = [3;1]$

$$\frac{1}{x} \rightsquigarrow \frac{1}{x-3} \rightsquigarrow \frac{4}{x-3} \rightsquigarrow 1 + \frac{4}{x-3}$$

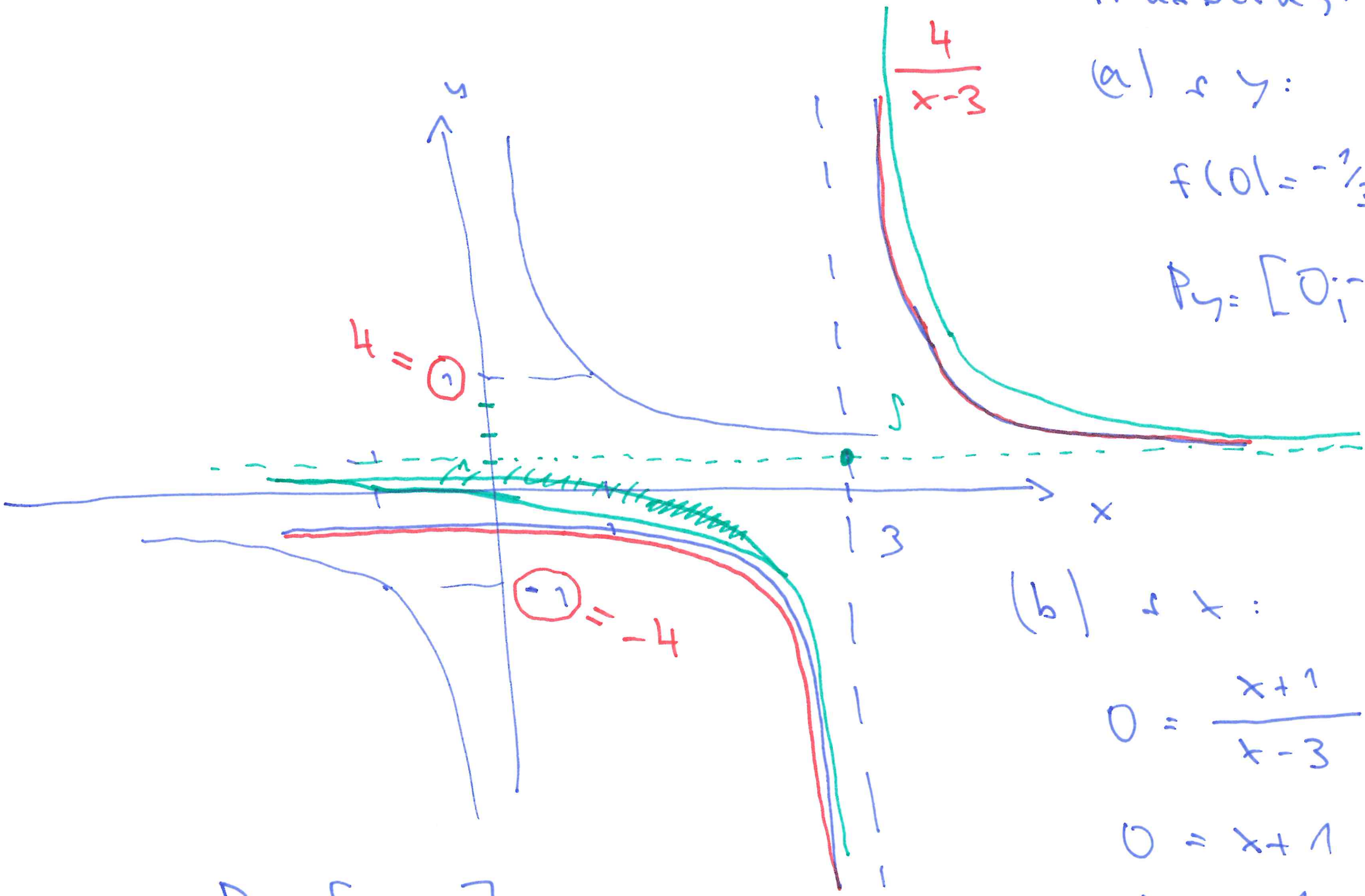
o 3 $\longrightarrow$ — přeškálovat y (4) — o 1 $\uparrow$

Průsečíky:

(a) s y:

$f(0) = -\frac{1}{3}$

$P_y = [0; -\frac{1}{3}]$



$P_x = [-1; 0]$

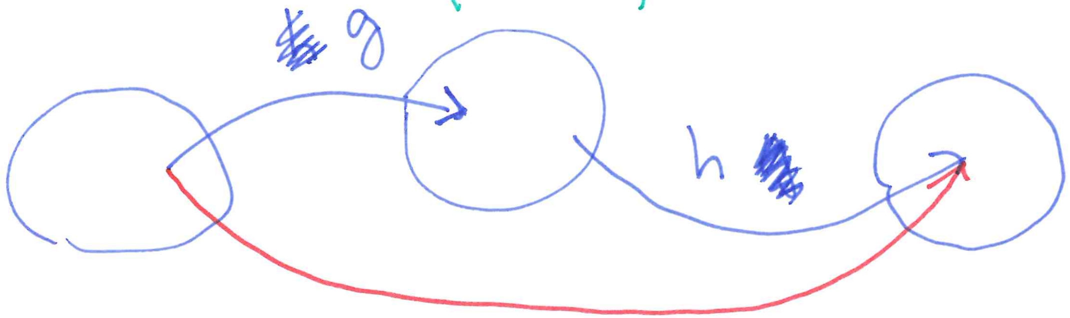
(b) s x:

$0 = \frac{x+1}{x-3}$

$0 = x + 1$

$\underline{\underline{x = -1}}$

Skládání fcí (kompozice)



$f = h \circ g = h(g)$

4a

$g(x) = \dfrac{x+1}{x-1}$ $\qquad h(\boxed{x}) = \sqrt{x}$

$f_1 = h \circ g$

$f_1(x) = h(g(x)) = h\left(\boxed{\dfrac{x+1}{x-1}}\right) = \sqrt{\dfrac{x+1}{x-1}}$

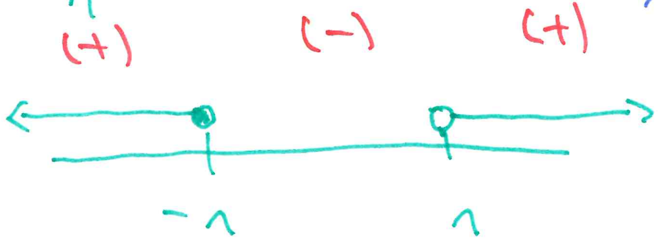
$f_2(x) = g(h(x)) = g(\sqrt{x}) = \dfrac{\sqrt{x}+1}{\sqrt{x}-1}$

$$f_1(x)=\sqrt{\frac{x+1}{x-1}}$$

$$D_{f_1}: \frac{x+1}{x-1}\geq 0 \wedge x-1\neq 0$$

-1 1

$$x\neq 1$$



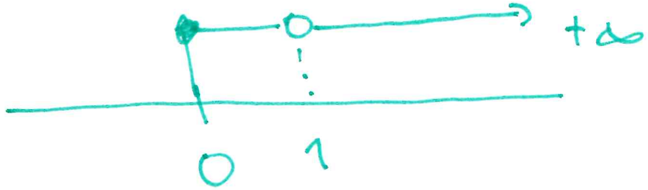
$$D_{f_1}=(-\infty,-1\rangle \cup (1;+\infty)$$

$$f_2(x)=\frac{\sqrt{x}+1}{\sqrt{x}-1}$$

$$D_{f_2}: x\geq 0 \wedge \sqrt{x}-1\neq 0$$

$$\sqrt{x}\neq 1 \quad /^2$$

$$x\neq 1$$



$$D_{f_2}=\langle 0;1) \cup (1;+\infty)$$

5a

$f(x) = \sqrt{\boxed{3x-4}}$

$f = h \circ g = h(g)$

$f(x) = h(\boxed{g(x)})$

$g(x) = 3x - 4$

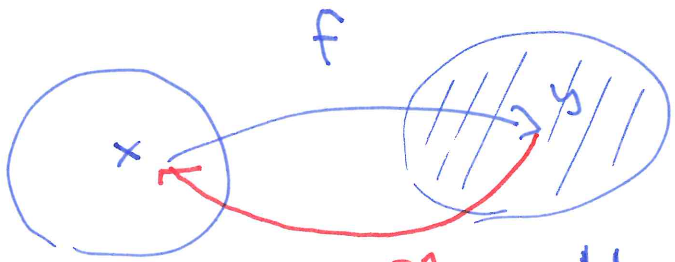
$h(x) = \sqrt{x}$

$D_f: \ 3x - 4 \geq 0$

$x \geq \frac{4}{3}$

$D_f = \langle 4/3 \, ; +\infty)$

6d Inverzní fce



$D_f = H_{f^{-1}}$ $H_f = D_{f^{-1}}$

f … prostá
f … zobrazení na
} bijekce

$$f(x) = 1 + \sqrt[5]{(x+2)^3} \qquad D_f = \mathbb{R} = H_{f^{-1}}$$

$$y = 1 + \sqrt[5]{(x+2)^3}$$

$$y - 1 = \sqrt[5]{(x+2)^3} \quad /^5$$

$$(y-1)^5 = (x+2)^3 \quad / \sqrt[3]{\ }$$

$$\sqrt[3]{(y-1)^5} = x + 2$$

$$\sqrt[3]{(y-1)^5} - 2 = x$$

$$f^{-1}(x) = \sqrt[3]{(x-1)^5} - 2$$

$$D_{f^{-1}} = \mathbb{R} = H_f$$

CV 4

CV2/PF6 e Výpočet f^{-1} :

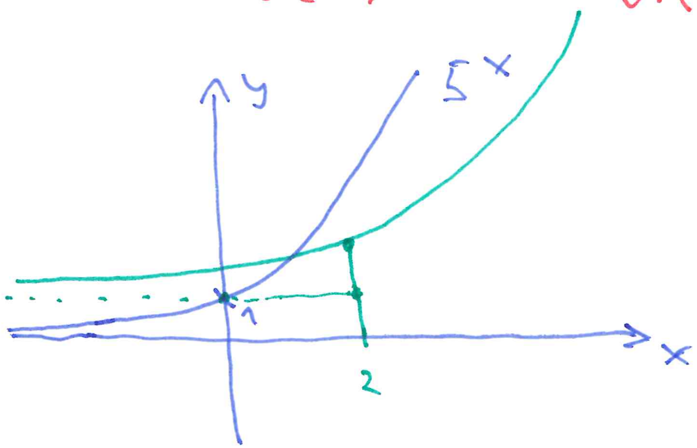
$f(x) = 1 + 5^{x-2}$

prostá :

$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

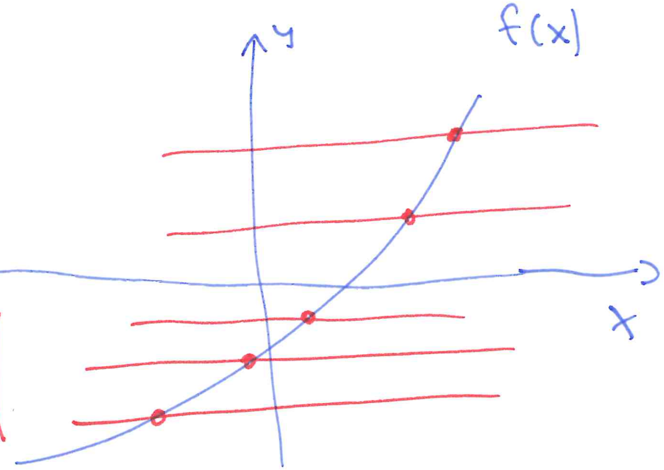
$5^x \longrightarrow 5^{x-2} \longrightarrow \boxed{5^{x-2} + 1}$

o 2 →   o 1 ↑



$D_f = \mathbb{R} = H_{f^{-1}}$

$H_f = (1; +\infty) = D_{f^{-1}}$



nejvýše v 1 bodě

$$y = 1 + 5^{x-2}$$

$$y - 1 = 5^{x-2} \quad / \log_5$$

$$\log_5(y-1) = \log_5(5^{x-2})$$

$$\log_5(y-1) = x - 2$$

$$\log_5(y-1) + 2 = x$$

$$f^{-1}: \underline{\underline{y = \log_5(x-1) + 2}}$$

$D_{f^{-1}}: x - 1 > 0$

$x > 1$

$D_{f^{-1}} = (1; +\infty)$

$$\boxed{\begin{aligned} D_f &= H_{f^{-1}} \\ H_f &= D_{f^{-1}} \end{aligned}}$$