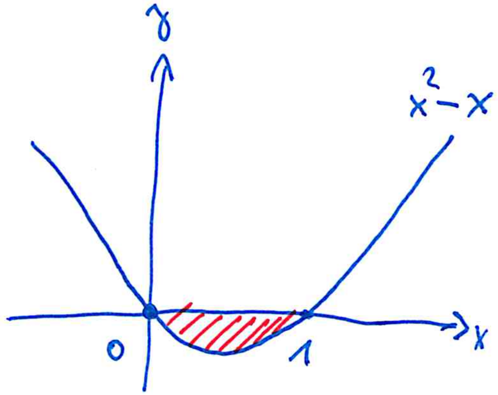
Určitý integrál – výpočet

$$\int_0^1 (x^2 - x)\,dx = \left[\frac{x^3}{3} - \frac{x^2}{2}\right]_0^1 = \frac{1}{3} - \frac{1}{2} - 0 = -\frac{1}{6}$$



Metoda per partes pro určitý integrál

Nechť funkce $u(x)$ a $v(x)$ mají v $\langle a, b\rangle$ spojité derivace. Potom

$$\int_a^b u \cdot v' \,dx = \left[u \cdot v\right]_a^b - \int_a^b u' \cdot v \,dx$$

$$\int_0^{\pi} x\sin x\,dx = \Big[-x\cos x\Big]_0^{\pi} + \int_0^{\pi}\cos x\,dx = \pi + \underbrace{\Big[\sin x\Big]_0^{\pi}}_{=0} = \underline{\underline{\pi}}$$

$$u = x \qquad u' = 1$$
$$v' = \sin x \qquad v = -\cos x$$

$$\int_1^{2} x\ln x\,dx = \left[\frac{x^2}{2}\ln x\right]_1^{2} - \int_1^{2}\frac{x}{2}\,dx = 2\ln 2 - \left[\frac{x^2}{4}\right]_1^{2} =$$

$$u = \ln x \qquad u' = \frac{1}{x}$$
$$v' = x \qquad v = \frac{x^2}{2}$$

$$= 2\ln 2 - \left(1 - \frac{1}{4}\right) = \underline{\underline{2\ln 2 - \frac{3}{4}}}$$

Substituce v určitém integrálu

Jsou-li funkce $t = g(x)$ a $g'(x)$ spojité v $\langle a, b \rangle$ a také $f(t)$ je spojitá pro všechna $t = g(x)$ pro $x \in \langle a, b \rangle$, platí

$$\int_a^b f(g(x)) \cdot g'(x)\, dx = \int_{g(a)}^{g(b)} f(t)\, dt$$

Provádíme substituci $t = g(x)$

$$dt = g'(x)\, dx$$

$$\int_0^1 \sqrt{x^2+1}\cdot \boxed{x\,dx} = \frac{1}{2}\int_1^2 \sqrt{t}\,dt = \frac{1}{2}\left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^2 = \frac{1}{3}\left(\sqrt{8}-1\right)$$

$$t = x^2+1$$

$$dt = 2x\,dx \quad / :2$$

$$\frac{1}{2}dt = \boxed{x\,dx}$$

$$\int_0^{\frac{\pi}{2}} \sin^3 x \cdot \cos x\,dx = \int_0^1 r^3\,dr = \left[\frac{r^4}{4}\right]_0^1 = \frac{1}{4}$$

$$r = \sin x$$

$$dr = \cos x\,dx$$

$$\int_0^1 \frac{x+1}{x^2+2x+2}\,dx = \frac{1}{2}\int_2^5 \frac{1}{t}\,dt = \frac{1}{2}\Big[\ln t\Big]_2^5 = \frac{1}{2}(\ln 5 - \ln 2)$$

$$t = x^2+2x+2$$

$$dt = (2x+2)\,dx \quad /:2$$

$$\frac{1}{2}\,dt = (x+1)\,dx$$