Derivace

$f: y = f(x)$, D_f $f': y = f'(x)$, $D_{f'}$

$$\boxed{D_{f'} \subset D_f}$$

$y = \ln x$, $D_f = (0; +\infty)$

$y' = (\ln x)' = \frac{1}{x}$, $D_f = (0; +\infty)$!

Základní vzorce

$(f \pm g)' = f' \pm g'$ ...aditivita

$(c \cdot f)' = c \cdot f'$ ...homogenita (c...konstanta)

$(f \cdot g)' = f' \cdot g + f \cdot g'$

$$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$$

1a/cv5

$$c' = 0$$

$$x' = 1$$

$$(x^n)' = n \cdot x^{n-1}$$

$$f(x) = \frac{5}{3}x - 2 + \frac{2}{3x^2} \quad / \; x \neq 0$$

$$f'(x) = \left(\frac{5}{3}x - 2 + \frac{2}{3x^2}\right)' =$$

$$= \left(\frac{5}{3} \cdot x\right)' - (2)' + \left(\frac{2}{3x^2}\right)' = \frac{5}{3}(x)' - 0 + \left(\frac{2}{3} \cdot x^{-2}\right)'$$

$$= \frac{5}{3} \cdot 1 + \frac{2}{3}(x^{-2})' = \frac{5}{3} + \frac{2}{3}(-2 \cdot x^{-3}) =$$

$$= \frac{5}{3} - \frac{4}{3}x^{-3} = \frac{5}{3} - \frac{4}{3x^3} \quad / \; x \neq 0$$

b) $f(x) = 6\sqrt[3]{x} - 4\sqrt[4]{x} + \dfrac{5}{3\sqrt[3]{x}}$, $x > 0$

$$f'(x) = 6\cdot\left(x^{1/3}\right)' - 4\cdot\left(x^{1/4}\right)' + \frac{5}{3}\cdot\left(x^{-1/3}\right)'$$

$$= 6\cdot\frac{1}{3}x^{-\frac{2}{3}} - 4\cdot\frac{1}{4}x^{-3/4} + \frac{5}{3}\cdot\left(-\frac{1}{3}\right)x^{-4/3}$$

$$= \frac{2}{\sqrt[3]{x^2}} - \frac{1}{\sqrt[4]{x^3}} - \frac{5}{9\sqrt[3]{x^4}}$$, $x > 0$

e) $f(x) = 3\sqrt[3]{x^2} - \frac{1}{3}\operatorname{cotg} x$

$\operatorname{cotg} x = \frac{\cos x}{\sin x}$

$\sin x \neq 0$

$x \neq 0$

$$f'(x) = 3 \cdot \left(x^{2/3}\right)' - \frac{1}{3} \cdot \left(\operatorname{cotg} x\right)'$$

$$= 3 \cdot \frac{2}{3} x^{-1/3} - \frac{1}{3} \cdot \left(\frac{-1}{\sin^2 x}\right)$$

$$= \frac{2}{\sqrt[3]{x}} + \frac{1}{3 \cdot \sin^2 x}$$

b

$$f(x) = x^2 \cdot \operatorname{tg} x \quad , \quad \cos x \neq 0$$

$$f'(x) = (x^2)' \cdot \operatorname{tg} x + x^2 \cdot (\operatorname{tg} x)'$$

$$= 2x^1 \cdot \operatorname{tg} x + x^2 \cdot \frac{1}{\cos^2 x}$$

$$= 2x \cdot \operatorname{tg} x + \frac{x^2}{\cos^2 x} \quad , \quad \cos x \neq 0$$

Součin tří funkcí – derivace

$$(f\cdot g\cdot h)' = ((f\cdot g)\cdot h)' = (f\cdot g)'\cdot h + f\cdot g\cdot h'$$

$$= (f'\cdot g + f\cdot g')\cdot h + f\cdot g\cdot h'$$

$$= f'\cdot g\cdot h + f\cdot g'\cdot h + f\cdot g\cdot h'$$

$(e^x)' = e^x$

$(\cos x)' = -\sin x$

$(\sin x)' = \cos x$

0

$$f(x) = x\cdot e^x\cdot \cos x \;,\quad x\in\mathbb{R}$$

$$f'(x) = 1\cdot e^x\cdot \cos x + x\cdot e^x\cdot \cos x + x\cdot e^x\cdot(-\sin x)$$

2a $$f(x) = \frac{x}{x+1}$$

$$f'(x) = \frac{(x)' \cdot (x+1) - x \cdot (x+1)'}{(x+1)^2} = \frac{1 \cdot (x+1) - x(1+0)}{(x+1)^2}$$

$$= \frac{x+1-x}{(x+1)^2} = \frac{1}{(x+1)^2} \quad , x \neq -1$$

$$f(x) = \frac{3}{4x}$$

$$! \quad \left(\frac{3}{4x}\right)' = \frac{0 \cdot 4x - 3 \cdot 4}{(4x)^2} \quad !$$

$$= \frac{-12}{16x^2} = \frac{-3}{4x^2}$$

$$f'(x) = \left(\frac{3}{4} \cdot x^{-1}\right)' = \frac{3}{4}(-1) \cdot x^{-2} = -\frac{3}{4x^2}$$

$$F(x) = \frac{3x}{4} \qquad \left(\frac{3x}{4}\right)' = \frac{3 \cdot 4 - 3x \cdot 0}{16} = \frac{12}{16} = \frac{3}{4}$$

$$F'(x) = \left(\frac{3}{4} \cdot x\right)' = \frac{3}{4} \cdot 1 = \underline{\underline{\frac{3}{4}}}$$

$$\left(\frac{3}{4} \cdot x\right)' = \underbrace{\left(\frac{3}{4}\right)'}_{=0} \cdot x + \frac{3}{4} \cdot (x)' = \frac{3}{4} \cdot 1 = \frac{3}{4}$$

CV 7

Derivace Funkce

$$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$$

2d/CV5

$$f(x) = \frac{1 + x - x^2}{1 - x + x^2}$$

$$f'(x) = \frac{(1+x-x^2)' \cdot (1-x+x^2) - (1+x-x^2) \cdot (1-x+x^2)'}{(1-x+x^2)^2}$$

$$= \frac{(1-2x)(1-x+x^2) - (1+x-x^2) \cdot (-1+2x)}{(1-x+x^2)^2}$$

$$= \frac{1 - x + \cancel{x^2} - 2x + \cancel{2x^2} - \cancel{2x^3} - [-1 - x + \cancel{x^2} + 2x + \cancel{2x^2} - \cancel{2x^3}]}{(1 - x + x^2)^2}$$

$$= \frac{2 - 4x}{(1 - x + x^2)^2}, \quad 1 - x + x^2 \neq 0$$

$$D < 0$$

... platí vždy

$$D_f = D_{f'} = \mathbb{R}$$

2g $\quad f(x) = \dfrac{x \cdot e^x}{1+x^2} \quad , x \in \mathbb{R}$

$$f'(x) = \frac{(x \cdot e^x)' \cdot (1+x^2) - x \cdot e^x \cdot (1+x^2)'}{(1+x^2)^2}$$

$$= \frac{(1 \cdot e^x + x \cdot e^x)(1+x^2) - x \cdot e^x \cdot 2x}{(1+x^2)^2}$$

$$= \frac{e^x + x \cdot e^x + \underline{x^2 \cdot e^x} + x^3 \cdot e^x - \underline{2x^2 \cdot e^x}}{(1+x^2)^2}$$

$$= \frac{e^x(1 + x - x^2 + x^3)}{(1+x^2)^2} \quad , x \in \mathbb{R}$$

Derivace složené funkce

$$[f(g)]' = f'(g) \cdot g'$$

$$[f(g(x))]' = f'(g(x)) \cdot g'(x)$$

$$[h(f(g))]' = h'(f(g)) \cdot f'(g) \cdot g'$$

$$h(x) = \underbrace{\sin}_{f}(\underbrace{5x}_{g})$$

$$h'(x) = [\sin(5x)]' = \cos(5x) \cdot 5$$

$$h(x) = \sin^{\overset{=f}{2}} x = g$$

$$h'(x) = [\sin^2 x]' = 2 \cdot \sin x \cdot \cos x (\cdot 1)$$

2k $f(x) = 3 \cdot \ln(5x)$

$$f'(x) = [3 \cdot \ln(5x)]' = 3 \cdot \frac{1}{5x} \cdot 5 = \frac{3}{x}$$

2l $f(x) = \ln(x^2 - 1)$

$$f'(x) = \frac{1}{x^2 - 1} \cdot 2x = \frac{2x}{x^2 - 1}$$

2m $F(x) = \arcsin\left(\frac{x-2}{2}\right)$

$$F'(x) = \frac{1}{\sqrt{1-\left(\frac{x-2}{2}\right)^2}} \cdot \left(\frac{x-2}{2}\right)'$$

$\frac{x}{2} - \frac{2}{2}$

$$= \frac{1}{\sqrt{1-\frac{(x-2)^2}{4}}} \cdot \left(\frac{1}{2}\cdot x - 1\right)'$$

$$= \frac{1}{\sqrt{\frac{\cancel{4}-x^2+4x-\cancel{4}}{4}}} \cdot \frac{1}{2} = \frac{1}{\frac{\sqrt{4x-x^2}}{2}} \cdot \frac{1}{2} = \frac{1}{\sqrt{4x-x^2}}$$

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$$f(x) = \frac{1}{(x^3-1)^2} = (x^3-1)^{-2}$$

$$f'(x) = \left[(x^3-1)^{-2}\right]' = -2 \cdot (x^3-1)^{-3} \cdot 3x^2$$

$$= \frac{-6x^2}{(x^3-1)^3}$$

$$\left[x^n\right]' = n \cdot x^{n-1}$$

Derivace f^g

$$[f(x)]^{g(x)} = e^{g(x) \cdot \ln(f(x))}$$

$$a^x = e^{x \cdot \ln a}$$

2y $\quad f(x) = x^{\sin x} = e^{\sin x \cdot \ln x}$

$$\left[e^{\sin x \cdot \ln x}\right]' = \underbrace{e^{\sin x \cdot \ln x}} \cdot \left(\sin x \cdot \ln x\right)'$$

$$= x^{\sin x} \cdot \left(\cos x \cdot \ln x + \sin x \cdot \frac{1}{x}\right)$$

$$= x^{\sin x} \cdot \left(\cos x \cdot \ln x + \frac{\sin x}{x}\right)$$