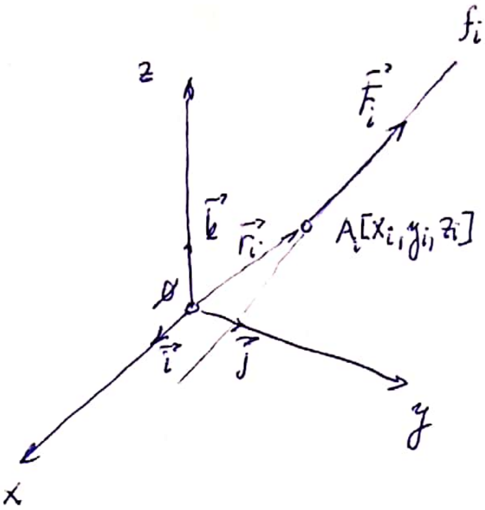
# Výsledné nahrazení (statický šroub)

| $i$ | $F_x$ [N] | $F_y$ [N] | $F_z$ [N] | $x_i$ [m] | $y_i$ [m] | $z_i$ [m] |
|---|---|---|---|---|---|---|
| 1 | 100 | −120 | −85 | 1,2 | −0,8 | −0,2 |
| 2 | 0 | 110 | 60 | 0,9 | 0,9 | −1,0 |
| 3 | −50 | 70 | −30 | −0,6 | 1,1 | 0,4 |
| 4 | −30 | −20 | 120 | −1 | −0,5 | −0,2 |




## 1) Nahrazení soustavy sil pro zvolený bod

Výslednice

$$\vec{F} = \vec{F_x} + \vec{F_y} + \vec{F_z} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k} = (20\vec{i} + 40\vec{j} + 65\vec{k})\,\mathrm{N}$$

$$F_x = \sum_{i=1}^{4} F_{ix} = 100 + 0 - 50 - 30 = 20\,\mathrm{N}$$

$$F_y = \sum_{i=1}^{4} F_{iy} = -120 + 110 + 70 - 20 = 40\,\mathrm{N}$$

$$F_z = \sum_{i=1}^{4} F_{iz} = 65\,\mathrm{N}$$

(nahrazujeme pro počátek souř. soustavy)

Výsledný moment soustavy k poč. souř. soustavě

$$\vec{M_{0i}} = \vec{r_i} \times \vec{F_i} = \ldots$$

$$\vec{M_0} = \sum_{i=1}^{4} \vec{M_{0i}}$$

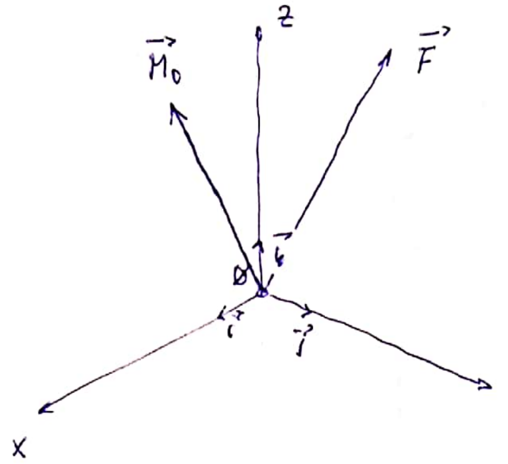
$i = 1$: 
$$\vec{M_{01}} = \vec{r_1} \times \vec{F_1} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1{,}2 & -0{,}8 & -0{,}2 \\ 100 & -120 & -85 \end{vmatrix} = \vec{i}\left(-0{,}8 \cdot 85 - (-0{,}2 \cdot (-120))\right) + -\vec{j}\left(1{,}2 \cdot (-85) - (-0{,}2 \cdot 100)\right) + \vec{k}\left(1{,}2\,(-120) - ((-0{,}8)\,100)\right) =$$

$$= (44\vec{i} + 82\vec{j} - 64\vec{k})\,\mathrm{Nm}$$

$i = 2$ 
$$\vec{M_{02}} = \vec{r_2} \times \vec{F_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0{,}9 & 0{,}9 & -1 \\ 0 & 110 & 60 \end{vmatrix} = \ldots$$

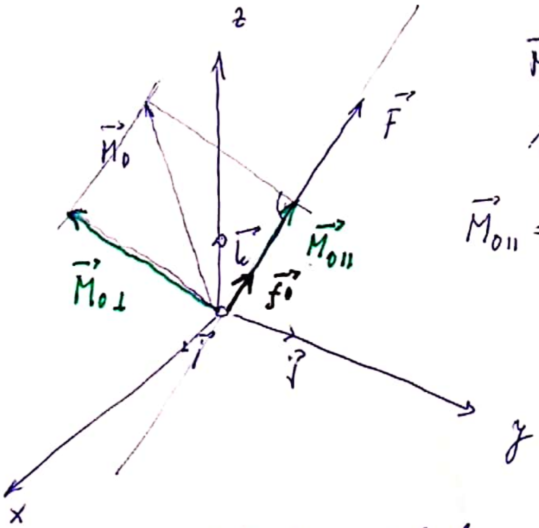
$i = 3$; $i = 4$

$$\vec{M}_0 = \sum_{i=1}^{4} M_{0i} = \vec{M}_{01} + \vec{M}_{02} + \vec{M}_{03} + \vec{M}_{04} = (83\vec{i} + 116\vec{j} + 53\vec{k})\ \mathrm{Nm}$$



Nahrazení silové soustavy pro zvolený bod redukční pár $\vec{M}_0, \vec{F}$

Výsledné nahrazení soustavy sil



$$\vec{M}_0 = \vec{M}_{0\perp} + \vec{M}_{0\parallel}$$

Výpočet $\vec{M}_{0\parallel}$:

$$\vec{M}_{0\parallel} = \vec{M}_0 \cdot \vec{f^0}$$

jednotkový vektor nositelky výslednice:

$$\vec{f^0} = \frac{F_x}{F}\vec{i} + \frac{F_y}{F}\vec{j} + \frac{F_z}{F}\vec{k}$$

velikost výslednice:

$$|\vec{F}| = F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{20^2 + 40^2 + 65^2} \doteq 78{,}9\ \mathrm{N}$$

$$\underline{\vec{f^0}} = \frac{20}{78{,}9}\vec{i} + \frac{40}{78{,}9}\vec{j} + \frac{65}{78{,}9}\vec{k} = 0{,}253\vec{i} + 0{,}507\vec{j} + 0{,}824\vec{k}$$

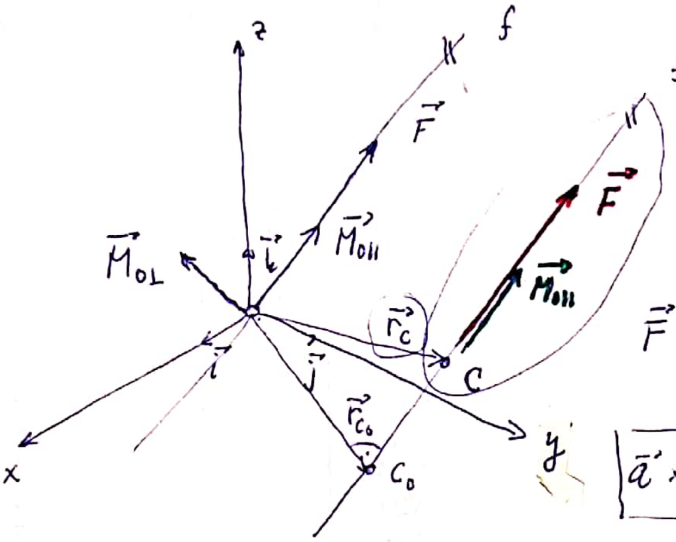
$$|\vec{M}_{0\parallel}| = M_{0\parallel} = (83;\ 116;\ 53)(0{,}253;\ 0{,}507;\ 0{,}824) =$$

$$= 83 \cdot 0{,}253 + 116 \cdot 0{,}507 + 53 \cdot 0{,}824 = 123{,}5\ \mathrm{Nm}$$

$$\vec{M}_{0\parallel} = M_{0\parallel} \cdot \vec{f^0} = 123{,}5 \cdot (0{,}253\vec{i} + 0{,}507\vec{j} + 0{,}824\vec{k}) =$$

$$= (31{,}25\vec{i} + 62{,}61\vec{j} + 101{,}76\vec{k})\ \mathrm{Nm}$$

$$\vec{M}_0 = \vec{M}_{0\parallel} + \vec{M}_{0\perp} \Rightarrow \vec{M}_{0\perp} = \vec{M}_0 - \vec{M}_{0\parallel} = (83;\ 116;\ 53) - (31{,}25;\ 62{,}61;\ 101{,}76) = \dots\ [\mathrm{Nm}]$$




$$\vec{M}_{0\perp} = \vec{r}_c \times \vec{F}$$

$$\vec{M}_0 - \vec{M}_{0\parallel} = \vec{r}_c \times \vec{F} \quad / \vec{F}\times$$

$$\vec{F} \times (\vec{M}_0 - \vec{M}_{0\parallel}) = \vec{F} \times (\vec{r}_c \times \vec{F})$$

$$\vec{F} \times \vec{M}_0 - \underbrace{\vec{F} \times \vec{M}_{0\parallel}}_{\vec{0}} = \vec{r}_c(\vec{F}\cdot\vec{F}) - \vec{F}(\vec{F}\cdot\vec{r}_c)$$

$$\boxed{\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}\cdot(\vec{a}\cdot\vec{c}) - \vec{c}(\vec{a}\cdot\vec{b})}$$

$$\vec{F} \times \vec{M}_0 = F^2 \vec{r}_c - \vec{F}\underbrace{(\vec{F}\cdot\vec{r}_c)}_{\emptyset} \qquad \vec{r}_{c0} \perp \vec{F}$$

$$\vec{F} \times \vec{M}_0 = F^2 \vec{r}_{c0} \Rightarrow \vec{r}_{c0} = \frac{\vec{F} \times \vec{M}_0}{F^2}$$

$$\vec{F} \times \vec{M}_0 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 20 & 40 & 65 \\ 83 & 116 & 53 \end{vmatrix} = \vec{i}(40\cdot 53 - 65\cdot 116) - \vec{j}(20\cdot 53 - 65\cdot 83) + \vec{k}(20\cdot 116 - 83\cdot 40) = (-5420,\ 4335,\ -1000)\ [\mathrm{N\cdot Nm}]$$

$$\vec{r}_{c0} = \frac{\vec{F} \times \vec{M}_0}{F^2} = (-0{,}87;\ 0{,}7;\ -0{,}16)\ \mathrm{m}$$

Silový šroub :