

1.

$(1, 0, 1)$ ,  $(3, 1, 0)$ ,  $(5, 2, 1)$   $\mathbb{R}^3$

$$\begin{pmatrix} 1 & 3 & 5 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{pmatrix} \xrightarrow{R_3 - R_1} \begin{pmatrix} 1 & 3 & 5 \\ 0 & 1 & 2 \\ 0 & -3 & -4 \end{pmatrix} \xrightarrow{R_3 + 3R_2} \begin{pmatrix} 1 & 3 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{pmatrix}$$

vektory jsou lin. nezav

tedy tvoří bázi  $\mathbb{R}^3$

$$V = \langle (1, 1, 1, 2, 0), (2, 1, -1, 1, 1), (1, 2, 1, -1, -1), (1, 3, 0, -2) \rangle$$

$$\begin{pmatrix} 1 & 2 & 1 & 1 & 0 \\ 1 & 1 & 2 & 3 & 0 \\ 1 & -1 & 1 & 0 & -1 \\ 1 & -1 & -1 & -2 & -2 \end{pmatrix} \xrightarrow{\substack{(1) \leftrightarrow (2) \\ (1) \leftrightarrow (3)}}} \begin{pmatrix} 1 & 2 & 1 & 1 & 0 \\ 0 & -1 & 1 & 2 & -1 \\ 0 & -5 & -1 & -2 & -1 \\ 0 & 1 & -1 & -2 & -2 \end{pmatrix} \xrightarrow{(5)} \begin{pmatrix} 1 & 2 & 1 & 1 & 0 \\ 0 & -1 & 1 & 2 & -1 \\ 0 & 0 & -6 & -12 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$h(V) = 3$$

3.

$$X \quad \{u, v, w\} = B$$

$$X = (1, 1, 1) \quad u = \begin{pmatrix} 1 & 0 & -1 \\ 4 & 2 & 2 \end{pmatrix} \quad v = (2, 1, -2) \quad w = (1, 2, 0)$$

$$X = k_1 u + k_2 v + k_3 w$$

$$k_1 \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + k_2 \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} + k_3 \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ -1 & -2 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

$$k_1 + 2k_2 + k_3 = 1$$

$$k_2 + 2k_3 = 1$$

$$k_1 - 6 + 2 = 1$$

$$k_3 = 2$$

$$k_2 = -3$$

$$X = (5, -3, 2)$$

$$k_1 = 5$$

4.

$$A = \begin{pmatrix} 4 & 2 & -1 & 2 \\ 3 & -7 & 1 & -8 \\ 2 & 4 & -3 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 2 & 3 \\ -3 & 0 \\ 1 & 5 \\ 3 & 1 \end{pmatrix}$$

$A+B$  ... není definováno

$$3B = \begin{pmatrix} 6 & 9 \\ -9 & 0 \\ 3 & 15 \\ 9 & 3 \end{pmatrix}$$

$$A \cdot B = \begin{pmatrix} 4 & 2 & -1 & 2 \\ 3 & -7 & 1 & -8 \\ 2 & 4 & -3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 3 \\ -3 & 0 \\ 1 & 5 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 9 \\ 4 & 6 \\ -8 & -8 \end{pmatrix}$$

$$B \cdot A = \begin{pmatrix} 2 & 3 \\ -3 & 0 \\ 1 & 5 \\ 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 4 & 2 & -1 & 2 \\ 3 & -7 & 1 & -8 \\ 2 & 4 & -3 & 1 \end{pmatrix}$$

... není definováno

$$A = \begin{pmatrix} 1 & 2 & 0 & -1 \\ 2 & 2 & 1 & -2 \\ -2 & -1 & 1 & -3 \\ 1 & 0 & 1 & -1 \end{pmatrix} \xrightarrow{\substack{(-2)R_1 \\ (-1)R_1}} \begin{pmatrix} 1 & 2 & 0 & -1 \\ 0 & -2 & 1 & 0 \\ 0 & 3 & 1 & -3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 0 & -1 \\ 0 & -2 & 1 & 0 \\ 0 & 3 & 1 & -3 \end{pmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{pmatrix} 1 & 2 & 0 & -1 \\ 0 & 3 & 1 & -3 \\ 0 & -2 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 0 & -1 \\ 0 & 3 & 1 & -3 \\ 0 & -2 & 1 & 0 \end{pmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{pmatrix} 1 & 2 & 0 & -1 \\ 0 & -2 & 1 & 0 \\ 0 & 3 & 1 & -3 \end{pmatrix} \xrightarrow{R_2 \times (-1)} \begin{pmatrix} 1 & 2 & 0 & -1 \\ 0 & 2 & -1 & 0 \\ 0 & 3 & 1 & -3 \end{pmatrix}$$

$$h(A) = 3$$

6.

$$A = \begin{pmatrix} 6 & -4 & -17 \\ -1 & 1 & 3 \\ 2 & -1 & -6 \end{pmatrix}$$

$$\left( \begin{array}{ccc|ccc} 6 & -4 & -17 & 1 & 0 & 0 \\ -1 & 1 & 3 & 0 & 1 & 0 \\ 2 & -1 & -6 & 0 & 0 & 1 \end{array} \right) \sim \left( \begin{array}{ccc|ccc} -1 & 1 & 3 & 0 & 1 & 0 \\ 6 & -4 & -17 & 1 & 0 & 0 \\ 2 & -1 & -6 & 0 & 0 & 1 \end{array} \right) \sim \begin{matrix} (-1) \\ \end{matrix}$$

$$\sim \left( \begin{array}{ccc|ccc} 1 & -1 & -3 & 0 & -1 & 0 \\ 6 & -4 & -17 & 1 & 0 & 0 \\ 2 & -1 & -6 & 0 & 0 & 1 \end{array} \right) \begin{matrix} (-6) \\ (-2) \\ \end{matrix} \sim \left( \begin{array}{ccc|ccc} 1 & -1 & -3 & 0 & -1 & 0 \\ 0 & 2 & 1 & 1 & 6 & 0 \\ 0 & 1 & 0 & 1 & 2 & 1 \end{array} \right) \sim \begin{matrix} (-6) \\ (-2) \\ \end{matrix}$$

$$\left( \begin{array}{ccc|ccc} 1 & -1 & -3 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 2 & 1 \\ 0 & 2 & 1 & 1 & 6 & 0 \end{array} \right) \begin{matrix} (-2) \\ (-2) \\ \end{matrix} \sim \left( \begin{array}{ccc|ccc} 1 & -1 & -3 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 1 & 2 & -2 \end{array} \right) \sim \begin{matrix} (-2) \\ (-2) \\ (-2) \\ \end{matrix}$$

7.

$$\begin{pmatrix} 1 & 0 & 0 & 3 & 5 & -6 \\ 0 & 1 & 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 1 & 2 & -2 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 0 & 0 & 3 & 7 & -5 \\ 0 & 1 & 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 1 & 2 & -2 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 3 & 7 & -5 \\ 0 & 2 & 1 \\ 1 & 2 & -2 \end{pmatrix}$$

Zkouška:

$$A \cdot A^{-1} = \begin{pmatrix} 6 & -4 & -17 \\ -1 & 1 & 3 \\ 2 & -1 & -6 \end{pmatrix} \begin{pmatrix} 3 & 7 & -5 \\ 0 & 2 & 1 \\ 1 & 2 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^{-1} \cdot A =$$

8.

$$B = \begin{pmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & 3 \\ -1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ -1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 2 & 3 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & -1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -2 & -1 & 1 & -1 & 1 & 0 \end{pmatrix}$$



9.

$$\begin{pmatrix} 1 & 0 & -1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & 2 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 & -2 & 0 \end{pmatrix} \begin{matrix} (I) \\ \\ \\ (II) \end{matrix}$$

$$\begin{pmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 2 & -1 & 1 & 3 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 3 & -2 & 3 & 4 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & -1 & -2 & 0 & -1 & 1 & -1 & -2 \end{pmatrix} \begin{matrix} \\ \\ \\ (III) \end{matrix}$$

$$B^{-1} \cdot B = \begin{pmatrix} 2 & -1 & 1 & 3 \\ 3 & -2 & 3 & 4 \\ 0 & 0 & 0 & 1 \\ -1 & 1 & -1 & -2 \end{pmatrix} \quad B^{-1} = \begin{pmatrix} 2 & -1 & 1 & 3 \\ 3 & -2 & 3 & 4 \\ 0 & 0 & 0 & 1 \\ -1 & 1 & -1 & -2 \end{pmatrix}$$

$$B \cdot B^{-1} = \begin{pmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & 3 \\ -1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & -1 & 1 & 3 \\ 3 & -2 & 3 & 4 \\ 0 & 0 & 0 & 1 \\ -1 & 1 & -1 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$