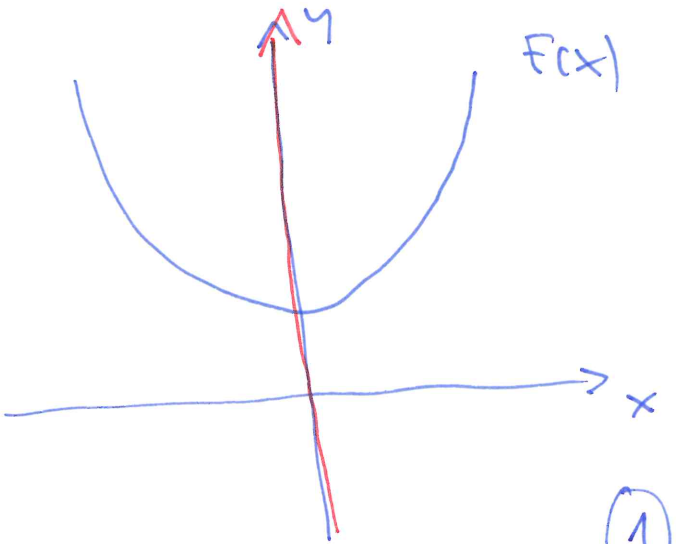
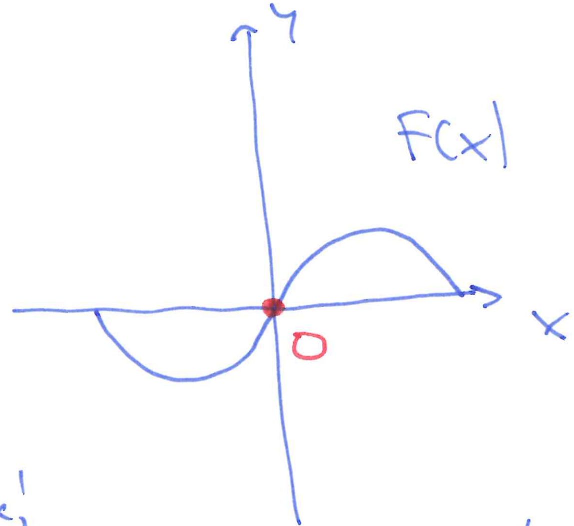
Parita fce (sudost/lichost)

sudá fce



osová symetrie podle y

lichá fce



středová symetrie podle O

1. D_f je symetrický podle O

$(x \in D_f \Rightarrow -x \in D_f)$

$\langle -1; 1)$ není symetrický

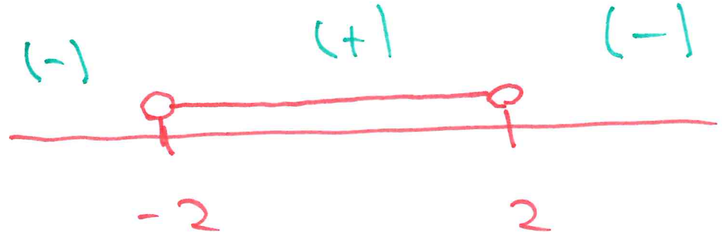
(2) $\boxed{f(-x) = f(x)}$ $\forall x \in D_f$ (SUDÁ) S

$\boxed{f(-x) = -f(x)}$ $\forall x \in D_f$ (LICHÁ) L

... nenastane-li výše $\Rightarrow$ ani L ani S

(1a) $f: y = \ln\left(\frac{2-x}{2+x}\right)$

$D_f: \frac{2-x}{2+x} > 0$ (2; -2)



$D_f = (-2; 2)$ ... je symetrický podle 0

$$f(\boxed{-x}) = \ln\left(\frac{2-\boxed{(-x)}}{2+\boxed{(-x)}}\right) = \ln\left(\frac{2+x}{2-x}\right) = \ln\left(\frac{2-x}{2+x}\right)^{-1}$$

$$= -\ln\left(\frac{2-x}{2+x}\right) = -f(x)\,, \quad \forall x \in D_f$$

$$\Downarrow$$

Fce $f(x)$ je ~~SUDÁ~~ LICHÁ!

Jak nepostupovat:

$F(1) = \ldots$

$-\ln\left(\frac{2-x}{2+x}\right)$ $\quad F(-1) = \ldots.$

1b

$$f: y = \frac{e^x + 1}{e^x - 1}$$

$$D_f: e^x - 1 \neq 0 \qquad D_f = \mathbb{R} \setminus \{0\}$$

$$e^x \neq 1$$

$$x \neq 0$$

je symetrický podle 0

$$f(-x) = \frac{e^{-x} + 1}{e^{-x} - 1} \cdot \frac{e^x}{e^x} = \frac{1 + e^x}{1 - e^x} =$$

$$= \frac{1 + e^x}{-(-1 + e^x)} = \underbrace{-\frac{e^x + 1}{e^x - 1}}_{-f(x)} \quad \forall x \in D_f$$

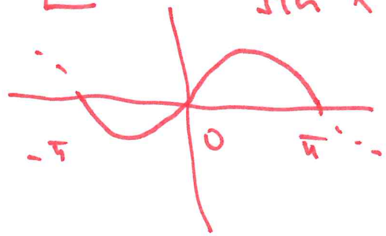
$$\Downarrow$$

LICHÁ

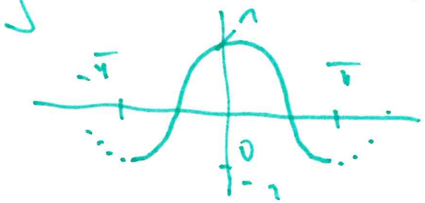
ⓔ $f: y = \sin x - \cos x$ $\qquad D_f = \mathbb{R}$

symetrie podle 0

L



S



$$f(-x) = \underline{\sin(-x)} - \underline{\cos(-x)}$$

$$= -\sin x - \cos x \quad \forall x \in D_f$$

$\Downarrow$

ani S, ani L

(F) $$y = \underbrace{\overset{L}{x}\cdot \overset{L}{\sin}(3x)}_{S} + \overset{S}{x^2}\cdot \overset{S}{\cos x} + \overset{S}{\frac{1}{x^2}}$$

$D_f = \mathbb{R} \setminus \{0\}$

je symetrický

$$f(-x) = -x \cdot \overset{(-1)}{\sin(-3x)} + (-x)^2 \cdot \cos(-x) + \frac{1}{(-x)^2}$$

$$= x \cdot \sin(3x) + x^2 \cdot \cos(x) + \frac{1}{x^2} = f(x) \quad \forall x \in D_f$$

$\Rightarrow$ SUDÁ

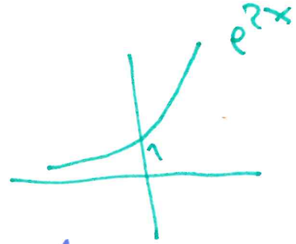
$L = (-)$

$S = (+)$

$$\frac{L}{S} = \frac{(-)}{(+)} = L$$

2b $f: \; y = \sqrt{1+e^{2x}}$

$D_f: \; 1+\underbrace{\boxed{e^{2x}}}_{\geq 0} \geq 0$



... platí vždy

$D_f = \mathbb{R}$

$y = \sqrt{1+e^{2x}} \quad / \boxed{2}$ není ekvivalentní!

$\boxed{y \geq 0}$

$y^2 = 1+e^{2x}$

$y^2-1 = e^{2x} \quad / \ln$

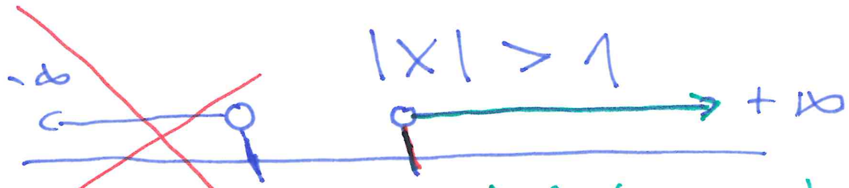
$\ln(y^2-1) = 2x$

$f^{-1}: \; \underline{\underline{y = \frac{1}{2}\ln(x^2-1)}}$

$\boxed{H_f = D_{f^{-1}}}: \; x^2-1 > 0$

$x^2 > 1 \quad / \sqrt{}$

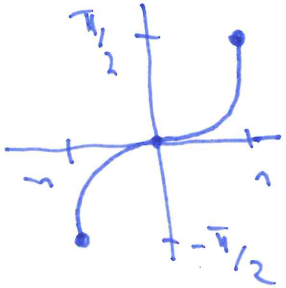
$|x| > 1$



$D_{f^{-1}} = (1; +\infty)$

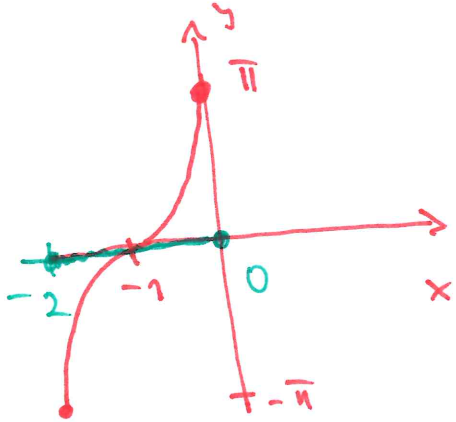
2d $f: y = 2\cdot\arcsin(x+1)$

arcsin(x)



o 1 ←

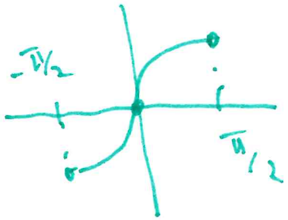
na y 2× (zdvojnásobit)



$D_f = \langle -2; 0\rangle = H_{f^{-1}}$

$H_f = \langle -\pi; \pi\rangle = D_{f^{-1}}$

sin x



$D_f: \underbrace{-1 \leq \overbrace{x+1}}\leq 1 \quad / -1$

$-2 \leq x \leq 0$

$y = 2\cdot\arcsin(x+1) \quad /:2$

$\boxed{\frac{y}{2}} = \arcsin(x+1) \quad / \sin$

$\sin\left(\frac{y}{2}\right) = x+1$

$f^{-1}: y = \sin\left(\frac{x}{2}\right) - 1$

$\frac{y}{2} \in \left\langle -\frac{\pi}{2}; \frac{\pi}{2}\right\rangle$

Limity posloupností

$\lim\limits_{n \to \infty} a_n =$

- neex.
- existuje
  - vlastní = reálné číslo (konečné) K
  - nevlastní = $\pm\infty$ D

$\lim\limits_{n \to \infty} c = c$, $c \in \mathbb{R}$

$\lim\limits_{n \to \infty} \frac{1}{n} = 0$

$\lim\limits_{n \to \infty} n = +\infty$

$\lim\limits_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e$

$\lim\limits_{n \to \infty} \left(1 - \frac{1}{n}\right)^n = e^{-1}$

3a

$$\lim_{n\to\infty}\frac{\boxed{-2}n^3-5n+7}{\boxed{5}n^2+n-8}=\left(\frac{-\infty-\infty+7}{\infty+\infty-8}=\frac{-\infty}{\infty}\ \ldots \text{není def.}\right)$$

Návod: Vytkneme nejvyšší mocninu ze <u>jmenovatele</u>.

$$=\lim_{n\to\infty}\frac{\cancel{n^2}\left[-2n-\frac{5}{n}+\frac{7}{n^2}\right]}{\cancel{n^2}\left[5+\frac{1}{n}-\frac{8}{n^2}\right]}=\lim_{n\to\infty}\frac{-2n}{5}=-\frac{2}{5}\cdot\infty=\underline{\underline{-\infty}}$$

$\frac{5}{n}\to 0$, $\frac{7}{n^2}\to 0$, $\frac{1}{n}\to 0$, $\frac{8}{n^2}\to 8\cdot\frac{1}{n}\cdot\frac{1}{n}\to 0$

3b

$$\lim_{n\to\infty}\frac{5n^2+8n+1}{7n^2+8n-1}\left(\frac{\infty}{\infty}\right)=\lim_{n\to\infty}\frac{\cancel{n^2}\left[5+\frac{8}{n}+\frac{1}{n^2}\right]}{\cancel{n^2}\left[7+\frac{8}{n}-\frac{1}{n^2}\right]}=\underline{\underline{\frac{5}{7}}}$$

3c $\lim_{n\to\infty} \frac{3n+3}{n^3-1} \left(\frac{\infty}{\infty}\right) = \lim_{n\to\infty} \frac{\cancel{n^3}\left[\frac{3}{n^2}+\frac{3}{n^3}\right]}{\cancel{n^3}\left[1-\frac{1}{n^3}\right]} = \frac{0}{1} = \underline{\underline{0}}$

Obecně

$$\lim_{n\to\infty} \frac{P(n)}{Q(n)} = \begin{cases} \pm\infty, & \text{st. } P > \text{st. } Q \\ \text{konečné číslo nenulové}, & \text{st. } P = \text{st. } Q \\ 0, & \text{st. } P < \text{st. } Q \end{cases}$$

CV5

3g/CV3

$$\lim_{n\to\infty}\left(\sqrt{9n^2-4} - 2n\right) = \left[\sqrt{\infty} - \infty = \infty - \infty\right] = \text{není def.}$$

Trik: Rozšíří výrazem: $\dfrac{\sqrt{9n^2-4}+2n}{\sqrt{9n^2-4}+2n}$

$$= \lim_{n\to\infty} \frac{\overset{A}{\boxed{\sqrt{9n^2-4}}} - \overset{B}{\boxed{2n}}}{1} \cdot \frac{\overset{A}{\sqrt{9n^2-4}} + \overset{B}{2n}}{\sqrt{9n^2-4}+2n} = \lim_{n\to\infty} \frac{\overset{A^2}{\left(\sqrt{9n^2-4}\right)^2} - \overset{B^2}{(2n)^2}}{\sqrt{9n^2-4}+2n}$$

$$= \lim_{n\to\infty} \frac{9n^2-4-4n^2}{\sqrt{9n^2-4}+2n} = \lim_{n\to\infty} \frac{5n^2-4}{\sqrt{9n^2-4}+2n} \left|\frac{\infty}{\infty+\infty}\right|$$

$$\sqrt{9n^2-4} = \sqrt{n^2\cdot\left(9-\frac{4}{n^2}\right)} = \sqrt{n^2}\cdot\sqrt{9-\frac{4}{n^2}}$$

$$|n| \qquad n\in\mathbb{N}$$

$$= n\cdot\sqrt{9-\frac{4}{n^2}}$$

$$= \lim_{n\to\infty} \frac{\cancel{n}\cdot\left(5n-\frac{4}{n}\right)}{\cancel{n}\cdot\left[\sqrt{9-\frac{4}{n^2}}+2\right]} = \lim_{n\to\infty} \frac{\cancel{5}n}{\cancel{5}} = \underline{\underline{\infty}}$$

2. způsob:

$$\lim_{n\to\infty}\left(\sqrt{9n^2-4}-2n\right)=\boxed{\lim_{n\to\infty} n}\cdot\underbrace{\left[\sqrt{9-\frac{4}{n^2}}-2\right]}$$

$$=\underbrace{\boxed{\lim_{n\to\infty} n}}_{\infty}\cdot\underbrace{\boxed{\lim_{n\to\infty}\sqrt{9-\frac{4}{n^2}}-2}}_{\sqrt{9}-2=1}=\infty\cdot 1=\infty$$

Př: $\lim_{n\to\infty}\left(\sqrt{9n^2-4}+2n\right)=\sqrt{\infty-4}+\infty=\infty+\infty=\infty$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$$

$$\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n = e^{-1}$$

substituce

$$\frac{1}{k} = \frac{1}{2n} \qquad n \to \infty$$

$$k \to 2\cdot\infty = \infty$$

$$\boxed{2n = k}$$

$$n = \frac{k}{2}$$

e) $$\lim_{n\to\infty}\left(1+\boxed{\frac{1}{2n}}\right)^{3n+6} = \lim_{k\to\infty}\left(1+\frac{1}{k}\right)^{3\cdot\frac{k}{2}+6}$$

$$\frac{1}{2n} = \frac{1}{k}$$

$$= \lim_{k\to\infty}\left(1+\frac{1}{k}\right)^{\frac{3}{2}k}\cdot\boxed{\left(1+\frac{1}{k}\right)^6} = \lim_{k\to\infty}\Big[\underbrace{\left(1+\frac{1}{k}\right)^k}_{\to e}\Big]^{3/2} = e^{3/2}$$

$$\left(1+\frac{1}{\infty}\right)^6 \to 1$$

Limita Fce

$$\lim_{x \to a} f(x) = b$$

neexistuje

existuje

vlastní limita ve vl. bodě
$a, b \in \mathbb{R}$

vlastní limita v nevl. b.
$b \in \mathbb{R}$, $a = \pm\infty$

nevlastní limita ve vl. b.
$b = \pm\infty$, $a \in \mathbb{R}$

nevlastní limita v nevl. b.
$a, b = \pm\infty$

4c $\boxed{CV3}$

$$\lim_{x \to \frac{\pi}{4}} \frac{\sin^4 x}{x} = \frac{\sin^4\left(\frac{\pi}{4}\right)}{\frac{\pi}{4}} = \frac{\left(\frac{\sqrt{2}}{2}\right)^4}{\frac{\pi}{4}} =$$

$$= \frac{\frac{4}{16}}{\frac{\pi}{4}} = \frac{16}{16\pi} = \frac{1}{\pi}$$

e

$$\lim_{x \to 2} \frac{x-2}{x^2-3x+2} = \left(\frac{0}{0}\right) \text{ není def.} = \lim_{x \to 2} \frac{\cancel{x-2}^{\,1}}{\cancel{(x-2)}(x-1)}$$

$$= \lim_{x \to 2} \frac{1}{x-1} = \frac{1}{2-1} = \frac{1}{1} = 1$$

(F) $\lim\limits_{x \to -2} \dfrac{x^3+3x^2+2x}{x^2-x-6} \left(\dfrac{0}{0}\right) = \lim\limits_{x \to \boxed{-2}} \dfrac{x(x^2+3x+2)}{(x-3)(x+2)}$

$(x-(-2))$

$= \lim\limits_{x \to -2} \dfrac{x\,\boxed{\cancel{(x+2)}}\,(x+1)}{(x-3)\,\boxed{\cancel{(x+2)}}} = \dfrac{-2(-2+1)}{(-2-3)} = \dfrac{-2\cdot(-1)}{-5} = \dfrac{-2}{5}$

h

$$\lim_{x\to -1} \frac{x^3+1}{\underbrace{\sqrt{x^2-3x}}_{A}+\underbrace{2x}_{B}} \left(\frac{0}{0}\right) \cdot \frac{\sqrt{x^2-3x}-2x}{\underbrace{\sqrt{x^2-3x}}_{A}-\underbrace{2x}_{B}}$$

$$=\lim_{x\to -1}\frac{(x^3+1)\left[\sqrt{x^2-3x}-2x\right]}{\underbrace{x^2-3x}_{A^2}-\underbrace{4x^2}_{B^2}}$$

$$=\lim_{x\to -1}\frac{\cancel{(x+1)}(x^2-x+1)\left[\sqrt{x^2-3x}-2x\right]}{\boxed{-3x^2-3x}} = -3x\cancel{(x+1)}$$

$$=\frac{\left((-1)^2-(-1)+1\right)\left[\sqrt{(-1)^2-3(-1)}-2(-1)\right]}{-3(-1)} = \frac{3(2+2)}{3} = \underline{\underline{4}}$$